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FERMION CONDENSATION UNDER ROTATION ON ANTI-DE SITTER SPACE*

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Due to the local curvature, the fermion condensate (FC) for a free Dirac field on anti-de Sitter (adS) space becomes finite, even in the massless limit. Employing the point splitting method using an exact expression for the Feynman two-point function, an expression for the local FC is derived. Integrating this expression, we report the total FC in the adS volume and on its boundary.

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1. Introduction

Over the past couple of decades, the analysis of quantum field theory (QFT) on the anti-de Sitter (adS) background space-time has received much attention due to the conjectured adS/CFT correspondence [1]. Through this conjecture, important insight into the properties of the quark–gluon plasma formed in relativistic heavy-ion collisions was drawn [2].

Recent experiments performed by the STAR Collaboration revealed the polarisation of the QGP in non-central collisions [3]. One mechanism that could lead to this polarisation is the chiral vortical effect, due to the spin–orbit coupling predicted through the Dirac equation [4].

In this contribution, we present a study of thermal states of fermions undergoing rigid rotation on the anti-de Sitter space. The focus of this study is the fermion condensate (FC) induced by the coupling to curvature. The discussion is restricted to massless particles in the absence of interaction.

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2. Finite temperature expectation values

The line element of adS can be written as

$$ds^2 = \frac{1}{\cos^2 \omega r}\left[-dt^2 + dr^2 + \frac{\sin^2 \omega r}{\omega^2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \qquad (1)$$

where $t \in (-\infty, \infty)$[1], $0 \le \omega r < \frac{\pi}{2}$ and the inverse radius of curvature ω is related to the Ricci scalar through $R = -12\omega^2$. We further employ the following Cartesian gauge tetrad [5]:

$$e_{\hat{t}} = \cos\omega r\, \partial_t\,, \qquad e_{\hat{\imath}} = \cos\omega r\left[\frac{\omega r}{\sin\omega r}\left(\delta_{ij} - \frac{x^i x^j}{r^2}\right) + \frac{x^i x^j}{r^2}\right]\partial_j\,, \qquad (2)$$

by which the local gamma matrices $\gamma^\mu = e^\mu_{\hat{\alpha}}\gamma^{\hat{\alpha}}$ are written in terms of the Minkowski ones, which satisfy $\{\gamma^{\hat{\alpha}}, \gamma^{\hat{\sigma}}\} = -2\eta^{\hat{\alpha}\hat{\sigma}}$. At finite temperature β_0^{-1} and in rigid rotation with angular velocity $\boldsymbol{\Omega} = \Omega \boldsymbol{k}$, we have [6]

$$\left\langle \hat{\overline{\Psi}}\hat{\Psi}\right\rangle_{\beta_0,\Omega} = \mathcal{Z}^{-1}\mathrm{tr}\left(\hat{\rho}\hat{\overline{\Psi}}\hat{\Psi}\right)\,, \qquad \hat{\rho} = e^{-\beta_0\left(\hat{H} - \Omega \hat{M}^{\hat{z}}\right)}\,, \qquad (3)$$

where $\hat{H} = i\partial_t$, $\hat{M}^{\hat{z}} = -i\partial_\varphi + S^{\hat{z}}$, $S^{\hat{z}} = \frac{i}{2}\gamma^{\hat{1}}\gamma^{\hat{2}}$ and $\mathcal{Z} = \mathrm{tr}(\hat{\rho})$.

To evaluate Eq. (3), we take the point-splitting approach, by which [7]

$$\left\langle \hat{\overline{\Psi}}\hat{\Psi}\right\rangle_{\beta_0,\Omega} = -\lim_{x'\to x}\mathrm{tr}\left[iS^{\mathrm{F}}_{\beta_0,\Omega}\left(x,x'\right)\Lambda\left(x',x\right)\right]\,, \qquad (4)$$

where $S^{\mathrm{F}}_{\beta_0,\Omega}(x,x')$ is the thermal two-point function and $\Lambda(x,x')$ is the bispinor of parallel transport, given by [11]

$$\begin{aligned}\Lambda\left(x,x'\right) = {} & \frac{\sec(\omega s/2)}{\sqrt{\cos\omega r\cos\omega r'}} \\ & \times\left[\cos\frac{\omega\Delta t}{2}\left(\cos\frac{\omega r}{2}\cos\frac{\omega r'}{2} + \sin\frac{\omega r}{2}\sin\frac{\omega r'}{2}\frac{\boldsymbol{x}\cdot\boldsymbol{\gamma}}{r}\frac{\boldsymbol{x}'\cdot\boldsymbol{\gamma}}{r'}\right)\right. \\ & \left. + \sin\frac{\omega\Delta t}{2}\left(\sin\frac{\omega r}{2}\cos\frac{\omega r'}{2}\frac{\boldsymbol{x}\cdot\boldsymbol{\gamma}}{r}\gamma^{\hat{t}} + \sin\frac{\omega r'}{2}\cos\frac{\omega r}{2}\frac{\boldsymbol{x}'\cdot\boldsymbol{\gamma}}{r'}\gamma^{\hat{t}}\right)\right]. \qquad (5)\end{aligned}$$

Using the property $\hat{\rho}\hat{\Psi}(t,\varphi)\hat{\rho}^{-1} = e^{-\beta_0\Omega S^{\hat{z}}}\hat{\Psi}(t + i\beta_0, \varphi + i\beta_0\Omega)$, together with the imaginary time anti-periodicity of the two-point function [8], it is possible to compute $S^{\mathrm{F}}_{\beta_0,\Omega}(x,x')$ via [9]

$$S^{\mathrm{F}}_{\beta_0,\Omega}\left(x,x'\right) = \sum_{j=-\infty}^{\infty}(-1)^j e^{-j\beta_0\Omega S^{\hat{z}}} S^{\mathrm{F}}_{\mathrm{vac}}\left(t + ij\beta_0, \varphi + ij\beta_0\Omega; t', \varphi'\right)\,, \qquad (6)$$

[1] We consider the covering space of adS.

where $S^{\rm F}_{\rm vac}(x,x')$ is the vacuum two-point function. The above expression is valid only when the vacua ($\beta_0 \to 0$) corresponding to the rotating (finite Ω) and non-rotating ($\Omega = 0$) cases coincide. This is ensured on adS when $|\Omega| \le \omega$ [10], which we assume to hold for the remainder of this paper.

Due to the maximal symmetry of adS, $S^{\rm F}_{\rm vac}(x,x')$ can be written as [12]

$$iS^{\rm F}_{\rm vac}\left(x,x'\right) = \left[\mathcal{A}(s) + \mathcal{B}(s)\not{n}\right] \Lambda\left(x,x'\right) \,, \tag{7}$$

where $n_\mu = \nabla_\mu s(x,x')$ is the normalised tangent at x to the geodesic connecting x and x', while the geodesic interval s is given through

$$\cos\omega s = \frac{\cos\omega\Delta t}{\cos\omega r\cos\omega r'} - \cos\gamma\tan\omega r\tan\omega r' \,, \tag{8}$$

where γ is the angle between $\boldsymbol{x}$ and $\boldsymbol{x}'$, such that $\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\Delta\varphi$. For massless fermions, the functions $\mathcal{A}$ and $\mathcal{B}$ are [11]

$$\mathcal{A}\rfloor_{M=0} = \frac{\omega^3}{16\pi^2}\left(\cos\frac{\omega s}{2}\right)^{-3} \,, \qquad \mathcal{B}\rfloor_{M=0} = \frac{i\omega^3}{16\pi^2}\left(\sin\frac{\omega s}{2}\right)^{-3} \,. \tag{9}$$

3. Analysis and conclusions

Without presenting the details of the computation, we find [10]

$$\left\langle :\hat{\bar{\Psi}}\hat{\Psi}:\right\rangle_{\beta_0,\Omega} = \sum_{j=1}^{\infty} \frac{(-1)^{j+1}\omega^3(\cos\omega r)^4\cosh\frac{\omega j\beta_0}{2}\cosh\frac{\Omega j\beta_0}{2}}{2\pi^2\left[\sinh^2\left(\frac{\omega j\beta_0}{2}\right) + \cos^2\omega r - \sin^2\omega r\sin^2\theta\sinh^2\left(\frac{\Omega j\beta_0}{2}\right)\right]^2} \,. \tag{10}$$

The total FC can be obtained by integrating Eq. (10) over the whole space

$$\begin{aligned} V^{\rm FC}_{\beta_0,\Omega} &= \int {\rm d}^3x\sqrt{-g}\left\langle :\hat{\bar{\Psi}}\hat{\Psi}:\right\rangle_{\beta_0,\Omega} = -\sum_{j=1}^{\infty}\frac{(-1)^j\cosh\left(\frac{\Omega j\beta_0}{2}\right)/\sinh\left(\frac{\omega j\beta_0}{2}\right)}{\cosh(\omega j\beta_0) - \cosh(\Omega j\beta_0)} \\ &\simeq \frac{3\zeta(3)T_0^3}{\omega\left(\omega^2 - \Omega^2\right)} - \frac{\left(3\omega^2 - \Omega^2\right)T_0}{6\omega\left(\omega^2 - \Omega^2\right)}\ln 2 + O\left(T_0^{-1}\right) \,. \end{aligned} \tag{11}$$

On the boundary, the following result is obtained:

$$\begin{aligned} S^{\rm FC}_{\beta_0,\Omega} &= \int {\rm d}\Omega\sqrt{-g}\left\langle :\hat{\bar{\Psi}}\hat{\Psi}:\right\rangle_{\beta_0,\Omega} \\ &\simeq \frac{7\pi^3T^4}{45(\omega^2 - \Omega^2)^{3/2}}\left[\frac{\omega}{\Omega}\tan^{-1}\left(\frac{\Omega/\omega}{\sqrt{1-\frac{\Omega^2}{\omega^2}}}\right) + \sqrt{1 - \frac{\Omega^2}{\omega^2}}\right] + O\left(T^2\right) \,. \end{aligned} \tag{12}$$

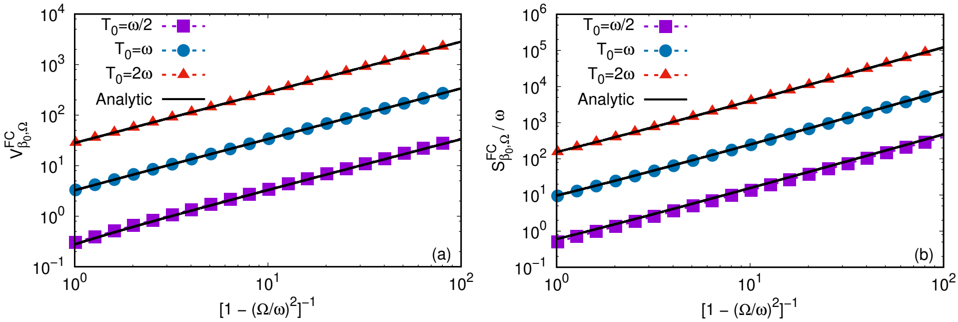

Fig. 1. Dependence of (a) $V^{\rm FC}_{\beta_0,\Omega}$ and (b) $S^{\rm FC}_{\beta_0,\Omega}/\omega$ with respect to $(1-\Omega^2/\omega^2)^{-1}$, in logarithmic scale. The dotted lines and symbols are numerical results obtained using Eq. (10), while the analytic curves correspond to Eqs. (11) and (12).

Both $V^{\rm FC}_{\beta_0,\Omega}$ (11) and $S^{\rm FC}_{\beta_0,\Omega}$ are amplified due to the rotation through the prefactors $(1-\Omega^2/\omega^2)^{-1}$ and $(1-\Omega^2/\omega^2)^{-3/2}$, respectively. Figures 1 (a) and (b) show the dependence of $V^{\rm FC}_{\beta_0,\Omega}$ and $S^{\rm FC}_{\beta_0,\Omega}$ on $(1-\Omega^2/\omega^2)^{-1}$, for various values of the temperature $T_0=\beta_0^{-1}$. It can be seen that the analytic results (11) and (12) (shown with solid black lines) match well the numerical results (dotted lines and symbols) computed using Eq. (10).

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