QUANTUM BIG BOUNCE SCENARIO
AND PRIMORDIAL GRAVITATIONAL WAVES*

Artur Miroszewski

National Centre for Nuclear Research, 00-681 Warszawa, Poland
artur.miroszewski@ncbj.gov.pl

The evolution of the quantum Friedmann–Lemaitre–Robertson–Walker model filled with radiation is studied. From the coherent states quantisation procedure, the non-singular behaviour of the very early universe is obtained. The model is expanded to the first order in tensor perturbations. Quantum perturbations evolving on a quantum spacetime produce primordial gravitational waves via parametric amplification mechanism. The spectrum of the primordial gravitational waves is presented.

1. Introduction

Here, we present a method to quantise and solve dynamics of gravitational waves in a quantum Friedmann–Lemaitre–Robertson–Walker (FLRW) spacetime filled with radiation. The classical model is formulated in the ADM formalism. Matter is introduced to the model and serves as a physical clock. The system is de-parametrised and a reduced phase space is found. With the use of a phase-space symmetry respecting quantisation map, the perturbed quantum FLRW cosmology is obtained. As a result of such a procedure, the initial singularity is replaced with a quantum bounce, which can act as a mechanism for generation of primordial gravitational waves. The obtained spectrum of gravitational waves indicates the possibility of observing the properties of the Quantum Big Bounce scenario.

The model setup is based on paper [1] and the details can be found there. We choose to use the convention of natural units $c = 1$, $\hbar = 1$.

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2. The classical system

As a background spacetime, we will use the flat FLRW model with radiation introduced with canonical formalism [2]. We introduce the radiation field as an internal clock and de-parametrise the theory (for details, see [1]). In the reduced formalism, one can find the true Hamiltonian which describes the evolution with respect to the radiation field and expand it to second order in perturbation variables. Utilising the result of the SVT decomposition, we set the scalar and vector perturbations to zero and analyse only the transverse, traceless tensor perturbations, also called gravitational waves. We obtain the following true Hamiltonian:

$$H = 2\kappa p^2 - \sum_{\lambda=+,\times} \sum_{\vec{k}} \left(A(q)\check{\pi}_\lambda^2\left(\vec{k}\right) + B(q)\check{w}_\lambda^2\left(\vec{k}\right) \right), \tag{1}$$

where p is a conjugate variable to $q = \gamma a$, $\gamma = 2\sqrt{6}$, and $A(q) = \frac{2\kappa\gamma^2}{q^2}$, $B(q) = \frac{k^2 q^2}{8\kappa\gamma^2}$.

3. The quantum system

We would like to quantise the classical spacetime developed in the previous section. Being aware of the fact that the $q \leq 0$ describes an unphysical universe with negative scale factor, we explicitly write down a domain of the phase space of the classical theory: $(q,p) \in \mathbb{R}_+ \times \mathbb{R}$, $(\check{w}_\lambda, \check{\pi}_\lambda) \in \mathbb{R} \times \mathbb{R}$. One can immediately see that the background phase-space variables cannot be quantised canonically as the Hamiltonian would not be a self-adjoint operator. To quantise the background degrees of freedom, we have to use a phase-space covariant quantisation map. We choose to employ coherent states quantisation method for this task [3] as it can map the phase space defined on the half plane to a Hilbert space on the half line $\chi = \mathbb{R}_+ \times \mathbb{R} \mapsto \mathcal{H} = L^2(\mathbb{R}_+, \mathrm{d}x)$. The perturbations $(\check{w}, \check{\pi})$ will be quantised with the use of canonical quantisation procedure.

The quantised Hamiltonian $\hat{H}$ reads

$$\hat{H} = 2\kappa \left(\hat{P}^2 + \frac{\beta}{\hat{Q}^2} \right) - \sum_{\lambda=+,\times} \sum_{\vec{k}} \left(A\left(\hat{Q}\right) \check{\pi}_\lambda^2\left(\vec{k}\right) + B\left(\hat{Q}\right) \hat{w}_\lambda^2\left(\vec{k}\right) \right), \tag{2}$$

where the additional repulsive potential term originates from the quantisation method and assures that for $\beta \geq 3/4$ the quantum Hamiltonian is essentially self-adjoint.

We decide to solve Hamiltonian (2) order by order. That means we constrain ourselves to find the dynamics with respect to the term in the first bracket of (2). The quantisation induces a singularity avoidance scenario.

First, the universe contracts and at some point the motion turns around, and from this moment the universe is expanding. This behaviour is called a Quantum Big Bounce. One can easily obtain conformal time evolution of position squared of such the system ($\kappa = 1/2$)

$$\hat{Q}^2 = 4\hat{H}(\eta - \eta_0)^2 + \hat{D}_0(\eta - \eta_0) + \hat{Q}_0^2 \,, \tag{3}$$

where the dilation operator is $\hat{D} = \frac{1}{2}(\hat{Q}\hat{P} + \hat{P}\hat{Q})$.

Solving the dynamics order by order meaning we do not take into account backreaction of the perturbations on the background. We integrate out background dynamics in the coupling operators: $A(\hat{Q}) \mapsto \langle A(\hat{Q})\rangle$ and $B(\hat{Q}) \mapsto \langle B(\hat{Q})\rangle$. Then we switch from a natural parametrisation presented in (2) to the Mukhanov–Sasaki parametrisation [4] obtaining the equation of motion for each polarisation and mode of tensor perturbations

$$\hat{v}_k'' + \left(\underbrace{\left\langle \hat{Q}^2 \right\rangle \left\langle \hat{Q}^{-2} \right\rangle}_{c_g^2} k^2 - \underbrace{\frac{\left[\left\langle \hat{Q}^{-2} \right\rangle^{-1/2} \right]''}{\left\langle \hat{Q}^{-2} \right\rangle^{-1/2}}}_{V} \right) \hat{v}_k = 0 \,. \tag{4}$$

For $c_g(\eta) = $ const and $V(\eta) = 0$, we obtain the standard wave equation for perturbations. The c_g parameter governs a speed of propagation of gravitational waves. One can set initial values in Eq. (3) such that $\lim_{\eta \to \pm\infty} c_g(\eta) = 1$. Remembering the chosen convention for units ($c = 1$), one can see that in the classical limit (late universe), the speed of gravitational waves coincides with the speed of light. It also does not depend on the polarization or mode. For $c_g = $ const, one can interpret equation (4) as a Schrödinger equation in the time domain for scattering of a particle of energy $E = c_g^2 k^2$ on the potential V

$$-\hat{v}_k'' + V\hat{v}_k = E\hat{v}_k \,. \tag{5}$$

The expectation value of a function of operators is not the same as a function of expectation values of operators, hence the potential is not equal to the usual term in the Mukhanov–Sasaki equation $V(\eta) \neq \frac{a''}{a}$.

Numerical simulations of the perturbation modes, which are assumed to be in a Minkowski vacuum long before the bounce produce the spectrum of perturbations are presented in Fig. 1. The small k modes become super-horizon and get amplified during the bounce, while large k modes do not feel the scattering potential at all. In the intermediate region, the bump in the spectrum indicates a quantum origin of primordial gravitational waves production.
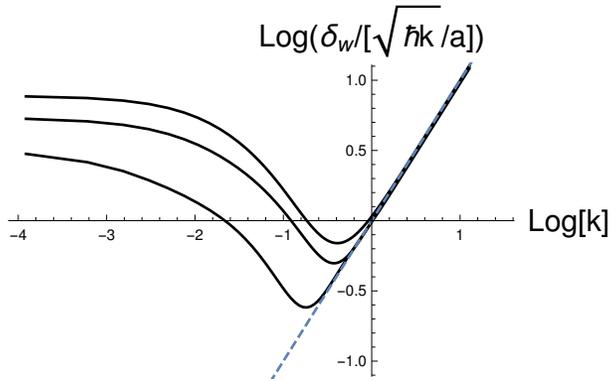
Fig. 1. Primordial gravitational waves spectra for different initial conditions. Long waves become superhorizon during the bounce, hence they get amplified during the Quantum Big Bounce scenario. The dashed, blue line indicates the Minkowski vacuum of the perturbation modes.

4. Conclusions

A phase-space covariant quantisation method of the FLRW cosmological background filled with radiation results in non-singular evolution of the universe. Using a linear approach to cosmological perturbations, one obtains a non-trivial coupling between the perturbations and the background. The approximate (no backreaction) analysis of the dynamics of perturbations leads to the conclusion that the Quantum Big Bounce scenario amplifies high k modes of the tensor perturbations, resulting in the production of primordial gravitational waves. The observation of primordial gravitational waves with the spectrum shown in Fig. 1 would indicate the existence of quantum effects in the very early universe.

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