

SOLAR SYSTEM CONSTRAINTS ON ANALYTIC PALATINI $f(R)$ GRAVITY*

JÚNIOR D. TONIATO

Núcleo de Astrofísica e Cosmologia (Cosmo-ufes), PPGCosmo
and Departamento de Física, Universidade Federal do Espírito Santo
Av. Fernando Ferrari 514, Vitória-ES, Brazil

(Received January 14, 2020)

The post-Newtonian equations-of-motion corrections to finite volume massive bodies and light rays for an arbitrary analytical Palatini $f(R)$ theory are derived. It is shown that, apart from a mass-energy redefinition that is explicitly found here, which cannot be constrained by solar system tests, the predictions are the same as in general relativity.

DOI:10.5506/APhysPolBSupp.13.255

1. Introduction

A very practical way to confront an alternative theory of gravity with solar system tests is through the Will and Nordtvedt parametrized post-Newtonian (PPN) formalism [1]. This framework considers a generic post-Newtonian (PN) metric as an expansion in terms of gravitational potentials and relates each coefficient of this expansion (the so-called PPN parameters) with specific phenomena by means of the equations of motion. Once derived the PN metric of a theory, its PPN parameters and solar system constraints are promptly obtained (see, for instance, [2]). It turns out that the number of gravitational potentials considered by the formalism is limited. This implies that, if a given theory does contain distinct potentials in its PN metric, one should deduce the PN equations of motion in order to obtain the influence of new potentials in the trajectories of planets and light rays. That is the case of Palatini $f(R)$ gravity, the alternative theory considered in this work [3].

The PN approximation of Palatini gravity has been considered previously in [4]. However, that work confronts Palatini gravity with solar system tests through the analysis of the PN metric, not considering the equations of motion. This issue we investigate here in detail considering an arbitrary analytic $f(R)$ Palatini gravity.

* Presented at the 6th Conference of the Polish Society on Relativity, Szczecin, Poland, September 23–26, 2019.

2. Palatini $f(R)$ gravity review

The Palatini $f(R)$ considers the spacetime metric $g_{\mu\nu}$ and an affine connection $\Gamma_{\mu\nu}^\lambda$ as independent objects of the manifold. Matter fields are minimally coupled with gravity and their Lagrangian does not depend on the connection. The action variations with respect to g and Γ lead, respectively, to the following field equations:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} = \kappa T_{\mu\nu} \quad \text{and} \quad \nabla_\lambda (\sqrt{-g}f'(R)g^{\mu\nu}) = 0, \quad (1)$$

where κ is some coupling constant, the prime indicates a derivative with respect to R , and ∇ represents a covariant derivative constructed with the affine connection. It is important to note that, similarly to GR, diffeomorphism invariance implies that $\nabla_C^\mu T_{\mu\nu} = 0$, where ∇_C^μ is the covariant derivative associated with the Christoffel symbol of metric g .

3. Palatini post-Newtonian metric

The PN framework is an approximation method for gravitational theories which considers a weak-field and slow-motion regime, a suitable approach to describe Solar System dynamics. To implement this framework to the Palatini $f(R)$, we follow Ref. [5]. Once the metric field is described as a small perturbation of a flat spacetime, we consider an analytical $f(R)$ and expand it around $R = 0$, $f(R) = \sum_{n=0}^\infty a_n R^n$, where a_n are constants. In order to proceed with PPN formalism, we impose that space should be asymptotic flat, $a_0 = 0$. Since κ is arbitrary, we can also set $a_1 = 1$. Solving the field equations order by order, we obtain the PN metric in Palatini gravity

$$g_{00} \approx -1 + 2(U + \psi) + \partial_{tt}\chi - 2U^2 + (3\tilde{a}_3 - 2\tilde{a}_2^2)\rho^{*2} + \tilde{a}_2(2\rho^* - 4\Phi_P - 10U\rho^* + v^2\rho^* - 2\Pi\rho^* + 6p), \quad (2)$$

$$g_{0i} \approx -4V^i, \quad g_{ij} \approx \delta_{ij} + 2(U + \tilde{a}_2\rho^*)\delta_{ij}, \quad (3)$$

where $\tilde{a}_2 = \kappa a_2$, $\tilde{a}_3 = \kappa^2 a_3$ and Latin indices run from 1 to 3. When $\tilde{a}_2 = \tilde{a}_3 = 0$, the GR solution is recovered. The quantities ρ^* , Π and p are the usual perfect fluid conserved mass density, internal energy density and pressure, respectively. The U , ψ , χ and V^i are standard PPN potentials, and Φ_P is a new potential particular to Palatini gravity, namely

$$\Phi_P = \int \frac{\rho^*(\mathbf{x}', t)^2}{|\mathbf{x} - \mathbf{x}'|} d^3x'. \quad (4)$$

We first note that our result is not in conflict with the previous analysis on the PN limit of Palatini gravity using scalar-tensor equivalence [4]. At the

Newtonian order, there is a Palatini correction term given by $\tilde{a}_2\rho^*$. This term is in general relevant for the internal stability of a given body, but it does not change the body center-of-mass trajectory, as will be clear later.

For the light propagation, PN corrections in the photon trajectories are given by the vacuum second order metric, which is equivalent to GR. Therefore, the PPN parameter γ within Palatini gravity is precisely 1, just like GR. To determine the equations of motion of massive, finite-volume bodies, further details are necessary. The first step is to examine the PN hydrodynamics in order to find the conserved quantities.

4. Conserved quantities

By manipulating the conservation of energy-momentum tensor it is possible to obtain conserved integrals. The time component leads to a total energy conservation

$$\frac{dE}{dt} = 0, \quad \text{with} \quad E \equiv \int \left(\frac{1}{2}\rho^*v^2 + \rho^*\Pi - \frac{1}{2}\rho^*U - \frac{\tilde{a}_2}{2}\rho^{*2} \right) d^3x. \quad (5)$$

The total mass-energy of the fluid is then defined as $M = m + E$ and it satisfies $dM/dt = 0$, where m is the material mass, $m = \int \rho^*d^3x$, and it is constant in time too. The vector equation of the conservation of energy-momentum tensor can also be integrated to define the total conserved momentum, $dP^i/dt = 0$, with

$$P^j = \int \rho^*v^j \left(1 + \frac{v^2}{2} - \frac{U}{2} + \Pi + \frac{3p}{\rho^*} - \tilde{a}_2\rho^* \right) d^3x - \frac{1}{2} \int \rho^*W^j d^3x. \quad (6)$$

The previous results show that Palatini $f(R)$ gravity does not violate total conservation of energy and momentum in the PN regime, although it redefines the conserved quantities. This is an expected outcome since any Palatini gravity model is a Lagrangian-based metric theory with matter action being independent from the affine connection and, as shown in [6], they should not violate PN conservation laws. In the context of the PPN formalism, the results obtained here directly show that the PPN parameters $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ and α_3 are all zero in the Palatini $f(R)$ gravity.

5. Equation of motion for massive bodies

In this section, we split the fluid description of the source into N separated bodies in order to obtain the PN equations of motion for the bodies center-of-mass positions. Each body indexed by A has a material mass and

center-of-mass acceleration given by

$$m_A = \int_A \rho^* d^3x, \quad \mathbf{a}_A(t) = \frac{1}{m_A} \int_A \rho^* \frac{d\mathbf{v}}{dt} d^3x. \quad (7)$$

The volume where the integration above is calculated is bounded in the inter-body vacuum region. The center-of-mass acceleration of each body will be a sum of three parts: the Newtonian acceleration $\mathbf{a}_A^{\text{Newt}}$, a PN correction $\mathbf{a}_A^{\text{PN}}$ and the structural contribution $\mathbf{a}_A^{\text{str}}$. The integrand of Eq. (7) is found from the PN Euler equation and all mathematical techniques are detailed in Ref. [5]. The final results read

$$\mathbf{a}_A^{\text{Newt}} = - \sum_{B \neq A} \frac{m_B}{r_{AB}^2} \mathbf{n}_{AB}, \quad \mathbf{a}_A^{\text{str}} = - \sum_{B \neq A} \frac{E_B}{r_{AB}^2} \mathbf{n}_{AB}, \quad (8)$$

$$\begin{aligned} \mathbf{a}_A^{\text{PN}} &= - \sum_{B \neq A} \frac{m_B}{r_{AB}^2} \left\{ \left[v_A^2 - 4(\mathbf{v}_A \cdot \mathbf{v}_B) + 2v_B^2 - \frac{3}{2}(\mathbf{n}_{AB} \cdot \mathbf{v}_B)^2 - \frac{(5m_A - 4m_B)}{r_{AB}} \right] \mathbf{n}_{AB} \right. \\ &\quad - [\mathbf{n}_{AB} \cdot (4\mathbf{v}_A - 3\mathbf{v}_B)] (\mathbf{v}_A - \mathbf{v}_B) + \sum_{C \neq A, B} \frac{7m_C r_{AB}}{2r_{BC}^2} \mathbf{n}_{BC} \\ &\quad \left. - \sum_{C \neq A, B} m_C \left[\frac{4}{r_{AC}} + \frac{1}{r_{BC}} - \frac{r_{AB}}{2r_{BC}^2} (\mathbf{n}_{AB} \cdot \mathbf{n}_{BC}) \right] \mathbf{n}_{AB} \right\}. \end{aligned} \quad (9)$$

In the above expressions, we use the definitions $\mathbf{r}_{AB} = \mathbf{r}_A - \mathbf{r}_B$, $r_{AB} = |\mathbf{r}_{AB}|$ and $\mathbf{n}_{AB} = \mathbf{r}_{AB}/r_{AB}$. These equations have no explicit dependence on either a_2 or a_3 and they are identical, in form, to the corresponding GR expressions. There is a single implicit difference inside the constant E_B , which is the energy associated with the planet indexed with B . However, the PPN parameters are not sensitive to this energy redefinition. Moreover, the above result implies that the remaining PPN parameters are once again equal to their GR values, $\beta = 1$ and $\alpha_1 = \alpha_2 = \xi = 0$. Hence, in spite of the appearance of non-standard PPN potentials in the metric expansion, we conclude that the values of all the PPN parameters are the same as of GR.

6. Conclusion

In this work, it was presented a post-Newtonian (PN) analysis of a class of analytic Palatini $f(R)$ gravity without making use of the equivalence with scalar-tensor theories and, more importantly, using the equations that describe light and planets trajectories to link theory and experiments. This

is because the PN metric in Palatini theories does not fit the PPN formalism. The results found is that the equations of motion are precisely the same as in GR. Using the PPN language, we show that GR and Palatini gravity share the same PPN parameters. The only difference is in the conserved mass-energy function, where Palatini theories contain a correction term. Nonetheless, solar system tests are insensitive to this kind of distinction, leading to the conclusion that the Palatini $f(R)$ gravity cannot be constrained by the current observations made at solar system. Further details on the present work can be found in Ref. [7].

The author thanks FAPES and CNPq (Brazil) for their support trough the Profix program.

REFERENCES

- [1] W.M. Clifford, «Theory and experiment in gravitational physics. 2nd Ed.», *Cambridge University Press*, Cambridge 1993.
- [2] J.D. Toniato, D.C. Rodrigues, Á.O.F. de Almeida, N. Bertini, *Phys. Rev. D* **96**, 064034 (2017), [arXiv:1706.09032 \[gr-qc\]](#).
- [3] T.P. Sotiriou, V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010), [arXiv:0805.1726 \[gr-qc\]](#).
- [4] G.J. Olmo, *Phys. Rev. D* **72**, 083505 (2005), [arXiv:gr-qc/0505135](#).
- [5] E. Poisson, C.M. Will, «Gravity: Newtonian, post-Newtonian, relativistic», *Cambridge University Press*, Cambridge 2014.
- [6] D.L. Lee, A.P. Lightman, W.T. Ni, *Phys. Rev. D* **10**, 1685 (1974).
- [7] J.D. Toniato, D.C. Rodrigues, A. Wojnar, *Phys. Rev. D* **101**, 064050 (2020), [arXiv:1912.12234 \[gr-qc\]](#).