TOP-DOWN APPROACH TO THE CURVED SPACETIME EFFECTIVE FIELD THEORY (cEFT) — THEORY AND EXAMPLES*

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The effective field theory (EFT) turns out to be an instrument of an immense value in all aspects of modern particle physics being theory, phenomenology or experiment. In the paper, I will show how to extend the systematic top-down approach to construction of the EFT proposed by Hitoshi Murayama (LBL, Berkeley) and separately by John Ellis (King's College London) groups to the curved spacetime. To this end, I will take advantage of the heat kernel method so far extensively used in obtaining the one-loop effective action in curved spacetime. After an introduction of the formalism, I will discuss its application to the problem of an influence of gravity on the stability of the Higgs effective potential.

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1. Introduction

In the most rudimentary sense, the Effective Field Theory is a tool that allows us to parametrize our lack of understanding. We use it when we do not have sufficient knowledge of details of investigated phenomena or when the full description is too complicated to be tracked precisely, or when at the current state this precise description is not needed yet. The ways of construction of the EFT may be separated into two main categories, namely the bottom-up approach and the top-down approach. In the top-down approach, we start with some extended theory that is presumably valid in the high-energy region, then we integrate out the high-energy degrees of freedom. In doing so, we end up with the EFT in which effects associated with the presence of the integrated out particles are encoded in the higher dimensional operators.

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In the process of extending the EFT to the curved spacetime Effective Field Theory (cEFT), we followed the top-down approach. This contribution was based on [1].

2. Some technical aspects and results

To extend the EFT to the cEFT, in other words to take into account the effects coming form the presence of the classical (non-quantum) non-trivial spacetime curvature, we decided to use the heat kernel method [2]. As an input to this method, we need to have a classical action functional for the matter and gravity fields $S_{\rm UV}$ that is valid at the high energy (UV stands for Ultra Violet). In our test case, we used the system of two scalar fields H which may represent Higgs field and X which represents heavy particles, for example heavy dark matter. The action is

$$\begin{aligned} S_{\rm UV} \;=\; & \int \sqrt{-g}\, \mathrm{d}^4x \left\{ -\frac{1}{2}\mathrm{d}_\mu H^\dagger \mathrm{d}^\mu H - \frac{1}{2}m_H^2|H|^2 - \frac{\lambda_H}{4!}|H|^4 - \xi_H R|H|^2 + \right. \\ & \left. -\frac{1}{2}\mathrm{d}_\mu X \mathrm{d}^\mu X - \frac{1}{2}m_X^2 X^2 - \xi_X R X^2 - \frac{1}{2}\lambda_{HX} X^2|H|^2 \right\} , \qquad (1) \end{aligned}$$

where d_μ is a covariant derivative (it can also contain gauge field-dependent part), R is the Ricci scalar, m_i, λ_i and ξ_i, where $i = H, X$, are mass parameters, quartic couplings and non-minimal couplings of the scalar fields. In the next step, our calculations follow closely the prescription of obtaining the one-loop effective action in curved spacetime [3, 4] with the added caveat that for now we consider only fluctuations of the heavy field X. After this step, we obtain the following action describing our low-energy cEFT (for details please see [1]):

$$\begin{aligned} S_{\rm cEFT} \;=\; & \int \sqrt{-g}\, \mathrm{d}^4x \Big\{ -\frac{1}{2}\mathrm{d}_\mu H^\dagger \mathrm{d}^\mu H - \frac{1}{2}m_H^2|H|^2 - \frac{\lambda_H}{4!}|H|^4 - \xi_H R|H|^2 + \\ & -\frac{1}{2}c_{dHdH}\mathrm{d}_\mu|H|^2\mathrm{d}^\mu|H|^2 - c_{GHH}G^{\mu\nu}\mathrm{d}_\mu|H|^2\mathrm{d}_\nu|H|^2 + \\ & -c_H|H|^2 - c_{HH}|H|^4 - c_6|H|^6 \Big\} . \qquad (2) \end{aligned}$$

The two last lines of (2) represent contributions of the higher dimensional and gravity-dependent operators to the physics of the Higgs field.

In what follows, we will focus on the gravity-mediated contributions to the Higgs quartic coupling. They are described by the coefficient c_{HH} in (2). These contributions are important from the standpoint of the vacuum stability of the Standard Model (SM). The c_{HH} coefficient as calculated for the model described by the action (1) is given by

$$c_{HH} = \frac{\hbar}{(4\pi)^2}\left[\frac{\lambda_{HX}^2}{4m_X^2}\left(2\xi_X - \frac{1}{6}\right)R - \frac{\lambda_{HX}^2}{8m_X^4}\left(2\xi_X - \frac{1}{6}\right)^2 R^2 + \right.$$
$$\left. -\frac{\lambda_{HX}^2}{720m_X^4}(\mathcal{K} - R_{\mu\nu}R^{\mu\nu}) + \frac{\lambda_{HX}^2}{m_X^4}\left(-\frac{1}{4}\xi_X + \frac{1}{40}\right)\Box R - \frac{\lambda_{HX}^2}{90m_X^4}\nabla_\mu\nabla_\nu R^{\mu\nu}\right], \quad (3)$$

where $\mathcal{K} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is a Kretschmann scalar and $\Box \equiv \nabla_\mu\nabla^\mu$ is a covariant d'Alembert operator.

The magnitude of this contribution to the Higgs quartic coupling for two selected fixed gravitational backgrounds is depicted in Figs. 1 and 2.

The first case is the one of the primordial black hole (PBH) modeled by the Schwarzschild metric. From Fig. 1, we see that for small black-hole mass $M_{\rm PBH} \sim 10^{10}$g the gravity-mediated contributions are comparable to the two-loop effects coming form the Higgs self-interaction. This implies that they should be taken into account if we want to analyze the problem of the SM vacuum stability near such a black hole beyond the one-loop approximation.

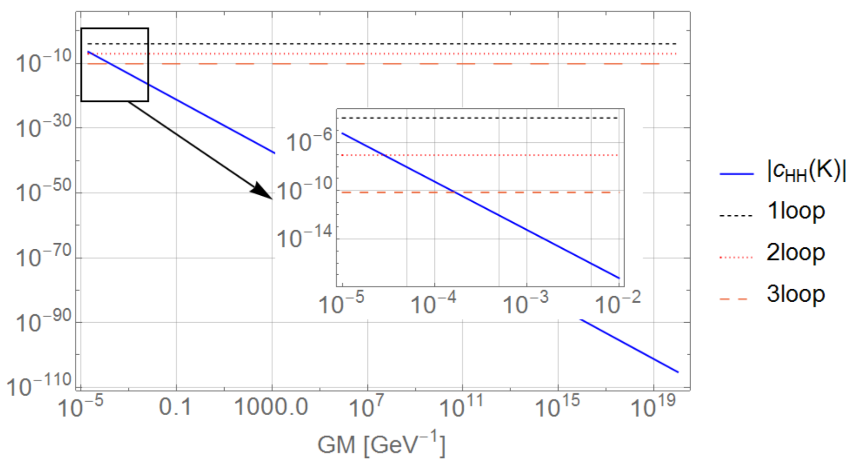

Fig. 1. Gravity-induced contribution to the Higgs quartic coupling in the black-hole background. $|c_{HH}(K)| = |-\frac{1}{(4\pi)^2}\frac{\lambda_{HX}^2}{720}\frac{\mathcal{K}}{m_X^4}|$, G is the Newton constant, M is the black-hole mass and loops prefactors are given by the formula $n\text{loop} = \frac{\lambda_H^{n+1}}{(16\pi^2)^n}$. For the plot, we chose $\lambda_{HX} = 0.25$, $\lambda_H = 0.13$ and $m_X = 10\,\text{TeV}$.

The second case is the gravity-mediated contribution in the de Sitter spacetime. Physically, this type of spacetime may describe short post inflationary reheating stage of the evolution of the Universe. From Fig. 2, we may see that if the temperature at this epoch is of the order of electroweak phase transition ($T_{\rm EW}$), then gravity-mediated effects are of the order of the two-, or even close to the one-loop effects and they should be included in the analysis of the vacuum stability.

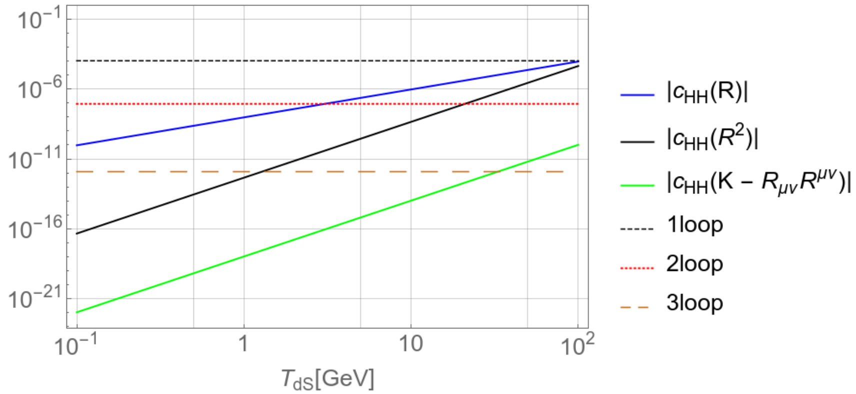

Fig. 2. Gravity-induced contribution to the Higgs quartic coupling in the de Sitter-like FLRW background. Loops prefactors are given by the formula $n\mathrm{loop} = \frac{\lambda_H^{n+1}}{(16\pi^2)^n}$ and $T_{\rm dS}$ is the temperature of the de Sitter spacetime. For the plot we chose $\lambda_{HX} = 0.25$, $\lambda_H = 0.13$, $m_X = 10\,\mathrm{TeV}$ and $\xi_X = 10$. For comparison $T_\odot \sim 10^{-13}$ GeV, and $T_{\rm EW} \sim 10^2$ GeV.

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