

EXOTIC SYMMETRY EFFECTS IN LIGHT $Z = N$ NUCLEI NEAR $^{80}\text{Zr}^*$

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In this article, we discuss the effects of the shape instability against the first order tetrahedral-symmetry nuclear shape deformation $t_1 \equiv \alpha_{32}$ for the $Z = N$ nuclei in the vicinity of $Z = 40$ using a deformed Woods–Saxon realistic mean-field Hamiltonian. We specifically focus on the effects of the tetrahedral deformation in its formally leading order, t_1 , since the recent discovery of the experimental evidence of the corresponding symmetry in the ^{152}Sm nucleus opens the new perspectives in experimental identification of the corresponding exotic nuclear configurations by proposing explicit unprecedented techniques for such applications.

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1. Introduction

Tetrahedral and octahedral symmetry configurations in atomic nuclei have attracted attention in the literature from both theoretical and experimental viewpoints during several years. The fact that nuclei presenting such symmetries may produce new spectroscopic features keeps this topic attractive especially from the viewpoint of identifying their presence. Recently, the first experimental evidence for the presence of a coexistence between tetrahedral and octahedral symmetries has been announced in Ref. [1]. A partial

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overview of the past publications on the subject can be found in Ref. [2]. In the present work, we wish to discuss some predictions related to these exotic shapes possibly present in the ground states of certain $N = Z$ nuclei in the vicinity of ${}^{80}_{40}\text{Zr}_{40}$.

Tetrahedral and octahedral symmetry studies focussing on the nuclear structure employ the combined methods of group and nuclear mean-field theories. In the context of the nuclear mean-field approach focusing on the nucleonic (thus fermion) degrees of freedom, the corresponding symmetry point groups are the so-called *double* tetrahedral T_d^D and octahedral O_h^D groups. Both of these symmetries generate four-fold degeneracies of the single-particle levels differing from the “traditionally” discussed $(2j + 1)$ -fold degeneracies for the spherical nuclei or 2-fold (Kramers or time-up, time-down) degeneracies for the non-spherical ones. These degeneracies reflect the mathematical properties of the structure of the irreducible representations of the corresponding double point groups. For instance, the T_d^D group has two 2-dimensional and one 4-dimensional irreducible representations implying that the ensemble of single-particle energies forms two families of two-fold and one family of four-fold degenerate levels.

The presence of four-fold degeneracies impacts the average density of the single-particle energies. In particular, the numbers of levels carrying the same number of nucleons in the presence (absence) of such symmetries will be, on average, lower (higher). Given the fact that the depth of the average potential well is to an approximation constant (it depends on the particle numbers only very weakly) implies that the single-nucleon spectra in the presence (absence) of tetrahedral and/or octahedral symmetries may produce on average bigger (smaller) single-particle shell gaps. Bigger gaps at certain nucleon numbers imply deeper tetrahedral (octahedral) total energy minima for the nucleon numbers at their vicinities and favour the exotic symmetry effects.

The choice of nuclei for the present studies around zirconium is, of course, not accidental. The previous studies in Ref. [3] already revealed tetrahedral magic gaps at the proton and neutron numbers at $Z/N = 32, 40, 56, 64, 70, 90, 112$ and 136 . On the other hand, studies of $N = Z$ nuclei with $A \sim 60\text{--}80$ have been carried out in the past in Ref. [4] using a Skyrme HF interaction. In the present article, we wish to examine the possible presence of the tetrahedral-symmetry effects using the realistic phenomenological deformed mean-field Hamiltonian in its Woods–Saxon realisation for the nucleon numbers close to the lightest tetrahedral magic gaps configurations.

2. Collective features and the issue of symmetry-identification

It is well-known that an arbitrary nuclear surface, Σ , can be described with the help of spherical harmonic basis $\{Y_{\lambda\mu}(\theta, \phi)\}$

$$\Sigma: R(\theta, \phi) \sim R_0 \left[1 + \sum_{\lambda} \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right], \quad (1)$$

where $\{\alpha_{\lambda\mu}\}$ represents the full set of deformations considered and $R(\theta, \phi)$ denotes the distance from the origin of the reference frame to the actual point on the surface. Let us consider the nuclear multipole moments defined by

$$Q_{\lambda\mu} = \int \rho_{\Sigma}(\vec{r}) r^{\lambda} Y_{\lambda\mu}(\theta, \phi) d\vec{r}, \quad (2)$$

where $\rho_{\Sigma}(\vec{r})$ is the nuclear density associated with the nucleons inside the surface Σ . It turns out that when calculating the quadrupole moments as functions of octupole deformations $\{\alpha_{3\mu}\}$ with the surfaces symmetric under the octahedral- and/or tetrahedral-symmetry groups, the implied quadrupole moments vanish and it follows that the $B(E2)$ transitions vanish as well. Consequently, the atomic nuclei in the high-rank symmetry states must not generate the collective E2 (neither E1 as one can show independently) transitions and it is expected that they may become isomeric what could greatly facilitate their experimental identification with an appropriately chosen instrumentation.

Another consequence of high-rank symmetries is that the implied rotational bands are composed of specific spin-parity sequences involving degenerate multiplets (doublets, triplets, ...) rather than the bands very well known from the nuclear physics literature. For instance: for the tetrahedral-symmetric even-even nuclei, the rotor Hamiltonian of the corresponding nucleus is invariant under the T_d -group which has 5 irreducible representations. The one containing the $I^{\pi} = 0^{+}$ ground state is usually denoted A_1 in the literature, *cf. e.g.* Refs. [1, 5] and references therein, and it turns out that the corresponding structure of the tetrahedral ground-state band of an even-even nucleus is given by the following sequence:

$$A_1: 0^{+}, 3^{-}, 4^{+}, (6^{+}, 6^{-}), 7^{-}, 8^{+}, (9^{+}, 9^{-}), (10^{+}, 10^{-}), 11^{-}, (2 \times 12^{+}, 12^{-}), \dots \quad (3)$$

Let us emphasise the presence of the degenerate spin doublets at $I = 6$, 9, 10 or triplets at $I = 12$; higher degeneracy is predicted for increasing spins. Such degeneracies are very characteristic spectral elements potentially facilitating the experimental identification of these structures. The main characteristic of these states is that they form approximately a common

parabola, with the usual energy dependence in the form of $E \sim I(I + 1)$ Refs. [5–8]. All these properties can be used nowadays to formulate the unique experimental identification criteria following their first successful application in the discovery article Ref. [1] for ^{152}Sm .

3. Total nuclear energies: Results and predictions

Deformed mean-field Hamiltonian selected for the calculations discussed in the present article is based on the phenomenological deformed Woods-Saxon (WS) potential, Ref. [9]. Our selection was justified by the fact that the corresponding Hamiltonian has significant prediction capacities as manifested by numerous applications over many years by other authors Ref. [10]. The definitions of the tetrahedral and octahedral deformations have become standard by now, *cf. e.g.* Ref. [11] and the reader is referred to the original articles. The mean-field Schrödinger equation has been solved employing among others the tetrahedral and octahedral deformation spaces. An illustration taking into account the properties previously detailed in Section 1 about the double tetrahedral group and the related specific nucleon-level degeneracies is presented in Fig. 1. The figure shows specifically the neutron

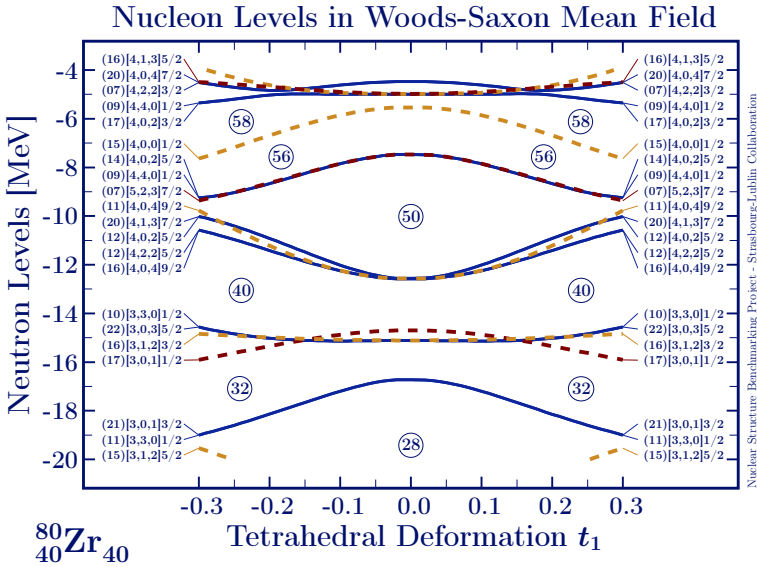


Fig. 1. (Colour on-line) Neutron single-particle energy spectrum for ^{80}Zr as a function of tetrahedral deformation $t_1 \equiv \alpha_{32}$ obtained with the Woods-Saxon mean-field potential of Ref. [9]. Solid lines represent 4-fold degenerate levels corresponding to four-dimensional irreducible representations of T_d^D group, whereas the dashed lines represent the two 2-fold representations, respectively. Significant gaps approaching 3 MeV opening at $N = 32, 40$ and $56/58$ deserve noticing.

single-particle energies for ^{80}Zr nucleus presented as functions of tetrahedral deformation $t_1 \equiv \alpha_{32}$. The presence of characteristic gap openings at $N = 32, 40, 56/58$ deserves noticing. Similar properties are valid for the protons (not shown). Guided by these results, we have studied the tetrahedral-gap influence on the $N = Z$ nuclei within $Z \in [28, 50]$. We have performed the total nuclear energy calculations using the standard Strutinsky approach with the phenomenological Woods–Saxon mean-field Hamiltonian of Ref. [9] in multidimensional deformation spaces including the quadrupole degrees of freedom α_{20} and α_{22} together with the hexadecapole one, α_{40} , octahedral one, α_1 , of Ref. [11], and various combinations involving octupole degrees of freedom $\alpha_{3\mu}$ (recall that α_{32} represents the lowest order tetrahedral symmetry deformation referred to as t_1). The nuclear energies projected on the quadrupole, α_{20} , and tetrahedral, $\alpha_{32} = t_1$, deformation plane are given in Fig. 2 showing the results minimised over octahedral deformation parameter α_1 , Ref. [11], for a number of selected $Z = N$ nuclei. The specific simultaneous combination of deformations t_1 and α_1 in this type of calculations is particularly relevant given the fact that tetrahedral group is a sub-group of the octahedral one, $T_d \subset O_h$.

The results for ^{80}Zr deserve particular attention in the present context since the corresponding ground-state minimum is predicted to have a strongly pronounced tetrahedral-symmetry configuration at $a_{32} \sim \pm 0.20$. This result reflects the presence of significant single-particle gaps which at increasing tetrahedral deformation approaches 4 MeV limit, as can be seen from Fig. 1. Competing quadrupole shape minimum at “super-deformation” of the order of $a_{20} \sim 0.40$ appears significantly higher in the energy scale, *cf.* also Ref. [12]. The low-lying, slightly excited tetrahedral minimum is predicted for the neighbouring ^{84}Mo nucleus in which the barriers surrounding the tetrahedral-symmetry minima, slightly in excess of 1 MeV, might be sufficiently high to generate an excited stable tetrahedral configuration. The lightest nucleus considered here, ^{64}Ge , is predicted to manifest a pronounced tetrahedral-symmetry instability, but the total-energy barriers separating the tetrahedral-symmetry minima from the prolate and oblate minima are most likely too low to provide the stable *e.g.* isomeric configurations. A similar scenario is likely for ^{68}Se in which the tetrahedral susceptibility is calculated to be even weaker.

Yet another scenario is predicted for ^{72}Kr and ^{76}Sr in which the ground-state configurations are calculated to be quadrupole-oblate, and prolate, respectively, *cf.* also Ref. [13]. In both nuclei mentioned, there exist stable $\alpha_{32} \neq 0$ excited configurations at $\alpha_{20} \neq 0$. Such minima do not produce any direct tetrahedral-symmetry mechanism, however, they should give rise to the octupole-type rotational bands with $K^\pi = 2^-$ accompanied by the

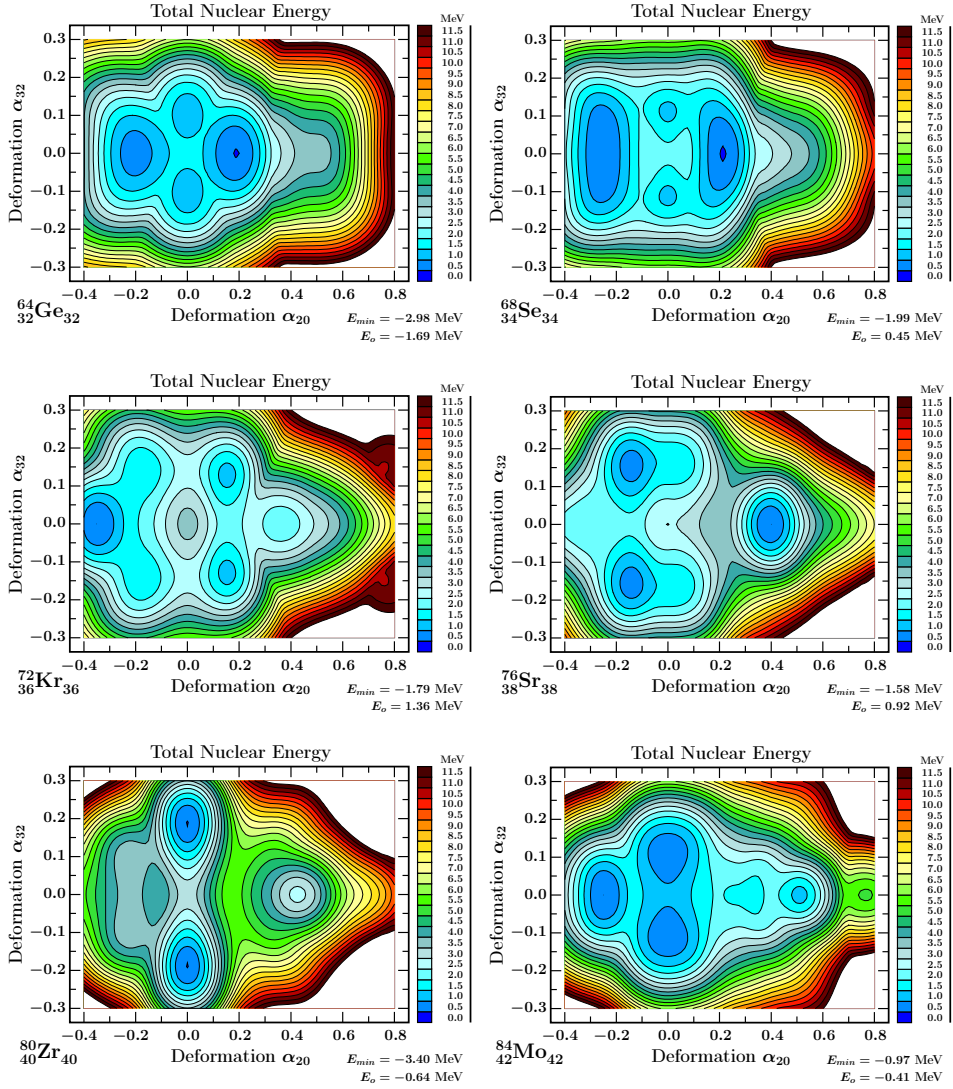


Fig. 2. (Colour on-line) Total nuclear energy surfaces for listed nuclei projected onto the $(\alpha_{20}, t_1 \equiv \alpha_{32})$ deformation plane and minimised over first order octahedral deformation o_1 whose definition was introduced in Ref. [11]. Except for ^{80}Zr , the ground states are predicted at some non-zero quadrupole deformations. However, prediction of a rich shape coexistence deserves noticing. It involves the prolate-oblate shape coexistence and competition, together with the competition with the tetrahedral-symmetry minima at $\alpha_{32} \neq 0$.

parity-doublet structures. The effects of this type render themselves much easier accessible via experiment as compared to the tetrahedral-symmetry configurations whose electromagnetic decay is strongly hindered.

4. Summary and conclusions

In the present article, we discuss the predicted shape coexistence and shape evolution in the nuclear ground states of $Z = N$ nuclei of Ge, Se, Kr, Sr, Zr and Mo. The discussed nuclear sequence has been selected because of its rich shape coexistence and shape evolution properties involving the tetrahedral-symmetry generating deformation $\alpha_{32} \equiv t_1$. In particular, we confirm earlier predictions pointing out that the shape of ^{80}Zr in its ground state is characterised by the well-pronounced tetrahedral symmetry. The neighbouring ^{84}Mo nucleus also manifests the presence of the tetrahedral-symmetry minima but at the energy comparable with the energy of the oblate-symmetry ground state. In ^{64}Ge , the tetrahedral symmetry minima are predicted at about 500 keV above the prolate ground-state minimum but the surrounding potential barriers of the order of 500 keV are most likely too low to lead to a stable minimum with measurable effects.

Except for ^{80}Zr , all the considered nuclei present the prolate–oblate shape coexistence with the prolate-deformed ground states for ^{64}Ge and ^{68}Se at $\alpha_{20} \approx 0.19$ and $\alpha_{20} \approx 0.21$, respectively, then super-oblate equilibrium for ^{72}Kr at $\alpha_{20} \approx -0.4$ which requires special attention for its exoticity due to an unusually strong “flatness” of the shape, followed by the super-prolate shape minimum for ^{76}Sr at $\alpha_{20} \approx +0.39$. Such super-deformed minima are also predicted for ^{80}Zr and ^{84}Mo at $\alpha_{20} \approx 0.41$ and $\alpha_{20} \approx 0.50$, respectively.

Finally, the ^{72}Kr and ^{76}Sr nuclei are predicted to produce the negative parity (exotic octupole) bands of the $K^\pi = 2^-$ band-head structure, the former nucleus with the slightly prolate quadrupole deformation superposed with pronounced component of $\alpha_{32} \approx \pm 0.15$ and the latter with the very exotic shape configuration composed of oblate-shape component at $\alpha_{20} \approx -0.18$, superposed with the strongly pronounced component of $\alpha_{32} \approx \pm 0.16$.

In our opinion, the majority of these predictions represent exoticities which seem very attractive due to the relative seldomness of the underlying mechanisms and can be tested experimentally either with the γ -multi-detector systems or with the mass spectrometry device like *e.g.* Fragment Separator (FRS) at the GSI, Darmstadt, Germany.

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REFERENCES

- [1] J. Dudek *et al.*, *Phys. Rev. C* **97**, 021302(R) (2018).
- [2] J. Dudek *et al.*, *Acta Phys. Pol. B* **50**, 685 (2019).
- [3] J. Dudek, A. Góźdz, N. Schunck, M. Miśkiewicz, *Phys. Rev. Lett.* **88**, 252502 (2002).
- [4] S. Takami, K. Yabana, M. Matsuo, *Phys. Lett. B* **431**, 242 (1998).
- [5] S. Tagami, Y.R. Shimizu, J. Dudek, *Phys. Rev. C* **87**, 054306 (2013).
- [6] S. Tagami, Y.R. Shimizu, J. Dudek, *Prog. Theor. Phys. Suppl.* **196**, 334 (2012).
- [7] S. Tagami, Y.R. Shimizu, J. Dudek, *J. Phys. G: Nucl. Part. Phys.* **42**, 015106 (2015).
- [8] S. Tagami, Y.R. Shimizu, J. Dudek, *Phys. Rev. C* **98**, 024304 (2018).
- [9] The *Universal Woods–Saxon Hamiltonian* and associated, the so-called “universal parametrization” has been developed in a series of articles: J. Dudek, T. Werner, *J. Phys. G: Nucl. Phys.* **4**, 1543 (1978); J. Dudek *et al.*, *J. Phys. G: Nucl. Phys.* **5**, 1359 (1979); J. Dudek, W. Nazarewicz, T. Werner, *Nucl. Phys. A* **341**, 253 (1980); J. Dudek, Z. Szymański, T. Werner, *Phys. Rev. C* **23**, 920 (1981); and has been summarized in: S. Ćwiok *et al.*, *Comput. Phys. Commun.* **46**, 379 (1987). This approach is being used without modifications by many authors also today.
- [10] Illustration of the use frequency of the phenomenological Woods–Saxon Hamiltonian with its “universal” parametrisation: the articles which appeared only in 2013 and only in one journal — *The Physical Review C*: J. Rissanen *et al.*, *Phys. Rev. C* **88**, 044313 (2013); W. Brodziński *et al.*, *Phys. Rev. C* **88**, 044307 (2013); S. Lalkowski *et al.*, *Phys. Rev. C* **88**, 024302 (2013); H.L. Liu *et al.*, *Phys. Rev. C* **87**, 067304 (2013); D.S. Delion *et al.*, *Phys. Rev. C* **87**, 041302 (2013); D.S. Delion *et al.*, *Phys. Rev. C* **87**, 034328 (2013); D.S. Delion *et al.*, *Phys. Rev. C* **87**, 024309 (2013); D. Deleanu *et al.*, *Phys. Rev. C* **87**, 014329 (2013); S. Takahara *et al.*, *Phys. Rev. C* **86**, 064323 (2012).
- [11] J. Dudek *et al.*, *Int. J. Mod. Phys. E* **16**, 516 (2007).
- [12] C.J. Lister *et al.*, *Phys. Rev. Lett.* **59**, 1270 (1987).
- [13] E. Ha, M.-K. Cheoun, H. Sagawa, *RIKEN Accel. Prog. Rep.* **52** (2019).