

THE ORIGIN OF MERGING BLACK HOLES*

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LIGO's discovery of gravitational waves from massive merging black hole binaries posed the fundamental question — what is the origin of these black holes? Two models have been proposed: *field stellar binaries* and *capture*. In the former, the binary was born as two massive stars. In this case, some level of alignment of the black holes spins with the orbital angular momentum is expected. In the latter, the two black holes evolve individually and were dynamically captured. The black holes' spins and the orbital angular momentum are not correlated and hence they are expected to be isotropically distributed. The effective spin, χ_{eff} , is probably the best parameter that can distinguish between the models. Recently, independent analysis of the LVC O1–O2 sample revealed, in addition to the original ten identified by LVC, eight new mergers. We present here a concise model for the spin evolution of field binaries and use it to estimate the expected χ_{eff} distribution. We compare this distribution as well as several isotropic distributions, reflecting *capture* scenarios, to the observations. While the current data slightly prefers *field binaries*, isotropic distributions or a combination of both origins are possible. Future detection in O3 and O4 of a few dozens to few hundred mergers will enable us to distinguish with sufficient statistical significance between the different models.

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1. Introduction

What is the origin of the merging massive binary black holes (BBHs) discovered by LIGO [1]? This is one of the most interesting yet unanswered questions that arose with this remarkable discovery. An answer to this can have far reaching implications to issues ranging from fundamental problems in stellar evolution (if it turns out that massive black holes with masses above $\sim 60M_{\odot}$ arose from massive stars) or to the nature of dark matter (if the majority of the BBHs are primordial [2]). The numerous models

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that have been put forward are divided into two groups: *field binaries* in which the BHs evolved from massive stellar binaries, *e.g.* [3–9], and *capture* scenarios in which each one of the BHs evolved on its own and the binary formed dynamically later. The *capture* scenarios are further divided into those that involve primordial black holes, *e.g.* [2, 10–13] and those in which the black hole originated from massive stars in various dense stellar environments [14–24].

The black holes’ spins are the most promising parameters that can enable us to distinguish between the two scenarios. The orientations of the spin vectors are randomly distributed within the different variants of the *capture* scenario. Within these scenarios, there is no clear physical mechanism¹ that favors alignment of the BHs spins with the orbital angular momentum. The BH spins are determined in earlier phases of the evolution, when the progenitors evolve on their own and the capture process is independent of the spins. On the other hand, numerous processes during a binary stellar evolution tend to align the individual stellar spins along the orbit’s angular momentum. These include, among others, the formation of the binary that arises from a single rotating cloud and tides, and mass transfer during different stages of the binary evolution. A common envelope phase, in which one of the stars orbits within its companion atmosphere will clearly operate to align the spins. While winds carry out angular momentum and decreases the spin and kicks may randomize it, there is no effect that preferably align the BH spins in a direction opposite to the orbital angular momentum.

The gravitational waves signal that arises during the spiral-in phase of the binary encodes the various parameters of the binary systems and those can be estimated at various levels of precision (see, *e.g.* [25]). Among those, the effective spin, χ_{eff} , the projection of the normalized spin component of the binary along the direction of the angular spin, is the easiest to recover among the different spin components. The χ_{eff} distribution is a natural source of information concerning the origin of the BBH population. In *capture* we expect an equal number of mergers with positive and negative χ_{eff} values. This equality should arise regardless of the average magnitudes of the spins and the masses involved. Given the requirement of triple coincidence between three vectors *capture*, scenarios also disfavor high absolute χ_{eff} values. *Field binaries* should show excess of positive over negative χ_{eff} values. The size of this excess depends on the specific binary evolution model. In particular, natal kicks during the BH formation randomize the spin orientation and may partially wash it out. However, this signature should be there, and depending on the size of the sample and the accuracy in determination of χ_{eff} it should be eventually detected.

¹ One can, of course, consider a system in which all the stars in the same dense region rotate in the same direction, but even then, the orbital spin will be tilted during the three body capture process.

We describe here a *field binary* model that focuses on the late stages of the binary evolution and compare the observed χ_{eff} distribution to its predictions and to the predictions of isotropic (*capture*) models. We consider a combined data-set that includes the original O1–O2 LVC catalog that contains ten events and ten additional mergers found in an independent analysis of the O1–O2 data [26–29]. We denote the additional data-set as the IAS sample and the combined data that we analyze as the LVC-IAS sample. The LVC data that included low χ_{eff} mergers whose values were roughly equally divided above and below zero, favored low-spin isotropic models [30] and hence *capture*. We show that with the addition of the new events, the LVC-IAS data-set tells a different story.

2. The field binary model

The complex evolution of massive stellar binaries, the progenitors of a massive field BBH (see Fig. 1) has been the subject of numerous studies (see *e.g.* [31]). Population synthesis models (see, *e.g.* [5]) attempt to follow each one of the different phases from birth to the formation of the binary and provide us with estimates of the distributions of different parameters (masses and spins) of the merging BBHs.

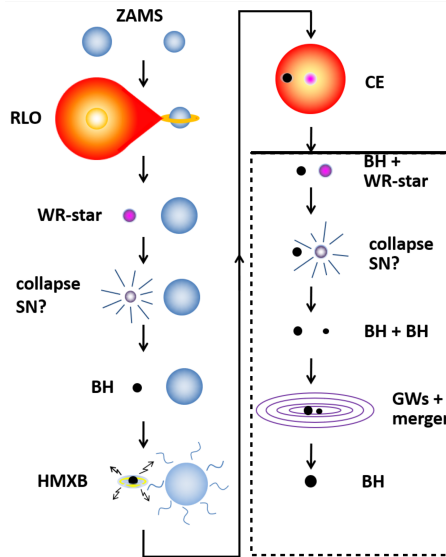


Fig. 1. The evolution of the a massive stellar binary system. We consider in the following only the last phases (marked in a box). To reflect the uncertainties in the earlier phases, we consider different initial conditions at the beginning of this phase marked by a bold face solid line. Figure credit: Thomas Tauris.

Here, we take a different approach. As we are interested in the spins of the merging BHs, we focus only on them. We begin from the observations of χ_{eff} and go backwards in time. We identify the last processes that influence the spin and consider only their effects.

At the merger, χ_{eff} is determined by the spin of the system just after the formation of the second BH (the secondary) as it does not change² during the spiral in phase. Just before the second collapse, the binary includes the primary BH and the secondary progenitor. The separation between the two must be sufficiently small so that the system merges in a Hubble time. Thus, the secondary is most likely a Wolfe–Rayet star (see *e.g.* [32, 33]), but other possible compact stars and, in particular, homogeneously mixed star [6] are possible. The compact configuration with an orbital separation not much larger than the stellar radius implies that the primary BH exerts a significant tidal force on the secondary. This tidal force tends to align the stellar spin along the orbital angular momentum and to synchronize it with the orbital period. While tidal forces tend to increase the spin, winds, that are common at the late phases of massive stellar evolution, carry out angular momentum and decrease it. These two counteracting effects are the dominant ones that are important at the late phases of the stellar evolution.

In principle, we can go further backwards and explore the effects of earlier phases. However, this is not essential. Instead, we consider different initial conditions at the onset of this last phase. If we find that the results are insensitive to different assumptions on these initial conditions, we can indeed disregard the earlier phases of the stellar evolution and consider just these two effects.

The essence of our model, that follows [32–34] and [35], is the following. We characterize the system by the BH masses and by the time it takes to merge, t_c . This time determines the initial separation, $a(t_c)$, which in turn determines the strength of the tidal force, which we describe by the synchronization time, $t_{\text{syn}}(t_c)$, and $\chi_{\text{syn}}(t_c)$, the normalized spin of a synchronized star. The wind is characterized by the time scale, t_w , in which the angular momentum is lost from the system. Overall, the combined effects of the tidal force and wind are described by a single equation for the stellar normalized spin [32, 33]

$$\frac{d\chi_*}{dt} = \frac{(\chi_{\text{syn}} - \chi_*)^{8/3}}{t_{\text{syn}}(t_c)} - \frac{\chi_*}{t_w}. \quad (2.1)$$

We evolve χ_* over the lifetime of the star t_* to obtain the final spin $\chi_*(t_*)$ (see Fig. 2).

² A rare situation in which the binary is a part of a triple system the tertiary star may influence χ_{eff} .

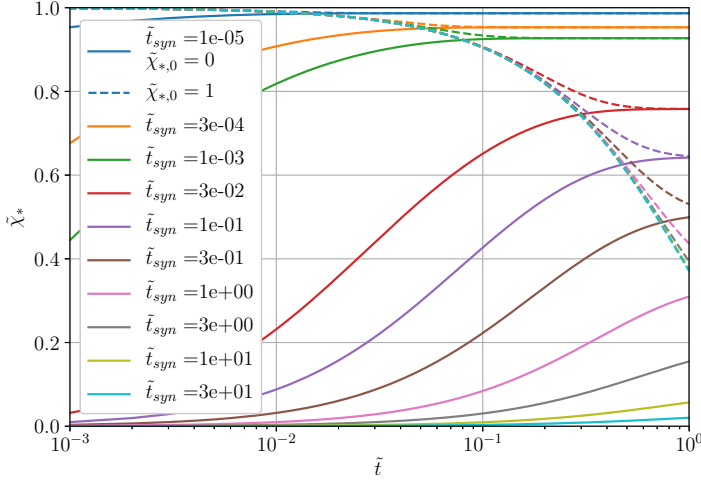


Fig. 2. The evolution of the ratio of the progenitors spin to the synchronized spin, $\tilde{\chi} \equiv \chi_*/\chi_{\text{syn}}$, as a function of $\tilde{t} \equiv t/t_w$ for different values of $\tilde{t}_{\text{syn}} \equiv \chi_{\text{syn}}^{-5/3}(t_{\text{syn}}/t_w)$.

After the collapse, the spin of the secondary (the second one to collapse) BH, $\vec{\chi}_2$, is determined by its progenitors spin just before the collapse, $\vec{\chi}_{2*}$ and by $\vec{\chi}_k$, the spin given to the BH due to any kick during the collapse

$$\vec{\chi}_{\text{BH},2} = \begin{cases} \vec{n} & \text{if } |\vec{\chi}_{2*} + \vec{\chi}_k| \geq 1, \\ \vec{\chi}_{2*} + \vec{\chi}_k & \text{if } |\vec{\chi}_{2*} + \vec{\chi}_k| < 1, \end{cases} \quad (2.2)$$

where $\vec{n}$ is a unit vector parallel to $\vec{\chi}_{2*} + \vec{\chi}_k$.

A kick during the collapse will be independent of the orbital motion. Hence, if the average size of the spin due to the kick $\bar{\chi}_k$ is larger than the average stellar spin χ_* , the secondary spin will have a random orientation. The resulting χ_{eff} will be the projected sum of two randomly oriented vectors, much like the situation in the capture scenario. However, evidence from the heaviest Galactic BHs suggests that those are born without the kick velocity [36]. Hence, in the following, we assume that $\bar{\chi}_k$ is negligible.

As stated earlier, to reflect the uncertainty in the earlier phases of the evolution, we consider different initial progenitors spins. We consider four models: (SA₀, SA_{syn}, DA₀, DA_{syn}). We set the initial progenitors spin either to zero (denoted by ₀) or to be fully synchronized (denoted by _{syn}). We further consider systems in which only the secondary (the lightest) BH is synchronized (those are labeled SA) or systems in which both progenitors are synchronized (denoted by DA). In the SA model, in which only the secondary is synchronized, we allocate a random spin to the primary.

To obtain the observed spin distribution, we sample a merger time t_c from the distribution

$$p_{\text{obs}}(t_c) \propto \begin{cases} t_c^{-1} & \text{if } t_c > t_{c,\min}, \\ 0 & \text{otherwise.} \end{cases} \quad (2.3)$$

Once t_c is chosen, we solve Eq. (2.1) and obtain χ_* , and using Eq. (2.2) we obtain $\chi_{\text{BH},2}$. In the DA models, we repeat the same procedure for $\chi_{\text{BH},1}$, while for the SA models, we choose $\chi_{\text{BH},1}$ randomly with a flat distribution in magnitude and random in direction. The effective spin is given by

$$\chi_{\text{eff}} = \frac{\chi_{\text{BH},1} + q\chi_{\text{BH},2}}{1 + q}. \quad (2.4)$$

Repeating this process, we obtain the theoretical χ_{eff} distribution $p_{\text{th}}(\chi_{\text{eff}}; \lambda)$, where λ denotes the parameters of the model used. To account for the observational error, we add a random noise to the distribution

$$p(\chi_{\text{eff}}; \lambda) = \int_{-1}^1 p_{\text{th}}(\chi'_{\text{eff}}; \lambda) \frac{e^{-\chi_{\text{eff}} - \chi'_{\text{eff}})^2 / 2\bar{\sigma}_{\chi_{\text{eff}}}^2}}{\sqrt{2\pi\bar{\sigma}_{\chi_{\text{eff}}}^2}} d\chi'_{\text{eff}}, \quad (2.5)$$

where $\bar{\sigma}_{\chi_{\text{eff}}} = 0.14$ is the standard deviation in estimations of χ_{eff} .

Figure 3 depicts the distribution of the four different models. The distributions are shown with best fitted parameters that fit the LVC-IAS data.

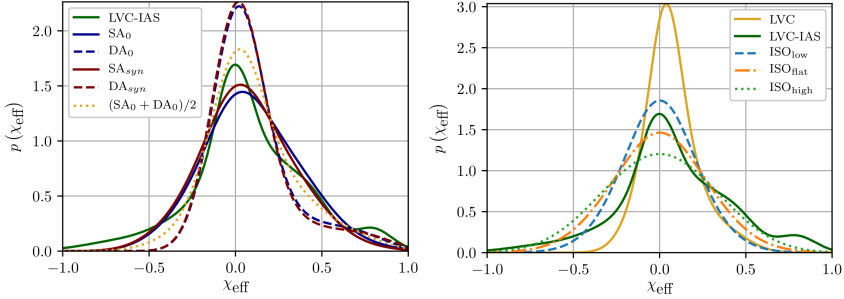


Fig. 3. The different models considered. Left: The probability density function, $p(\chi_{\text{eff}})$, for the different field binary models (using the best fitted parameters $(t_{c,\min}, t_w)$ Myr: SA_0 : (20, 0.1), DA_0 : (100, 1), SA_{syn} : (20, 0.05), DA_{syn} : (50, 0.05), and a mixed model, $(\text{SA}_0 + \text{DA}_0)/2$, taken with SA_0 parameters). Also shown is the LVC-IAS data. Note the excess of intermediate and high positive χ_{eff} events in these distributions. Right: A comparison of the probability density function, $p(\chi_{\text{eff}})$, of the *low*, *flat* and *high* isotropic models with the LVC-IAS data. In both panels, the theoretical distribution has been convolved with the observational error, $\bar{\sigma}_{\chi_{\text{eff}}}$.

3. The isotropic models

The χ_{eff} distributions for the isotropic models are given by a weighted sum of two randomly oriented (isotropic) normalized spin vectors $\mathbf{s}_i$

$$\chi_{\text{eff-iso}} = \frac{\mathbf{s} \cdot \hat{\mathbf{L}} + q \mathbf{s}_2 \cdot \hat{\mathbf{L}}}{1 + q}. \quad (3.1)$$

Following [30], we consider three distributions defined by the distribution of $|\mathbf{s}_i|$: *flat*, or dominated by either *low* or *high* spins. The probability for a given $\mathbf{s}$ value is

$$p(|\mathbf{s}_i|) = \begin{cases} 2(1 - |\mathbf{s}_i|) & \text{low;} \\ 1 & \text{flat;} \\ 2|\mathbf{s}_i| & \text{high.} \end{cases} \quad (3.2)$$

We use $q = 1$ (varying q has a minor effect).

The different distributions are shown in Fig. 3, together with the LVC-IAS data. One can easily see that the isotropic distributions, as expected, are symmetric with equal probability of positive and negative events.

4. The data

We consider the combined data of the LVC analysis of the O1–O2 runs that revealed ten BBH mergers [37] and the IAS analysis that provided additional eight events [26–29]. Figure 4 reveals the distributions and their parameters.

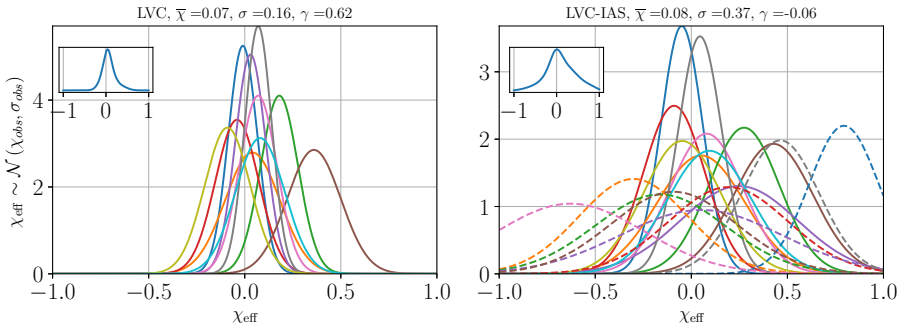


Fig. 4. The distribution of the observed χ_{eff} in the LVC (left) and LVC-IAS (right) data. We have approximated each observation as a Gaussian whose mean value and 90% credible interval are the values given in [37] and [29] respectively. The inserts show the average distribution. The title indicates the mean $\bar{\chi}$, the standard deviation σ_{χ} and the skewness γ_{χ} .

In the following analysis, we neglect possible mass/spin correlations. We evaluate the models over a fixed mass and compare them to the unweighted³ observed distribution. This is natural in the isotropic scenario and valid for field binary scenarios if tidal locking and winds operate in the same manner across the progenitors mass range. Further, given the small size of the sample, such an assumption is essential.

A possibly significant observational bias may arise from the dependence of the GW horizon on the spin with larger horizons for positive spins and smaller for negative ones [38, 39]. We have modeled this effect assuming that overall the detection volume is related to χ_{eff} as

$$V(\chi_{\text{eff}}) \propto \frac{\chi_{\text{eff}}^2}{4} + \frac{3\chi_{\text{eff}}}{4} + 1, \quad (4.1)$$

namely the detection volume of an event with $\chi_{\text{eff}} = 1$ is larger than the volume for $\chi_{\text{eff}} = 0$ by a factor of 2 and than the volume for $\chi_{\text{eff}} = -1$ by a factor of 4. This is probably an overestimate. Figure 5 depicts the distribution with and without such a correction.

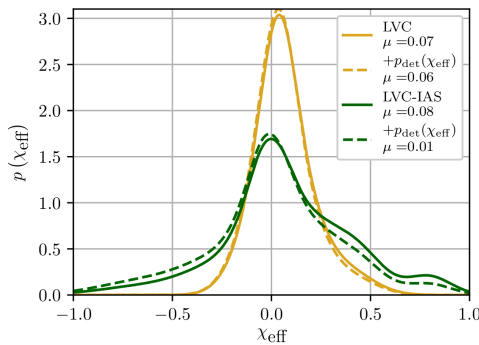


Fig. 5. The effect of the observational bias on the χ_{eff} distributions. Distribution of the observed χ_{eff} in the LVC and IAS data. Shown is the effect of the correction due to the observational bias.

5. Analysis and results

Before turning to a detailed comparison of the expected and observed distributions, we note that one can use a rather simple intuitive test in comparing isotropic to field binary models. As already noted, the former should have a symmetric distribution around zero, while the latter should have an excess of positive events, some of which could have high χ_{eff} values. The average χ_{eff} in the LVC and the LVC-IAS samples is 0.07 and 0.08

³ With respect to the mass.

respectively. The probability to get such a value of larger from 10 or 18 flat isotropically distributed spins (with an observations error of $\bar{\sigma}_{\chi_{\text{eff}}} = 0.14$) is 0.21 and 0.11, respectively. However, if we consider the bias corrected χ_{eff} distribution, $\bar{\chi}_{\text{eff}} \approx 0$ and with this value this test is meaningless.

While such a simple test could be indicative, it does not use all the available information within the χ_{eff} distribution. To use the full information, we have compared [35] the different models to the data. We carry out this comparison using the [40] statistic (denoted hereafter AD).

The comparison to the three isotropic models was simple, as they do not have any free parameters (apart from the nature of the $|\chi_{\text{eff}}|$ distribution (shown in Eq. (3.2)). We find that all isotropic models are acceptable. However, the *flat* model is favored, whereas the *low* model was the most favorable with the LVC data [30].

The *field binary* models depend on three time-parameters, t_* , $t_{c,\text{min}}$ and t_w . We take $t_* = 0.3 \text{ Myr}$ as the typical⁴ and using maximal-likelihood determine the best $t_{c,\text{min}}$, t_w values. Once we find the best fit parameters we compare, again using the AD statistical test, the models to the data. We find good fits for all models. The two single aligned (SA) models, initially unsynchronized and synchronized, result in almost identical distributions (using different parameters and with different likelihood values). Similarly, the two DA models give almost identical distributions.

Among the *field binary* models, the single synchronized initially unaligned one, SA_0 , stands out as the preferred model with the highest likelihood and the most reasonable physical parameters, see, [34]: $t_{c,\text{min}} = [c] \times 10 - 100 \text{ Myr}$ (corresponding, for $m_i \approx 30M_\odot$, $q = 1$, to $a = 4 - 7 \times 10^{11} \text{ cm}$) and $t_w = 0.1 - 5 \text{ Myr}$, reflecting a wide range of winds. The Maximum Likelihood of SA_{syn} is comparable to the one of SA_0 but the former requires somewhat stronger winds ($t_w < 0.1 \text{ Myr}$) and is valid at a more confined range. SA_0 and DA_0 have a comparable broad range of allowed physically acceptable parameters but the latter has a smaller maximal likelihood. The DA_{syn} model has the smallest feasible parameter phase space and it seems least likely. We also consider, as an example, a model that combines two different initial conditions — $0.5(\text{SA}_0 + \text{DA}_0)$ — using the best fit parameters of the SA_0 model. Even without optimizing the ratio of SA_0 relative to DA_0 and the model parameters, this model fits the data slightly better than all others. Overall the acceptance level (using the AD statistic) of the *field binary* models is better than the acceptance level of the isotropic ones. The acceptance levels for the models are shown now in Table I. Interestingly, the probabilities remain the same when we compare the models to the data that has been corrected for the observational bias in χ_{eff} .

⁴ Variation of t_* will amount to scaling of the two other time scales.

TABLE I

AD acceptance probabilities for the different models. For the field binaries, these are for the best fitted parameters. The numbers in brackets correspond to a comparison of the model to the data corrected to the observational bias in χ_{eff} (see Fig. 5). Note that the values with and without the χ_{eff} detection bias corrections are almost identical.

Model	ISO _{low}	ISO _{flat}	ISO _{high}	SA ₀	DA ₀	SA _{syn}	DA _{syn}	0.5(SA ₀ + DA ₀)
Acceptance	0.28	0.5	0.5	0.93	0.54	0.95	0.54	0.96
Probability	(0.28)	(0.5)	(0.5)	(0.93)	(0.54)	(0.96)	(0.54)	(0.96)

The current data-set is insufficient to distinguish between the models. Even the least preferred model is consistent at $\sim 30\%$ with the data. Assuming that one of the models considered here is the correct one, we can ask now how many mergers are needed to rule out the others? To do so, we [35] choose one of the models as the fiducial one and test the others against it. Within the *field evolution* models, we consider the SA₀ model and the DA₀ with the best fit parameters, and we compare those to the three isotropic models, *low*, *flat* and *high*.

We find that 50–100 (150–250) mergers are required to distinguish the SA₀ model from the isotropic ones at the 5% (1%) confidence level. The DA₀ model includes more positive high-spin events and fewer negative spin ones. Hence, as expected, it is easier to distinguish it from the isotropic models. 30–60 (50–120) mergers are sufficient to distinguish between the DA₀ model and the different isotropic models at the 5% (1%) confidence level. The situation is more complicated when we consider mixed models that combine both field binaries and capture. A few hundred mergers are needed to distinguish between these models and “pure” field binaries or “pure” capture models.

If in future mergers we will be able to estimate χ_{eff} better (lower $\bar{\sigma}_{\chi_{\text{eff}}}$), then fewer events would suffice. If $\bar{\sigma}_{\chi_{\text{eff}}}$ is a quarter of its current value 25–50 (80–120) events will be sufficient needed to distinguish at the 5% (1%) confidence level between the SA₀ model and the isotropic ones.

6. Conclusions

The observed low effective spins, that were centered around zero, in the LVC O1–O2 sample favored low spin isotropic distributions [30] and hence *capture* scenarios. We have shown here that while the combined LVC-IAS data-set that includes a high χ_{eff} binary cannot rule out any model, it favors field binaries over capture.

Within the *field binary* models, the high χ_{eff} merger implies a significant fraction of short ($t_c \sim 20$ Myr) mergers, namely BBHs that at formation had small, but reasonable ($4\text{--}7 \times 10^{11}$ cm), separations. Overall the LVC-IAS sample brackets nicely the phase space of the *field binary* model with $10 \text{ Myr} \lesssim t_{c,\text{min}} \lesssim 100 \text{ Myr}$, $0.05 \text{ Myr} \lesssim t_w \lesssim 5 \text{ Myr}$.

While the isotropic scenario is somewhat disfavored, it is not ruled out. Among those models, the *flat* variant becomes the most favorable and the *low* the least. It is interesting to note that recently [41] have shown that the eccentricity of all the events in the LVC sample are smaller than 0.02 to 0.05, whereas a *capture* scenario suggests that 5% of the events should have larger eccentricity. While this result does not rule out the *capture* scenarios, they support our findings. Clearly, a mixture of *field binaries* and *capture* is possible and even likely. In this case, the current LVC-IAS sample hints that the former will be dominant. However, given the limited data we did not explore this possibility here. Considering future observations, we note that the hallmark of the *field binaries* scenario is a preferably positive χ_{eff} distribution with a few large positive χ_{eff} mergers. At the same time, unless kicks are very significant and dominate the BHs spin distribution, large negative χ_{eff} will pose a problem for the *field binary* model. We have shown that for the models considered here, we will need 30–250 events, depending on the details of the model and the level of confidence required to distinguish between the two scenarios. Higher S/N data that has a better determined χ_{eff} value would require a fewer events. Hundreds of events will be needed to determine the ratio of capture to field evolution events in mixed model that includes both or to distinguish those from pure *capture* or pure *field binary* models.

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