MASSLINE AND OTHER RECENT RESULTS OF CDT QUANTUM GRAVITY*

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With the basis of CDT quantum gravity, we implemented a dynamical particle in the form of a massline, which is minimally coupled to the geometry via the action $S_m = m\,L$, where m is the bare mass and L is the length of the line. During our simulations, we measured the radial distribution of the volume and curvature around the line, which resulted in nontrivial findings. Furthermore, we measured the length of the line in the function of the mass and found agreement with the expected theoretical value.

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1. Introduction

Causal Dynamical Triangulations (CDT) is a framework which attempts to quantize gravity via a nonperturbative formalism [1]. It defines a Feynman path integral on a piecewise simplical manyfold built up from d-dimensional simplices. CDT introduces a globally hyperbolic foliation to enforce causal structure. The partition function of the theory is defined as

$$\mathcal{Z}_{\rm CDT} = \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}}\, {\rm e}^{-S_{\rm E}[\mathcal{T}]}\,, \tag{1}$$

where $S_{\rm E}[\mathcal{T}]$ is the Euclidean Einstein–Hilbert action which is defined on the $\mathcal{T}$ triangulation, and $C_{\mathcal{T}}$ is a sum factor of a configuration. The 4-dimensional model cannot be solved analytically so one has to use numerical simulations such as Monte Carlo Metropolis algorithm to create and sum over the ensemble of configurations to calculate the expectation values of various observables. The Euclidean Einstein–Hilbert action which drives the dynamics of simulations can be written in the Regge formalism as

$$S_{\rm R}^{\rm E} = -(K_0 + 6\varDelta)\, N_0 + K_4 N_4 + \varDelta N_{41}\,, \tag{2}$$

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where K_0, K_4 and Δ plays the role of the bare gravitational coupling, the cosmological coupling and asymmetry parameters, respectively. After fixing the volume of configurations [2], the parameter-space can be limited to a 2-dimensional subspace called the phasestructure (shown in Fig. 1). Analyzing

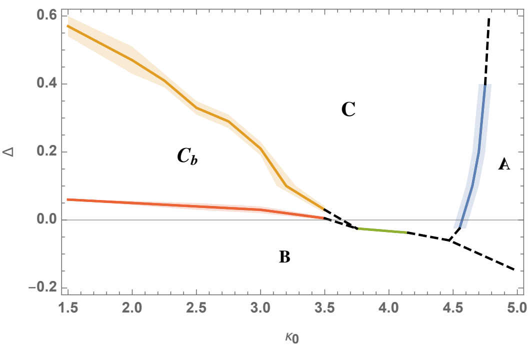

Fig. 1. The phase structure of CDT with its 4 distinct phases.

this subspace, one can find the existence of four distinct phases which seems to be a topology-independent feature of the theory [3]. Phase C is the region where the effective action agree with the Hartle–Hawking Minisuperspace model [4]. The results shown in the following sections were achieved for coupling values: $(K_0, \Delta, V) = (4.0, 0.2, 160\,\text{k})$, where V is the number of simplices and the spatial topology was that of a sphere and torus.

2. Massline

The notion of a massive particle in this theory could be imagined as a line in the 4-dimensional configuration. The massline is a closed timelike loop which is minimally coupled via the action $S_m = m\,L$, where m is the value of the mass and L is the length of the loop. Since CDT is a coordinate free model, there is no notion of location only the distances between simplices can be measured. Looking at the radial distribution of the 3-dimensional volume around the massline $V^m(d,m)$ and around a random point $V^r(d,m)$ as a function of distance d (defined via the centers of tetrahedra) and mass m shows difference shown in Fig. (2). The local environment of the massline with a value m_{crit} shows nontrivial deviation from other values. The 3-dimensional curvature is located around links, thus the formula of curvature will be

$$C(d,m) \approx \sum_d \frac{1}{o(l_d)}\,, \tag{3}$$

where $o(l_d)$ is the order of the link l of a tetrahedron at distance d, and m dependence is due to different expected profiles for various m (Fig. (2)).

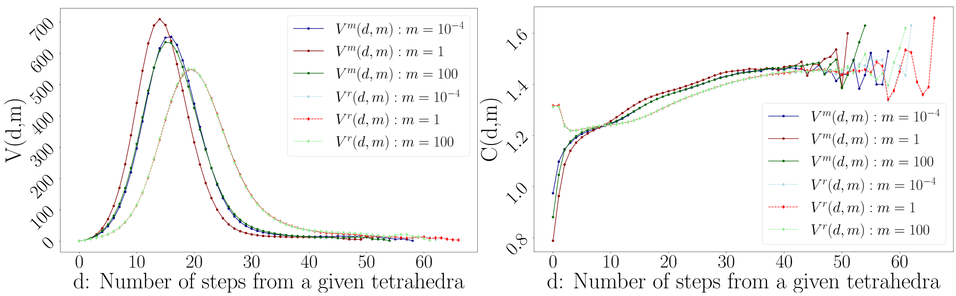

Fig. 2. $V^m(d,m)$ and $V^r(d,m)$ (left) and $C^m(d,m)$ and $C^r(d,m)$ (right).

The behavior of $C^m(d,m)$ can be extracted by plotting the value of curvature in m. Figure (3) shows the values of $C^m(d,m)$ at given distances and masses. When m is close to a critical value m_{crit}, the value of $C^m(d,m) < 1$, which means negative (attractive) curvature.

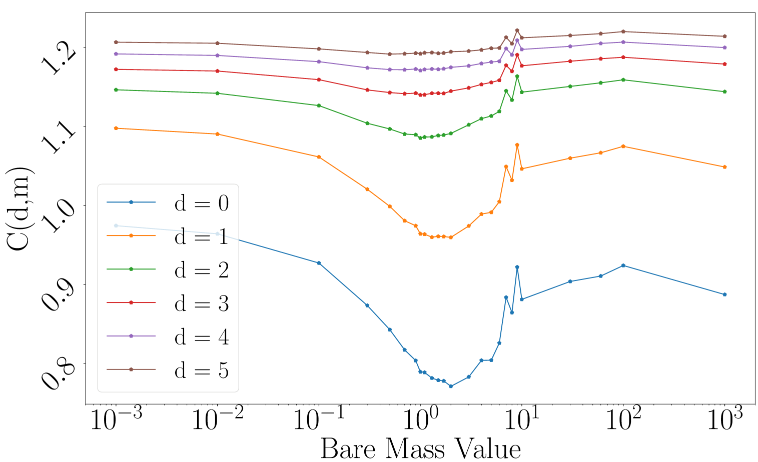

Fig. 3. (Color online) The values of $C(d,m)$; the color denotes the distance from the line.

The effect is stronger in the vicinity of the line and dies off around $d > 10$. Furthermore, the same shape of distribution was observed independently of the system size, topology or position in the phase diagram. The results coming from $C(d,m)$ and $V(d,m)$ are in agreement. Another natural measure is $l(m)$ which should approach the minimal length for large m and raise to ∞ as the mass approaches a non-zero critical value (which is not equal to the above described m_{crit}). In Fig. (4), the length of the line function of m can be seen. For large m, the function approaches $m_0 = 4T$, where T is the number of timeslices, which was $T = 10$ in our simulations. The solution which was predicted theoretically is independent of the topology and the total volume of the measured configuration.

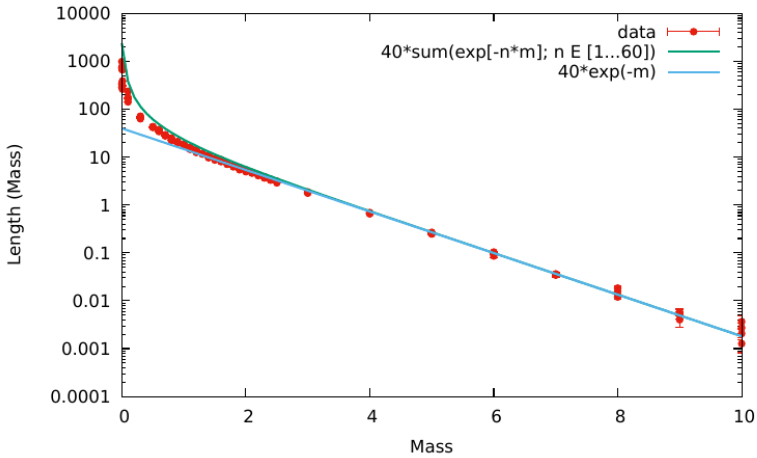

Fig. 4. The length of the line function of m. The minimal length $l_0 = 40$ is subtracted from the measured value. $l(m)$ behaves as the sum of exponentials with the limit: $40\exp(-m)$.

3. Conclusions

In this article, the first results on implementing a point particle in the CDT setup were presented. The radial volume distribution and the curvature were found to be meaningful observables around the massline. These values have a strong dependence on m and a critical value of mass was found to be $m_{\rm crit} \approx 1.5 \pm 0.5$. The function $l(m)$ was measured with high accuracy.

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