TEMPORAL SELF-INTERFERENCE OF A RELATIVISTIC PARTICLE*

Andrzej Góźdź, Marek Góźdź

Faculty of Mathematics, Physics and Computer Science
Maria Curie-Skłodowska University, Lublin, Poland

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We discuss the problem of time in quantum systems and stress the need of introducing it as an observable and not just a numerical parameter. This approach allows to redefine the quantum evolution in the form of a series of projections onto the spaces of new states. Within this framework, we describe the temporal version of the double-slit experiment in which a particle interferes with itself from a different instance of time.

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1. Introduction

In the traditional formulation of quantum mechanics, time is introduced as a parameter which numbers the subsequent changes of the quantum state. The time parameter is unbound and continuous. Since the energy may be bounded from below and discrete, Pauli pointed out that it is impossible to construct a time operator canonically conjugated to the Hamiltonian [1]. This observation was based on the assumption that all observables have to be represented by Hermitian operators. In the following years, an extensive experimental study of quantum systems was performed. It revealed many non-classical features connected with the entanglement [2] and the delayed choice setups [3]. It has also been observed that quantum states depend on time in a similar manner as they depend on the spatial variables [4], including a possible interference in time [5]. Theoretical description of these phenomena is inconsistent within the traditional quantum mechanics.

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Recent theoretical studies showed that the Hermitian operator is just a special case of a wider class, called the positive operator valued measure (POVM) and that POVMs can also represent measurable quantities. It turned out that one may define the time operator in the form of a POVM [6], a fact that opened the possibility to discuss the evolution of quantum systems in a consistent way.

In this paper, we present the evolution scheme of a quantum system based on the projection evolution principle. As an example, we discuss the temporal version of the double-slit experiment, in which a particle interferes with itself from a different time instance, as was observed in the experiment reported in Ref. [5].

2. The projection evolution principle

Whenever we mention time, even in the context of quantum systems, we mean the macroscopic laboratory time. The experiments (see [4, 5] and references therein) suggest that this time is different from the "quantum time" in the microscopic scale. From the theoretical point of view, this means that the time-dependent unitary Schrödinger evolution of the wave function, given by the $\exp(-iHt)$ factor, is just a semi-quantal approximation.

To construct a more general rule, it is convenient to have a parameter which allows to number the subsequent steps of the evolution. We cannot use time, so let us introduce a parameter τ, for which the only requirement is that it belongs to an ordered set. It is not an observable and the final physical result will not depend on it. As this parameter is not connected with the macroscopic chronology of events, it cannot be directly mapped onto the macroscopic time.

A quantum state evolves, if a change of the state can be observed. We account for that fact, postulating that the new state is obtained by projecting the old one onto the space of all possible new states. The existence of such projection operators, which act between state spaces, has been proven in Refs. [7, 8]. If more than one new state is possible, the system chooses one of them according to certain probability distribution.

We write the density matrix at the evolution step τ_n as a normalized projection of the state at the evolution step τ_{n-1}

$$\rho(\tau_n, \nu_n) = \frac{\mathbb{E}_n \rho(\tau_{n-1}, \nu_{n-1}) \mathbb{E}_n^\dagger}{\mathrm{Tr}\left[\mathbb{E}_n \rho(\tau_{n-1}, \nu_{n-1}) \mathbb{E}_n^\dagger\right]}, \tag{1}$$

where ν_n are the quantum numbers, $\mathbb{E}_n$ is the operator which projects the state $\rho(\tau_{n-1}, \nu_{n-1})$ onto the space of states labeled τ_n, and the denominator gives the proper normalization. The evolution $\tau_0 \to \tau_1 \to \cdots \to \tau_n$ starting from the initial state $\rho(\tau_0, \nu_0)$ takes the form of subsequent projections

$$\rho(\tau_n, \nu_n) = \frac{\mathbb{P}_n \mathbb{P}_{n-1} \dots \mathbb{P}_1 \rho(\tau_0, \nu_0) \mathbb{P}_1^\dagger \dots \mathbb{P}_n^\dagger}{\mathrm{Tr}\left[\mathbb{P}_n \mathbb{P}_{n-1} \dots \mathbb{P}_1 \rho(\tau_0, \nu_0) \mathbb{P}_1^\dagger \dots \mathbb{P}_n^\dagger\right]} \,. \tag{2}$$

A special case of a density matrix is a pure state $|\psi\rangle$, for which the corresponding density matrix is constructed in the standard way: $\rho = |\psi\rangle\langle\psi|$. For each step of the evolution τ, there is in general a whole family of the evolution operators $\mathbb{P}(\tau;\nu)$, whith ν describing uniquely the state $\rho(\tau, \nu)$.

The projection operators may have different forms, for example a special choice of the evolution generator leads to the Schrödinger evolution scheme. An alternative form is the resolution of unity, in which case one has

$$\text{(Hermitian)} \quad \mathbb{P}(\tau;\nu)^\dagger = \mathbb{P}(\tau;\nu)\,, \tag{3}$$

$$\text{(orthogonal)} \quad \mathbb{P}(\tau;\nu)\mathbb{P}(\tau;\nu') = \delta_{\nu\nu'} \mathbb{P}(\tau;\nu)\,, \tag{4}$$

$$\text{(resolution of unity)} \quad \sum_\nu \mathbb{P}(\tau;\nu) = \mathbb{I}\,, \tag{5}$$

where $\mathbb{I}$ is the unit operator.

The main consequence of the projection evolution approach is that on the quantum level, the evolution is not ordered by time t but by the parameter τ. The density matrix ρ at each step of the evolution depends on the spacetime coordinates. It means that the quantum object is always described in the whole space and at all times. Therefore, the notions of *past*, *present*, and *future* are not directly applicable, because each change of the state affects it in the whole spacetime, including to certain extend its past and future, as seen from the laboratory frame of reference. The connection of the quantum system to the laboratory time t can be established by interactions of the object with the macroscopic environment. Between the measurements, the system evolves non-classically, but each interaction with the environment changes the state and may be interpreted as a tick of a classical clock.

Another consequence is that zero-time events are not physical, so every event should have some "width" Δt along the time axis. It follows that for a time operator which is canonically conjugated to the temporal component of the four-momentum, the lower bound on the product $(\Delta t)(\Delta p_0)$ can be established. The uncertainty relation between the energy and time can then be formulated by using the equations of motion.

3. The temporal interference

In the project reported in Ref. [5], a modified version of the double-slit experiment was conducted. A single photon source was used and the detector registered the energy spectrum of the particles. A rotating wheel with slits was placed between the source and the detector. The speed of the rotation was adjusted in such a way that the photon could have passed at least two different slits in the wheel. The detector registered an energy modulation in the form of an interference pattern. The most natural interpretation of this observation is that the photon was in a superposition of states, corresponding to passing different slits in different times, and that these states interfered. This interpretation requires, however, the extension of the traditional formalism to include time as an observable.

Let us build a description of a similar process on the example of a relativistic spin-zero particle. The rotating wheel is represented by a spacetime region in which a single slit opens twice. When the slit is closed, the particle is blocked and cannot reach the detector.

In the projection evolution, we have to start from the initial state of the particle and construct operators which describe the passing of the slit, the free propagation to the detector, and the detection process.

The emission of the particle does not happen in zero-time, which introduces some spread in its energy. We assume that the emitted mass m_0 is distributed around some mean value $\bar{m}_0$

$$m_0 \in \mathit{\Delta}_{\bar{m}_0} = \left[\bar{m}_0 - \frac{\mathit{\Gamma}}{2}, \bar{m}_0 + \frac{\mathit{\Gamma}}{2}\right] , \tag{6}$$

where the mass spread $\mathit{\Gamma}$ comes from the duration of the emission process and the particle's half-life. It follows that the particle's four-momentum must belong to the set $B_{\bar{m}_0}$,

$$k \in B_{\bar{m}_0} \Leftrightarrow \left(\bar{m}_0 - \frac{\mathit{\Gamma}}{2}\right)^2 \leq k^2 \leq \left(\bar{m}_0 + \frac{\mathit{\Gamma}}{2}\right)^2 , \tag{7}$$

with some distribution $a(k)$. The simplest profile $a(k)$ is rectangular, with equal distribution of the momenta within the set $B_{\bar{m}_0}$, but other choices are also possible. The initial state of the particle is given by

$$|\psi_0\rangle = \int\limits_{B_{\bar{m}_0}} \mathrm{d}^4k \; a(k)|k\rangle \, , \tag{8}$$

where $\langle x|k\rangle = \exp(-ik_\mu x^\mu)/(4\pi^2)$. This state is unnormalized. In the following, we skip the normalization, as it does not change qualitatively our final result, but in principle a proper normalization should be done after each step of the evolution.

Let us denote by $\Delta_{\mathrm{T}} = \Delta_1 \cup \Delta_2$ the spacetime intervals which represent the opened slit. The spatial parts of $\Delta_{1,2}$ are the same, but the temporal parts differ. The passing of the slits is given by the evolution operator

$$\mathcal{E}_{\mathrm{S}}(\tau_1) = \int\limits_{\Delta_{\mathrm{T}}} \mathrm{d}^4x \; |x\rangle\langle x| \,, \tag{9}$$

which projects the state onto the intervals $\Delta_{1,2}$. At this stage of the evolution, the state of the particle contains two parts, one projected onto Δ_1 and another projected onto Δ_2.

The free propagation to the detector is just the Klein–Gordon condition, *i.e.*, we have to project the state onto correct four-momentum using the operator

$$\mathcal{E}_{\mathrm{F}}(\tau_2) = \int\limits_{B_{\bar{m}_0}} \mathrm{d}^4k' \; |k'\rangle\langle k'| \,. \tag{10}$$

Finally, the detector measures the four-momentum κ of the incoming particle, so the last evolution operator takes the form of

$$\mathcal{E}_{\mathrm{D}}(\tau_3) = |\kappa\rangle\langle\kappa| \,. \tag{11}$$

Taking all this into account, the evolution of the initial state is given by

$$\begin{aligned} &\mathcal{E}_{\mathrm{D}}(\tau_3)\mathcal{E}_{\mathrm{F}}(\tau_2)\mathcal{E}_{\mathrm{S}}(\tau_1)|\psi_0\rangle \\ &= |\kappa\rangle \int\limits_{B_{\bar{m}_0}} \mathrm{d}^4k' \; \delta^4(\kappa - k') \int\limits_{B_{\bar{m}_0}} \mathrm{d}^4k \; a(k) \int\limits_{\Delta_{\mathrm{T}}} \mathrm{d}^4x \; \langle\kappa|x\rangle\langle x|k\rangle \,. \end{aligned} \tag{12}$$

Due to the fact that $\Delta_{\mathrm{T}} = \Delta_1 \cup \Delta_2$, the integration over the spacetime coordinates is

$$\int\limits_{\Delta_{\mathrm{T}}} \mathrm{d}^4x \; \langle\kappa|x\rangle\langle x|k\rangle = \int\limits_{\Delta_1} \mathrm{d}^4x \; \langle\kappa|x\rangle\langle x|k\rangle + \int\limits_{\Delta_2} \mathrm{d}^4x \; \langle\kappa|x\rangle\langle x|k\rangle \,. \tag{13}$$

The probability that the detector registers a particle with the momentum κ is given by $||\mathcal{E}_{\mathrm{D}}(\tau_3)\mathcal{E}_{\mathrm{F}}(\tau_2)\mathcal{E}_{\mathrm{S}}(\tau_1)|\psi_0\rangle||^2$, which generates the temporal interference term between the Δ_1 and the Δ_2 parts.

4. Conclusions

Time on the quantum level is not the same as the macroscopic time. This triggers the need of a proper extension of the traditional formulation of quantum mechanics. We have presented the projection evolution approach in which the state at the previous step of the evolution is projected onto the space of all possible new states. This mechanism is capable of recreating the Schrödinger picture but is not limited to this particular choice only — the discussed above resolution of unity being another example. In the projection evolution approach, one treats the microscopic time as an observable and a coordinate, for which a Hermitian operator may be constructed.

We have presented a theoretical description of the temporal version of the double-slit experiment. The derived formula contains an interference term for the time coordinate, which explains in a consistent way the outcome of the experiment reported in Ref. [5]. This description falls outside the traditional formulation of the quantum theory in which it is impossible to discuss time as a physical observable.

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