

QUANTUM GRAVITY ON A TORUS*

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We analyze four-dimensional quantum gravity model defined by Causal Dynamical Triangulations (CDT). One of the key features of CDT is that the geometry of quantum space-time can be globally foliated into spatial slices with fixed three-dimensional topology. We show that CDT with toroidal spatial topology (T^3) has rich phase structure, including the semi-classical phase C consistent with Einstein's general relativity. Some of the phase transitions are also found to be second (or higher) order which makes a possibility of taking continuum limit viable. These findings are consistent with earlier results obtained in CDT with spherical spatial topology (S^3).

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1. Introduction

One of the most successful attempts of the lattice formulation of *quantum gravity* (QG) is that of Causal Dynamical Triangulations (CDT) (for a recent review, see [1]). CDT is based on the path integral formalism and makes only a few assumptions on the geometry of quantum space-time, namely it requires that the geometry can be globally foliated into three-dimensional space-like hypersurfaces, each with the same fixed topology. CDT uses the discretization of space-time geometry following the method proposed by Regge [2]. In this approach, the (discretized) geometry is constructed by gluing together two types of elementary simplicial building blocks, called the $(4, 1)$ and the $(3, 2)$ simplex, to obtain piecewise-linear manifolds (also called triangulations \mathcal{T})¹. In CDT, the path integral of QG is defined by a sum over all possible triangulations \mathcal{T} which obey the requested topological conditions

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¹ For more details, see [1].

$$\mathcal{Z}_{\text{QG}} = \int \mathcal{D}_{\mathcal{M}}[g] e^{iS_{\text{HE}}[g]} \longrightarrow \mathcal{Z}_{\text{CDT}} = \sum_{\mathcal{T}} e^{iS_{\text{R}}[\mathcal{T}]}, \quad (1)$$

where S_{R} is the Regge discretization [2] of the continuum gravitational Hilbert–Einstein action S_{HE} for a triangulation \mathcal{T}

$$S_{\text{R}}[\mathcal{T}] = -(\kappa_0 + 6\Delta) N_0 + \kappa_4 (N_{(4,1)} + N_{(3,2)}) + \Delta N_{(4,1)}. \quad (2)$$

The Regge action in equation (2) is a linear combination of the total number of the (4,1)-simplices: $N_{(4,1)}$, the (3,2)-simplices: $N_{(3,2)}$ and vertices: N_0 in the triangulation \mathcal{T} , and three dimensionless bare coupling constants κ_0 , Δ and κ_4 . The coupling constants are related to Newton’s constant, the cosmological constant and the asymmetry between lengths of space-like and time-like links in the triangulation, respectively. The existence of the global foliation means that each triangulation can be analytically continued between the Lorentzian and the Euclidean geometry. In the Euclidean formulation, the memory of the time direction and foliation is preserved. Wick rotation changes the Lorentzian action into the Euclidean action and the path integral \mathcal{Z}_{CDT} becomes a partition function of (triangulated) random geometries which can be studied numerically using Monte Carlo methods.

Earlier results were mainly obtained for the fixed spatial topology of the 3-sphere and the studied systems were also assumed to be periodic in the (Euclidean) time. In such a case, CDT showed rich phase structure (see Fig. 1) including the *semiclassical* phase C , where one observes dynamical emergence of 4-dimensional background geometry consistent with general

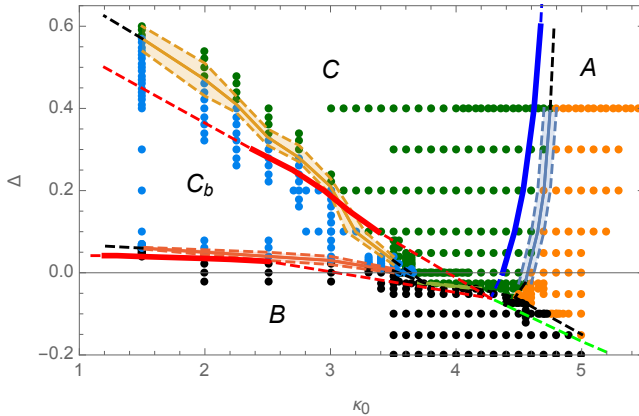


Fig. 1. Phase structure of 4-dimensional CDT. Thin solid lines are phase transitions measured for the toroidal spatial topology T^3 (shaded regions represent error bars). Thick solid lines are phase transitions measured for the spherical spatial topology S^3 .

relativity [3], and quantum fluctuations of spatial volume are described by the Hartle–Hawking minisuperspace action [4, 5]. Another important result was the existence of phase transitions of higher order [6, 7] which makes a possibility of taking continuum limit viable [8, 9].

2. CDT with toroidal spatial topology

Below, we present new results obtained for the fixed spatial topology of the 3-torus (and the time periodic boundary conditions), which has been studied recently. In [10, 11], it was shown that in such a case, one can observe the analogue of phase C , albeit the quantum fluctuations occur around a different semiclassical background geometry than in the spherical topology². In the toroidal case, the quantum fluctuations of spatial volume are again well-described by the minisuperspace action, and one can also observe a quantum correction in the effective potential, which could not be measured in the spherical CDT.

By analyzing the behaviour of four order parameters in various points on the toroidal CDT phase diagram, one can as well observe the analogues of all other phases discovered in the spherical CDT [12]. One can also measure precisely the position and order of the phase transitions. The phase diagram in Fig. 1 shows that the toroidal CDT phase transitions are only slightly shifted *versus* the spherical CDT case which most likely results from different finite size effects in these topologies³. As an additional bonus, in the toroidal case, one was able to perform precise numerical simulations in the most interesting region of the parameter space where the phases meet, which was not possible in the spherical CDT. The measurements showed that instead of the conjectured “quadruple” point, where all four phases were supposed to meet, one observes two separate “triple” points, where three phases meet. In between the two “triple” points, one can also measure the direct C – B phase transition, which was shown to be first order [13], albeit with some untypical properties suggesting that the end points can be higher order.

The recent studies of the toroidal CDT confirmed that the A – C transition is first order [14] and that the B – C_b transition is second (or higher) order, exactly as it was observed in the spherical topology. For the C – C_b transition, which was shown to be second (or higher) order in the spherical case, in the toroidal CDT, one observes strong hysteresis, suggesting that the order of the transition might have changed. So far, due to the hysteresis, the numerical algorithms used do not allow us to make precise finite size

² The average spatial volume profile in phase C changes from that of the (Euclidean) de Sitter space (in the spherical CDT) to the flat profile (in the toroidal CDT).

³ The minimal possible triangulation of the 3-torus is much larger than the minimal triangulation of the 3-sphere [10].

scaling analysis of that transition, so one can neither prove nor disprove this hypothesis. The order of measured phase transitions is summarized in the table below.

Phase transition	Topology: $S^1 \times T^3$	Topology: $S^1 \times S^3$
$A-C$	1 st order	1 st order
$B-C$	1 st order	?
$B-C_b$	higher order	higher order
$C-C_b$?	higher order

3. Summary and conclusions

We have summarized results of the phase structure and phase transitions studies in CDT with the toroidal spatial topology (T^3) and compared them with the results earlier obtained in the spherical spatial topology (S^3). Our findings confirm that although the detailed features of quantum geometries observed in different topologies may change, *e.g.* a different background geometry emerges in the semiclassical phase C in each of the topologies, the key properties of CDT remain universal independently of the (fixed) spatial topology choice.

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