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LEVEL DENSITY PARAMETERS OF HEAVIEST NUCLEI*

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The nuclear level densities of superheavy nuclei at the ground state and at the saddle point are calculated using the single-particle energies obtained with the Woods–Saxon potential. The level density parameters are calculated by fitting the obtained results with the Fermi gas expression. The energy and shell correction dependencies of the level density parameter at the ground state and at the saddle point are studied and compared with phenomenological expressions. The ratios of the level density parameters at the saddle point to those of the ground state are calculated.

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1. Introduction

Nuclear level density is of special importance in the cross-section calculations and nuclear structure studies. Going from low to higher energies, the nuclear system reverts from a paired system to a system of noninteracting fermions which is successfully described with the well-known Fermi gas model. In this phenomenological model, the pairing effect is taken into account with a temperature-dependent parameter Δ. In the Fermi gas model, the average value of the level density parameter, which establishes the connection between the excitation energy and the nuclear temperature, is often assumed to depend linearly on the mass number [1]. In real situation, the level density parameter is energy-dependent and gradually reaches an asymptotic value at the energies higher than neutron separation energy. A phenomenological expression has been introduced by Ignatyuk for the energy and shell correction dependencies of the level density parameter [2].

In this work, we have used the BCS model to calculate the intrinsic level densities of superheavy nuclei with $Z = 112$–120. Similar approach was used in Ref. [3]. This formalism is successful in description of nuclear quantities

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such as level density and isomeric ratio [4, 5]. The level density parameter is calculated by fitting the obtained results with the Fermi gas expression. The single-particle energies are obtained in a microscopic–macroscopic model based on the deformed single-particle Woods–Saxon potential and Yukawa-plus exponential macroscopic energy [6, 7], at the ground state and at the saddle point. The energy and shell correction dependencies of the level density parameter of superheavy nuclei at the ground state and at the saddle point are studied and compared with the Ignatyk expression. The ratio of the level density parameter at the saddle point to that of the ground state, which is an important factor in indication of the probability of fission compared to neutron emission, is calculated for the considered superheavy nuclei.

2. BCS formalism

For a nucleus with Z protons and N neutrons, the pairing gaps are obtained with the even–odd staggering in nuclear binding energies B of neighboring isotopes [8, 9]

$$\Delta_n(N,Z) = -\tfrac{1}{2}[B(N-1,Z)+B(N+1,Z)-2B(N,Z)]\,, \quad (1)$$

$$\Delta_p(N,Z) = -\tfrac{1}{2}[B(N,Z-1)+B(N,Z+1)-2B(N,Z)]\,. \quad (2)$$

Here, calculated binding energies taken from microscopic–macroscopic model [6, 7] have been used. If data were not available, the pairing gaps were taken as for the neighboring isotopes. Corresponding Fermi energies ($\lambda_{N,Z}$) and pairing strengths ($G_{N,Z}$) were calculated in the BCS approach. Using the obtained values, pairing gaps and Fermi energies were determined by solving superfluid equations at given temperatures ($T=\frac{1}{\beta}$)

$$N(Z) = \sum_k \left(1-\frac{\varepsilon_{N(Z),k}-\lambda_{N(Z)}}{E_{N(Z),k}}\tanh\frac{\beta E_{N(Z),k}}{2}\right), \quad (3)$$

$$\frac{2}{G_{N(Z)}} = \sum_k \frac{1}{E_{N(Z),k}}\tanh\frac{\beta E_{N(Z),k}}{2}\,. \quad (4)$$

The quasi-particle energies $E_k=\sqrt{(\varepsilon_k-\mu)^2+\Delta^2}$ were calculated using the single-particle energies (ε_k) extracted with the Woods–Saxon potential. The number of single-particle levels considered in our calculations are equal to Z for the proton system and N for the neutron system of each nucleus.

The calculations were repeated using the single-particle level energies obtained with the Woods–Saxon potential at the saddle point. For the saddle point, the pairing constants were taken the same as for the ground state.

After a critical temperature ($T_{\rm cr}$), the pairing gap vanishes and all thermodynamical quantities revert to those of a Fermi gas system. Generally, larger density of states close to the Fermi surface at the saddle point leads to larger pairing correlation and larger critical temperature in comparison to the ground state. In the mass region considered in our calculations, the critical temperatures averaged over neutron and proton systems are up to 0.48 MeV for the ground state and 0.6 MeV for the saddle point. The corresponding total excitation energies are $U_{\rm cr} \approx 6.23$ MeV for the ground state and $U_{\rm cr} \approx 13.44$ MeV for the saddle point.

Using obtained values of $\Delta(T)$ and $\lambda(T)$, excitation energies ($U = U_Z + U_N$), entropies ($S = S_Z + S_N$) and intrinsic level densities (ρ) are calculated as

$$E_{Z,N}(T) = \sum_k \varepsilon_k \left(1 - \frac{\varepsilon_k - \lambda_{Z,N}}{E_k} \tanh \frac{\beta E_k}{2}\right) - \frac{\Delta^2_{Z,N}}{G_{Z,N}}\,, \tag{5}$$

$$U_{Z,N}(T) = E_{Z,N}(T) - E_{Z,N}(0)\,, \tag{6}$$

$$S_{Z,N}(T) = \sum_k \left\{ \ln[1 + \exp(-\beta E_k)] + \frac{\beta E_k}{1 + \exp(\beta E_k)} \right\}, \tag{7}$$

$$\rho = \frac{\exp(S)}{(2\pi)^{\frac{3}{2}}\sqrt{D}}\,, \tag{8}$$

where D is the determinant of the matrix comprised of the second derivatives of the entropy with respect to β and $\mu = \beta\lambda$ [4, 5]. As an example, we performed calculations for ^{164}Dy for which the experimental data are

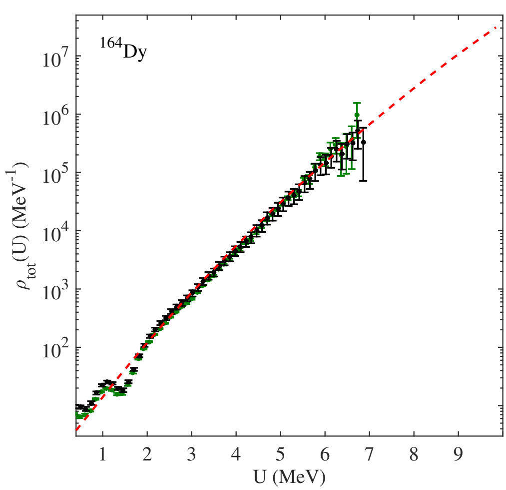

Fig. 1. (Color online) Calculated level density of ^{164}Dy as a function of excitation energy (dashed red line) is compared with the corresponding experimental data (symbols).

available [10, 11]. The results presented in Fig. 1 are in a good agreement with the experiments. The spin distribution and collective effects are taken into account in this calculation [12].

3. Level density parameters

Fitting calculated values of the intrinsic level density at given excitation energies with the Fermi gas expression

$$\rho_{\mathrm{FG}}(U) = \frac{\sqrt{\pi}}{12a^{\frac{1}{4}}U^{\frac{5}{4}}} \exp\left(2\sqrt{aU}\right) , \tag{9}$$

the level density parameter is obtained as a function of excitation energy $a(U)$. Energy and shell correction (δE_{sh}) dependencies of the level density parameter can be described by the following phenomenological expression [2]:

$$a(A,U) = \tilde{a}(A) \left[1 + \frac{1-\exp\left(-\frac{U}{E'_{\mathrm{D}}}\right)}{U} \delta E_{\mathrm{sh}}\right] , \tag{10}$$

where damping parameter E'_{D} indicates how fast the effect of shell structure fades with excitation energy. We obtained E'_{D} and the asymptotic value of the level density parameter ($\tilde{a}$) by analyzing the calculated energy-dependent level density parameter with Eq. (10) for each nucleus. The value of $\tilde{a}$ smoothly depends on the mass number and can be fitted as [2]

$$\tilde{a} = \alpha A + \beta A^2 . \tag{11}$$

Our calculations show that the expression of $a(A,U)$ in Eq. (10) gives a good agreement with the BCS calculations at the ground state, in which the values of the shell corrections are significant. As examples, in Fig. 2, comparisons of $a(A,U)$ values calculated for $^{294}115$,$^{298}116$ and $^{299}118$ isotopes by fitting of the BCS calculations with Eq. (9) and the result of Eq. (10) are displayed. As seen, a very good agreement is obtained at $U > 15$ MeV. The values of $\alpha = 0.128$ MeV^{-1} and $\beta = -1.098 \times 10^{-4}$ MeV^{-1} are found for the ground state. The damping parameter values vary between $E'_{\mathrm{D}} = 5.26$ and 20.05 MeV. However, for the saddle point with shell correction values less than $|\delta E_{\mathrm{sh}}| = 1.7$ MeV, the expression of Eq. (10) cannot describe well the calculated values of $a(A,U)$. In these cases, replacing the shell correction parameter in Eq. (10) with $(\delta E_{\mathrm{sh}} - \Delta_n - \Delta_p)$ can lead to better agreement with the results of BCS calculations.

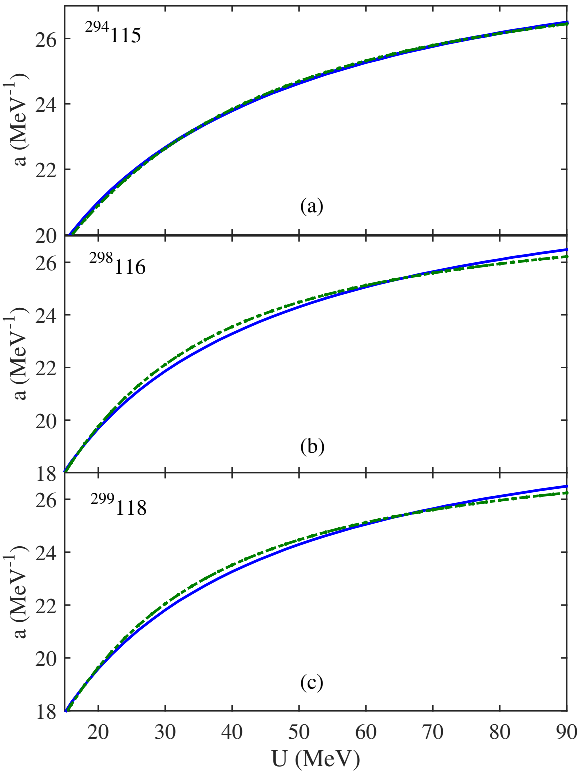

Fig. 2. (Color online) Calculated level density parameter for $^{294}115$, $^{298}116$ and $^{299}118$, at the ground state deformations (solid blue lines). The results of fit with Eq. (10) are shown by dash-dotted green lines.

In Fig. 3, the comparisons between the $a(A, U)$ values calculated by fitting of BCS calculations at the saddle point with Eq. (9) and the results of Eq. (10) are displayed for $^{294}115$, $^{298}116$ and $^{299}118$, with and without account of pairing. As seen in the figure, taking into account the pairing effect in Eq. (10) as a parameter along with the shell correction has improved the agreement. The values of $\alpha = 0.075$ MeV^{-1} and $\beta = 0.69 \times 10^{-4}$ MeV^{-1} are found for the saddle point with inclusion of the pairing effect for the isotopes with $|\delta E_{\rm sh}| < 1.7$ MeV. The obtained values of the damping parameter for the saddle point vary from $E'_{\rm D} = 0.1$ to 13.68 MeV.

In Fig. 4, the ratio of the calculated saddle-point level density parameters of $Z = 114$ isotopic chain at $U - B_{\rm f}$ to those of the ground state at $U = 10$, 30, 60 MeV is displayed as a function of the mass number along with the corresponding shell correction values. Here, $B_{\rm f}$ is the fission barrier energy. Strong influence of the shell structure which is evident in the a ratios at $U = 10$ MeV is decreased at higher energies. Besides, the magnitude of

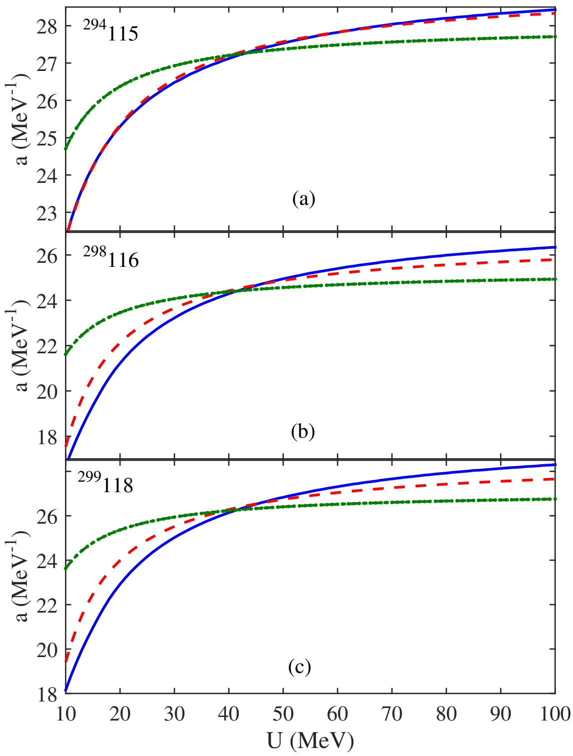

Fig. 3. (Color online) The same as in Fig. 2, but for the saddle point. The calculations (solid blue lines) are compared with the phenomenological model without (dash-dotted green lines) and with (dashed red lines) taking into account the pairing effect.

$a_{\rm SP}(U - B_{\rm f})/a_{\rm GS}(U)$ increases from 0.8 of $U = 10$ MeV up to ~ 1.1 of $U = 30$ MeV. No significant change is observed with further increase of excitation energy. In order to obtain the ratio of the average level density parameter at the saddle point to that of the ground state, the calculated intrinsic level densities in the energy interval between $U = 10$ and 40 MeV were fitted to the back-shifted Fermi gas expression to adjust the average level density parameter (a) of each nucleus

$$\rho_{\rm FG}(U) = \frac{\sqrt{\pi}}{12a^{\frac{1}{4}}(U - \Delta)^{\frac{5}{4}}} \exp\left(2\sqrt{a(U - \Delta)}\right) . \tag{12}$$

In these calculations, the energy shifts were taken as $\Delta = 24/A, 12/A, 0$ for even–even, odd and odd–odd isotopes, respectively. In the energy region considered, the average level density parameter is slightly dependent on the energy interval included in the calculations. However, this does not have significant effect on the calculated ratios. The lowest panel of Fig. 4 shows the values of $a_{\rm SP}/a_{\rm GS}$ ratio calculated for $Z = 114$ isotopic chain as a function of the mass number. A clear shell and pairing effect is evident in the figure.

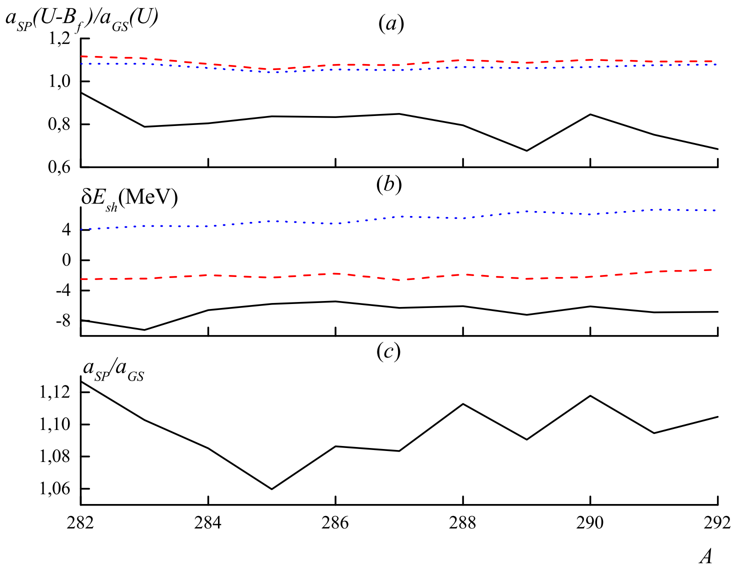

Fig. 4. (Color online) For $Z = 114$ (a), the ratio of the level density parameter at the saddle point over the one at the ground state with excitation energy 10 MeV (solid black line), 30 MeV (dashed red line), and 60 MeV (dotted blue line), respectively. At the saddle point, fission barrier energy $B_{\rm f}$ is taken into account; (b) value of shell corrections for the ground state (black solid line) and saddle point (dashed red line) are presented as functions of mass number A. Corresponding fission barriers are indicated by dotted blue line; (c) the same as (a) the ratio of the average level density parameter at the saddle point over the one at the ground state in the energy interval between $U = 10$ and 40 MeV.

The ratio of fission width $\Gamma_{\rm f}$ to the neutron evaporation width Γ_n, which indicates the probability of fission compared to neutron emission, effectively depends on the difference between level density parameter at the saddle point and at the ground state [13]

$$\frac{\Gamma_{\rm f}}{\Gamma_n} \approx \frac{10a_n\left[2a_{\rm f}^{1/2}(U-B_{\rm f})^{1/2}-1\right]}{4A^{2/3}a_{\rm f}(U-B_n)}\,{\rm e}^{\left[2a_{\rm f}^{1/2}(U-B_{\rm f})^{1/2}-2a_n^{1/2}(U-B_n)^{1/2}\right]}\,. \quad (13)$$

The values of the neutron level density parameter, which is defined as the ratio of the level density parameter values of mother nucleus at the saddle point to that of the daughter nucleus after neutron emission,

$$\frac{a_{\rm f}}{a_n} = \frac{a_{\rm SP}(Z,N)}{a_{\rm GS}(Z,N-1)}\,, \tag{14}$$

are calculated for the considered superheavy nuclei. In these calculations, the fission barrier energy ($B_{\rm f}$) has been included for the saddle point, and neutron binding energy (B_n) has been included for the daughter nucleus at the ground state. The obtained values of $a_{\rm f}/a_n$ vary between 1.06 and 1.3.

4. Conclusions

Using the BCS formalism, the intrinsic level densities and level density parameters of superheavy nuclei with $Z = 112$–120 were calculated. As shown, the excitation energy and shell correction dependencies of the level density parameters of these nuclei at the ground state is well-described with the phenomenological expression. One can also use it at the saddle point taking into account the pairing effect in the cases with small shell-correction values. In the mass region considered in our calculations, $a_{\rm f}/a_n$ ratio varies between 1.06 and 1.3.

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