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STABILITY OF POLYTROPIC STARS IN PALATINI GRAVITY*

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We will briefly discuss the necessary conditions for stability of polytropies in $f(\hat{R})$ Palatini gravity and the differences with the GR ones.

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1. Introduction

Palatini $f(\hat{R})$ gravity [1–4], in the similar manner as other theories of gravity, usually is a background for astrophysical objects such as black holes and neutron stars (NS). However, because of a degeneracy in the mass–radius profiles (the NS composition in the central core is still under debate), NS are not yet ideal objects to test theories of gravity.

It turns out that non-relativistic stars, particularly dwarf ones, can also be used to test gravitational theories since the non-gravitational physics giving their properties, such as metallicity and opacity for example, is not modified by them [5–8]. Thus, we would like to focus on the stability problem of such objects in the context of the Palatini $f(\hat{R})$ gravity.

2. Palatini stellar objects

2.1. Relativistic and non-relativistic stars

It was shown [9] that for a spherical-symmetric object, whose matter is described by the perfect fluid energy-momentum tensor $T_{\mu\nu}$ and an equation of state given by the barotropic relation $p = p(\rho)$, that the Tolman–Oppenheimer–Volkoff (TOV) equation can be written in the case of $f(\hat{R})$ Palatini gravity as

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Pi(r)}{\Phi^2(r)}\right) = -\frac{\tilde{A}G\mathcal{M}}{r^2}\left(\frac{\Pi+Q}{\Phi(r)^2}\right)\left(1+4\pi r^3\frac{\Pi}{\Phi(r)^2\mathcal{M}}\right), \tag{1}$$

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where $\Phi = f'(\hat{R})$, $\tilde{A} = \Phi^{-1}(1 - 2G\mathcal{M}/r)^{-1}$, while $\Pi = \rho + \frac{c^4 U}{2\kappa^2 G}$ and $Q = \rho - \frac{c^2 U}{2\kappa^2 G}$ are generalized pressure and energy density, respectively, with $U = U(\Phi)$ being a scalar field potential in the Einstein frame, depending on the $f(\hat{R})$ model. It should be noticed that the coordinate r is a conformal one thus the conformal transformation must be taken into account $r^2 \to \Phi r^2$.

Since the Palatini gravity introduces modifications to the Newtonian hydrostatic equilibrium describing non-relativistic stellar objects such as dwarf stars or stars from the main sequence, applying non-relativistic limits to (1) $p \ll \rho$ with $4\pi r^3 p \ll \mathcal{M}$ and $\frac{2G\mathcal{M}}{r} \ll 1$, one gets

$$-r^2 \Phi(r) p' = G\mathcal{M}(r)\rho(r)\,, \qquad \mathcal{M}(r) = \int_0^r 4\pi\tilde{r}^2 \frac{Q(\tilde{r})}{\Phi(\tilde{r})^2} \mathrm{d}\tilde{r}\,. \tag{2}$$

From now on, we will focus on the particular gravitational model, that is, the quadratic one $f(\hat{R}) = \hat{R} + \beta\hat{R}^2$, while the matter part will be described by the polytropic equation of state, that is, $p = K\rho^{\Gamma}$. In that case, it was shown [10] that the conformal transformation preserves the polytropic character of EoS in the Einstein frame. Thus, one simplifies further the mass function (2) to $\mathcal{M}(r) = \int_0^r 4\pi\rho\tilde{r}^2 \mathrm{d}\tilde{r}$. That makes possible to write down the generalized Lane–Emden equation for the quadratic model [11]

$$\xi^2 \theta^n \Phi^{3/2} + \frac{1}{1 + \frac{\xi\Phi_\xi}{2\Phi}} \frac{\mathrm{d}}{\mathrm{d}\xi} \left(\frac{\xi^2 \Phi^{3/2}}{1 + \frac{\xi\Phi_\xi}{2\Phi}} \frac{\mathrm{d}\theta}{\mathrm{d}\xi} \right) = 0\,, \tag{3}$$

where $\Phi = 1 + 2\alpha\theta^n$, $\Phi_\xi = \mathrm{d}\Phi/\mathrm{d}\xi$, and $\alpha = \kappa^2 c^2 \beta \rho_{\mathrm{c}}$ with ρ_{c} being the star's central density and $\kappa^2 = 8\pi G c^{-4}$. Equation (3) possesses two exact solutions for $n = \{0, 1\}$ [12]. The solutions (exact or numerical) allow to get the star's mass

$$\mathcal{M} = 4\pi r_{\mathrm{c}}^3 \rho_{\mathrm{c}} \omega_n\,, \tag{4}$$

as well as central density, radius, and temperature, with

$$\gamma_n = (4\pi)^{\frac{1}{n-3}} (n+1)^{\frac{n}{3-n}} \omega_n^{\frac{n-1}{3-n}} \xi_R\,,$$

and they are

$$\rho_{\mathrm{c}} = \delta_n \left(\frac{3\mathcal{M}}{4\pi R^3} \right), \qquad R = \gamma_n \left(\frac{K}{G} \right)^{\frac{n}{3-n}} \mathcal{M}^{\frac{n-1}{n-3}} \xi_R\,, \qquad T = \frac{K\mu}{k_{\mathrm{B}}} \rho_{\mathrm{c}}^{\frac{1}{n}} \theta_n\,, \tag{5}$$

where k_{B} is Boltzmann's constant and μ the mean molecular weight. However, ω_n and δ_n depend not only on the solutions of the LE equation [12] but also on Φ and Φ_ξ, such that

$$\omega_n = -\frac{\xi^2\Phi^{\frac{3}{2}}}{1+\frac{1}{2}\xi\frac{\Phi_\xi}{\Phi}}\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\bigg|_{\xi=\xi_R}, \qquad \delta_n = -\frac{\xi_R}{3\frac{\Phi^{-\frac{1}{2}}}{1+\frac{1}{2}\xi\frac{\Phi_\xi}{\Phi}}\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\Big|_{\xi=\xi_R}}. \tag{6}$$

2.2. Stability of polytropic stars in Palatini gravity

The stability analysis for an arbitrary Lagrangian functional $f(\hat{R})$ performed in [9] shows that the stability condition of Palatini relativistic stars is similar to the one in GR: the condition depends on a given equation of state. Not very surprising, when the Palatini gravity is considered, one also needs to specify a model, that is, to examine the problem case by case.

In what follows, we would like to have a closer look on the above statement. Thus, let us limit ourselves to the non-relativistic hydrostatic equilibrium equations to which we apply the polytropic equation of state such that we will use the Lane–Emden formalism to the stability analysis.

It is well-known (see *e.g.* [13]) that a necessary condition for stability can be expressed as

$$\frac{\partial\mathcal{M}}{\partial\rho_\mathrm{c}} > 0\,. \tag{7}$$

Applying it to (4) at the star's center with $\Phi = 1 + 2\alpha\,\theta^n$ gives us an inequality of the form of

$$n - 3 - 2\bar{\beta}(n+6)\rho_\mathrm{c} - 4\bar{\beta}^2(3+2n)\rho_\mathrm{c}^2 > 0\,, \tag{8}$$

while the equality will be satisfied by

$$\rho_{\mathrm{c}_1} = -\frac{1}{2\bar{\beta}}\,, \qquad \rho_{\mathrm{c}_2} = \frac{3-n}{3+2n}\rho_{\mathrm{c}_1}\,, \tag{9}$$

where $\bar{\beta} = c^2\kappa^2\beta$. The first difference between GR and the Palatini gravity is noticed immediately: the latter one introduces to the inequality a dependence on the model parameter β apart from the polytropic parameter n only as it happens in GR. The stability criterion crucially depends here on the sign of the parameter $\bar{\beta}$: For positive values of the parameter, it may happen that stability occurs for the negative values of the central densities which is unphysical. Thus, in order to have ρ_{c_2} positive and to be a stationary point, we immediately are left with the stable region for densities values from the range $(0;\rho_{\mathrm{c}_2})$ but with $(\Gamma = 1 + 1/n)$

$$n > 3\,, \quad \left(\Gamma < \frac{4}{3}\right). \tag{10}$$

In GR, we deal with unstable stellar configurations for such values of n; the Palatini model allows stable ones, though.

For the negative value of the parameter $\bar{\beta}$, the central density ρ_{c_1} is a stationary point. In order for ρ_{c_2} to be also a stationary point, the following condition must be satisfied for the polytropic index:

$$n < 3, \quad \left(\Gamma > \frac{4}{3}\right) \tag{11}$$

giving also the range (ρ_{c_2}, ρ_{c_1}) of the possible (and positive) central densities for which a star can be a stable configuration. However, in the case of the negative parameter $\bar{\beta}$, we will always deal with some range of the stable configuration for each $n \geq 0$, that is, $(0, \rho_{c_1})$, but with the only one stationary point ρ_{c_1}.

It should be mentioned that for the polytropic index $n = 3$, which gives the EoS describing a white dwarf star, the stationary point exists only for the negative values of the parameter $\bar{\beta}$.

Let us notice that the stability analysis of the non-relativistic stellar objects allows to constraint the parameter $\bar{\beta}$: for example, the typical density of a white dwarf is of the order of 10^9 kg/m^3, therefore, the parameter $\bar{\beta} \sim 10^{-9}$ m^3/kg.

3. Conclusions

The aim of this short letter was to demonstrate that in the case of Palatini quadratic model one deals with the similar situation as in GR, that is, the stability depends on the EoS. Though, as shown here for polytropic EoS, in the given gravitational model, a necessary condition for stability allows to consider a wider range of polytropic index, being however dependent on the sign of the theory parameter β. A stationary point, that is, a central density ρ_c for which $\frac{\partial \mathcal{M}}{\partial \rho_c} = 0$, for polytropic white dwarfs ($n = 3$) exists only for negative values of the parameter β.

Furthermore, the stationary points obtained from stability analysis can also provide additional and independent of the theory of gravity, constraints for the model's parameter due to the reasonable values of energy density which are given by the non-gravitational physics. Non-relativistic stars, although still having many secrets to be revealed, are stellar objects much better known and understood than neutron stars, and thus giving interesting opportunities to test gravitational theories.

All together, this simple example shows that stability conditions should be reanalyzed in the context of modified gravity.

REFERENCES

[1] A. Palatini, *Rend. Circ. Mat. Palermo* **43**, 203 (1919).

[2] T.P. Sotiriou, V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010).

[3] S. Capozziello, M. De Laurentis, *Phys. Rep.* **509**, 167 (2011).

[4] M. Ferraris, M. Francaviglia, I. Volovich, *Class. Quantum Grav.* **11**, 1505 (1994).

[5] G. Olmo, D. Rubiera-Garcia, A. Wojnar `arXiv:1912.05202 [gr-qc]`.

[6] J. Sakstein, *Phys. Rev. Lett.* **115**, 201101 (2015).

[7] J. Sakstein, *Phys. Rev. D* **92**, 124045 (2015).

[8] G.J. Olmo, D. Rubiera-Garcia, A. Wojnar, *Phys. Rev. D* **100**, 044020 (2019).

[9] A. Wojnar, *Eur. Phys. J C* **78**, 421 (2018).

[10] A. Mana, L. Fatibene, M. Ferraris, `arXiv:1505.06575v2 [gr-qc]`.

[11] A. Wojnar, *Eur. Phys. J. C* **79**, 51 (2019).

[12] A. Sergyeyev, A. Wojnar, *Eur. Phys. J. C* **80**, 313 (2020).

[13] N.K. Glendenning, «Compact stars», *Springer US*, New York 1996.