NUCLEAR SYMMETRY ENERGY WITH FINITE NUCLEON VOLUMES*

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We present the calculation of the symmetry energy E_{sym}, its first L_{sym} and second derivative K_{sym}. To achieve it, we will further extend the Relativistic Mean Field (RMF) model already developed with finite nucleon volumes inside Nuclear Matter (NM). The correction to the nucleon energy ε_A is proportional to the product of nucleon volume V_N and pressure. It gives the first order differential equation for energy ε_A and we derive here a similar differential equation for E_{sym}. The resulting Equations of State (EoS) are softer.

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1. Introduction

Our objective is to examine the important approximation in the nuclear mean-field calculations, namely the absence of nucleon volumes in almost all models of RMF [1]. The nuclear EoS, in particular compressibility, depends on the NN interaction but also should depend on the compressibility K_N^{-1} of quark matter confined inside nucleons. The novel feature of our model [2, 3] is a direct volume contribution of pressure on the energy of a nucleon, missed in previous works [4–6] on excluded volume effects with the constant nucleon radius [7] and also absent in Quark–Meson Coupling (QMC) models [8, 9]. For positive pressure p_H, the nucleon bag decreases its radius (8) due to the work of the meson field on the bag surface. For negative pressure p_H, the nucleon bag increases its radius (8), so the energy is transfered in opposite direction — from bags to the meson field. The simplest (σ, ω) model [10, 11] with constant couplings via scalar and vector mesons in NM, plus ρ meson exchange, which contribute to symmetry energy will be used to obtain clear

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conclusions. We will neglect nuclear pion contributions above the saturation point. Dirac–Brueckner calculations show that the pion effective cross section, in the reaction of two nucleons $N + N = N + N + \pi$, is strongly reduced at higher nuclear densities above the threshold [12] (also with RPA insertions to the self-energy of N and Δ-resonance [13]). We restrict our degrees of freedom to interacting nucleons.

In order to show the main features of our model, let us compare the chemical potential μ_N^q for nucleons with volumes V_N with analogous chemical potential μ_N for point-like nucleons as are considered in the standard RMF models. The chemical potential in the system with the uniform pressure $p = \mathrm{d}U/\mathrm{d}V$ and volume V is

$$\mu_N = \left[\frac{\partial U}{\partial A}\right]_V \quad \text{where} \quad \mathrm{d}U = -p\,\mathrm{d}V + \mu_N \mathrm{d}A\,. \tag{1}$$

It can be interpreted as an energy increase created by the additional particle. When the part of the total volume V is occupied by the nucleon volumes AV_N, then the resulting pressure $p_H = p/(1 - AV_N/V)$ is higher because the accessible volume is respectively smaller. Let us compare the chemical potential μ_N^{rmf} for point-like nucleons with chemical potential for finite nucleon volumes — μ_N^{bag}. Let us check it using the Hugenholz–van Hove theorem (HvH) with respective nucleon energies: $U = \varepsilon_N^{\mathrm{rmf}}$ for point-like nucleons and $U = \varepsilon_N^{\mathrm{bag}}$ for nucleons as quark bags with volumes V_N. We have, for created pressures p and p' in these two systems, the following HvH relations:

$$\begin{aligned} \mu_N^{\mathrm{rmf}} &= \varepsilon_A^{\mathrm{rmf}} + p'/\varrho\,, & (2)\\ \mu_N^{\mathrm{bag}} &= \varepsilon_A^{\mathrm{rmf}} + p_H/\varrho = \varepsilon_A^{\mathrm{bag}} + p_H((V - AV_N)/A + V_N) = \varepsilon_A^{\mathrm{bag}} + p/\varrho\,. & (3) \end{aligned}$$

The second HvH relation (3) is satisfied when the single-particle energies for nucleon bags are diminished

$$\varepsilon_A^{\mathrm{bag}} = \varepsilon_A^{\mathrm{rmf}} - p_H V_N \tag{4}$$

by the energy transfer $\Delta E(\varrho) = p_H V_N(\varrho)$ from the meson field to the quarks inside the nucleon bag. Thus, the volume energy $p_H V_N(\varrho)$ weakens (4) "effectively" the repulsion between nucleons and, consequently, the pressure is smaller in a system with nucleon bags ($p < p'$). It implies that chemical potential is smaller for an RMF model with nucleons described by quark bags. Consequently, the Fermi energy (equal to the chemical potential $E_{\mathrm{F}}^{\mathrm{bag}} = \mu_N^{\mathrm{bag}}$ (3) at saturation density) is related to $E_{\mathrm{F}}^{\mathrm{rmf}} = \mu_N^{\mathrm{rmf}}$ calculated for point-like nucleons (3), as follows:

$$\begin{aligned} \mu_N^{\mathrm{bag}} &< \mu_N^{\mathrm{rmf}} \quad \text{for} \quad p > 0\,, & (5)\\ \mu_N^{\mathrm{bag}} &= \mu_N^{\mathrm{rmf}} \quad \text{at the saturation density } \varrho_0\,. & (6) \end{aligned}$$

Note that at the saturation point the derivative $\mathrm{d}\varepsilon^q_A(\varrho)/\mathrm{d}\varrho|_{\varrho=\varrho_0} = 0$ but derivative of the Walecka part $\varepsilon^{\mathrm{rmf}}_N$ is $\mathrm{d}\varepsilon^{\mathrm{rmf}}_A(\varrho)/\mathrm{d}\varrho|_{\varrho=\varrho_0} > 0$. Differentiating equation (4), one obtains relation (7)

$$K^{-1}(\varrho_0) = 9\frac{(1-\varrho_0 V_N(\varrho_0))}{V_N}\left(\varepsilon^{\mathrm{rmf}}_A(\varrho)\right)'\Big|_{\varrho=\varrho_0}, \tag{7}$$

$$R_0/R(\varrho) = 1 + \Delta E(\varrho)/M_N(\varrho), \tag{8}$$

$$\text{where} \quad \Delta E(\varrho) = p_H V_N = \frac{\varrho^2 {\varepsilon^q_A}'(\varrho) V_N(\varrho)}{(1-\varrho V_N(\varrho))} \tag{9}$$

which determines the new saturation density, slightly bigger ($\varrho_0 = 0.162\,\mathrm{fm}^{-3}$ for the compressibility $K^{-1}(\varrho_0) = 250$ MeV), then $\varrho_0 = 0.149$ fm^{-3} in a basic RMF model [11] with point-like nucleons. In that way, the nuclear compressibility K^{-1} determines the saturation density where the pressure vanishes by $\mathrm{d}\varepsilon^q_A(\varrho)/\mathrm{d}\varrho|_{\varrho=\varrho_0} = 0$. As a result, the saturation density inside NM depends from compressibility and nucleon volume. The energy of NM in density is shown for different models in Fig. 1.

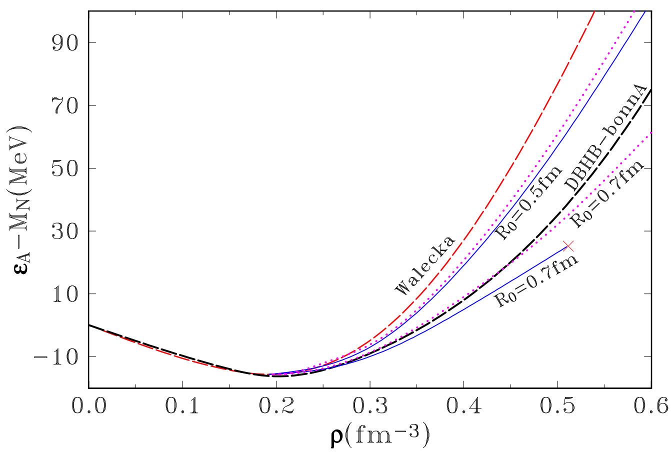

Fig. 1. Energy of NM above the equilibrium density for different models [3]. Walecka [10] and Dirac–Bruckner–Hartree–Fock (DBHF) [14] calculations with the Bonn A interaction are shown as long dashes. Results for constant nucleon mass M_N are denoted by dotted lines (for $R = 0.5$ fm and $R = 0.7$ fm). Solid lines are for constant nucleon radii (see scenario B in [2]).

2. Symmetry energy in dense nuclear matter

The symmetry energy $E_{\rm sym}$ is defined as the coefficient of the quadratic term in the expansion of the energy per nucleon $\varepsilon^q_N(\varrho)$ in neutron excess given by the relative difference of neutron and proton densities $t = (\varrho_n - \varrho_p)/\varrho$. Since energy differentiation with respect to density is alternating with differentiation with respect to the parameter t, the following equation for symmetry energy has the similar structure to equations (4), (8) in NM with Fermi momentum $P_{\rm F}$ and effective nucleon mass M^{*2}_N:

$$E_{\rm sym} = \frac{\partial^2 \varepsilon_A^{\rm bag}(\varrho)}{2\partial t^2} = E_{\rm sym}^{\rm rmf} - \frac{L}{3}\frac{V_N(\varrho)}{(1-\varrho V_N(\varrho))}\bigg|_{\varrho=\varrho_0}, \qquad \frac{L}{\varrho_0} = 3\frac{{\rm d}E_{\rm sym}}{{\rm d}\varrho}\bigg|_{\varrho=\varrho_0}, \tag{10}$$

$$\text{where} \quad E_{\rm sym}^{\rm rmf} = \frac{\partial^2 \varepsilon_N^{\rm rmf}}{\partial t^2} = \frac{g_\rho^2}{8m_\rho^2}\varrho + \frac{P_{\rm F}^2}{6\sqrt{P_{\rm F}^2 + M_N^{*2}}}\,. \tag{11}$$

It is straightforward to determine the additional coupling g_ρ of the ρ meson [15, 16], which contributes only to the $E_{\rm sym}$ of NM (11) and correct the energy of asymmetric neutron matter. The σ and ω meson exchange set down the energy of nuclear matter $\varepsilon_N^{\rm bag}$ (4), therefore, the value of the sum $(E_{\rm s} + \frac{L}{3}\frac{V_N(\varrho)}{(1-\varrho V_N(\varrho))})$ in (11) is determined by the value of ρ contributions for a given couplings of scalar and vector meson. The parameter $L_{\rm sym} = 88$ MeV of symmetry energy slope calculated without nucleon volume corrections is much higher then the phenomenologically extrapolated value $L_{\rm sym}^{\rm exp} \simeq 55$ MeV. If we include nucleon volumes and solve our equation (11) numerically starting from the saturation density with the meson ρ coupling $((g_\rho/m_\rho)^2 = 3.8\ {\rm fm}^2)$, we get [17] $E_{\rm s} = 31.5$ MeV and the slope $L_{\rm sym} = 55$ MeV — in very good agreement with the phenomenological estimate $E_{\rm s}^{\rm exp} = 31.5 \pm 3$ and $L^{\rm exp} = 58.5 \pm 16$ MeV [18]. In addition, their density dependence shown in Fig. 2 agrees with terrestrial and astrophysical constrains [18, 19].

Differentiating equation (11), we get a following expression for the second derivative of symmetry energy $K_{\rm sym}$:

$$K_{\rm sym} = \varrho\frac{\partial E_s^{\rm rmf}}{\partial \varrho}\frac{(1-\varrho V_N)}{\varrho V_N} - \frac{\varrho}{1-\varrho V_N}\frac{\partial E_{\rm sym}}{\partial \varrho}\left(1 + \varrho^2\frac{\partial V_N(\varrho)}{\partial \varrho}\right)\bigg|_{\varrho=\varrho_0}. \tag{12}$$

We present the plot of the second derivative $K_{\rm sym}$ given by equation (12) in Fig. 3. The broken curves show the $K_{\rm sym}$ for the solution started from the saturation point with a choice — $E_{\rm s} = 31.5$ MeV and slope $L_{\rm sym} = 55$ MeV. Note that $L_{\rm sym}$ reaches high positive value at equilibrium, then the function goes down when the density is increasing or decreasing to zero. However, this

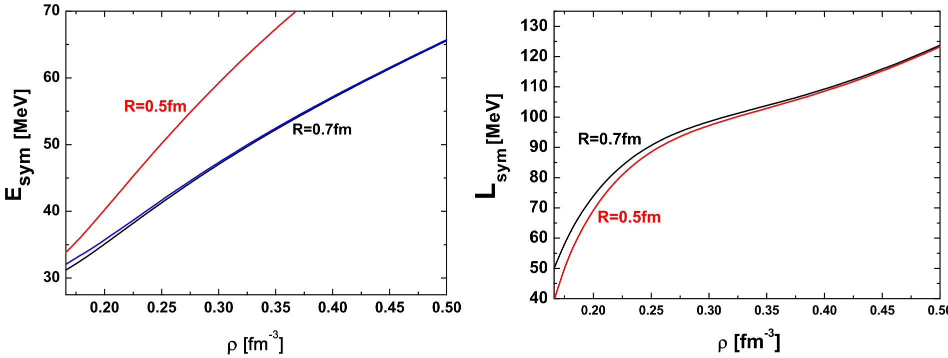

Fig. 2. Symmetry energy of NM $E_{\rm sym}$ (left) and the symmetry energy derivative $L_{\rm sym}$ (right) above the equilibrium density as a function of the nuclear density for two nucleon radii $R = 0.7$ fm and $R = 0.5$ fm.

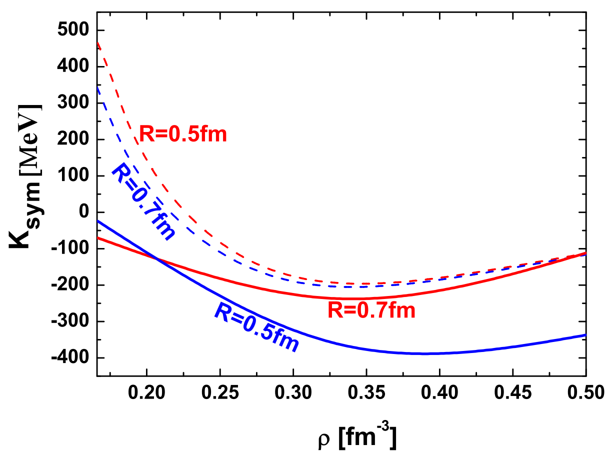

Fig. 3. The second derivative of symmetry energy $K_{\rm sym}$ as a function of the nuclear density for nucleon radii $R = 0.7$ fm and $R = 0.5$ fm.

solution is not unique, we can fit a bigger value of $L_{\rm sym}$ increasing ρ coupling. In order to have a unique solution, it is better to start our integration of (11) from small $\varrho_0 = 0.02$ fm^{-3}, where the $L_{\rm sym}$ is well-approximated by RMF expression (low pressure). Now, we fix $E_{\rm sym} = 31.5$ MeV with higher value of $((g_\rho/m_\rho)^2 = 4.3$ fm$^2)$ and we get the monotonic slope of $L_{\rm sym}$ depending on the nucleon radii (Figs. 4 and 5). At the saturation density, we obtain $L_{\rm sym} = (65$–$80)$ MeV which strongly depends on nucleon volumes excluding the radii $R < 0.6$ fm. The previous high, positive peak of $K_{\rm sym}$ at equilibrium density is absent for solid lines in Fig. 3, which shows the solution of our differential equation (11) starting from very low density, where the slope parameter $L_{\rm sym}$ has minimum at $\varrho_0 = 0$ and increases with density.

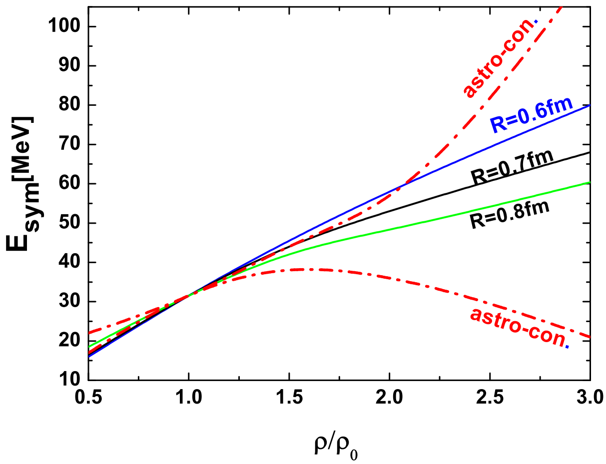

Fig. 4. Symmetry energy of NM for different nucleon radii $R = 0.6$ fm, $R = 0.7$ fm, $R = 0.8$ fm. Astrophysical constrains (astro-con.) from [18] are marked by two dash-dotted lines.

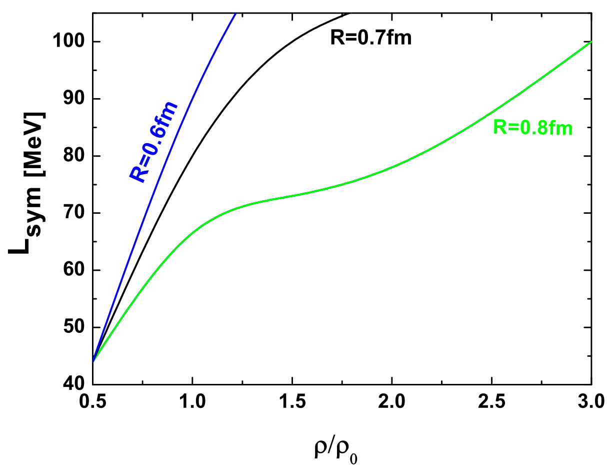

Fig. 5. The symmetry energy derivative $L_{\rm sym}$ as a function of nuclear density for the nucleon radii $R = 0.5$ fm, $R = 0.6$ fm, $R = 0.7$ fm.

3. Results and conclusions

Our results, shown in Fig. 4, agree with astrophysical constrains [18] for the nucleon radii $R > 0.6$ fm. In our model, the pressure contribution to energy originated from the finite sizes of nucleons with quarks degrees of freedom. Consequently, the derivative of the symmetry energy $L_{\rm sym}$ at the saturation density can get from our equation (11) different values presented in Fig. 5: from 65 MeV for $R = 0.8$ fm to 80 MeV for $R = 0.7$ fm. It allows to connect the nuclear compressibility with the saturation density and determine their values from equation (7). In addition, the value of the symmetry slope L is well-fitted for $R > 0.6$ fm with the help of derivative term in equation (11). Therefore, the presented model of compressed nucleons in dense NM is suitable for studying Equation of State of nuclear matter and properties of neutron stars [20]. Further work for finite temperature is planned.

REFERENCES

[1] O. Lourenço, M. Dutra, D.P. Menezes, *Phys. Rev. C* **95**, 065212 (2017).

[2] J. Rożynek, *J. Phys. G: Nucl. Part. Phys.* **42**, 045109 (2015).

[3] J. Rożynek, *Int. J. Mod. Phys. E* **27**, 1850030 (2018).

[4] D.H. Rishke, M.I. Gorenstein, H. Stöcker, W. Greiner, *Z. Phys. C* **51**, 485 (1991).

[5] R. Costa, A.J. Santiago, H. Rodrigues, S.J. Borges, *Commun. Theor. Phys.* **46**, 1052 (2006).

[6] A.S. Rocha, C.A. Vasconcellos, H.T. Coelho, *Int. J. Mod. Phys. E* **20**, 160 (2011).

[7] K. Johnson, *Acta Phys. Pol. B* **6**, 865 (1975); A. Chodos *et al.*, *Phys. Rev. D* **9**, 3471 (1974).

[8] X. Jin, B.K. Jennings, *Phys. Rev. C* **54**, 1427 (1996); H. Müller, B.K. Jennings, *Nucl. Phys. A* **626**, 966 (1997).

[9] Guo Hua, *J. Phys. G: Nucl. Part. Phys.* **25**, 1701 (1999); K. Saito, A.W. Thomas, *Phys. Lett. B* **327**, 9 (1994).

[10] B.D. Serot, J.D. Walecka, «The Relativistic Nuclear Many-Body Problem», in: J.W. Negele, E. Vogt (Eds.) «Advances in Nuclear Physics, Vol. 16», *Plenum Press*, New York 1986.

[11] R.J. Furnstahl, B.D. Serot, *Phys. Rev. C* **41**, 262 (1990).

[12] B. ter Haar, R. Malfliet, *Phys. Rev. C* **36**, 1611 (1987); E.G.P. Rowe, G.T. Rowe, *Phys. Rep.* **149**, 287 (1987).

[13] E. Oset, L.L. Salcedo, *Nucl. Phys. A* **468**, 631 (1987); M. Baldo (Ed.) «The Nuclear Methods and the Nuclear Equation of State», *World Scientific*, 1999.

[14] T. Gross-Boelting, C. Fuchs, A. Faessler, *Nucl. Phys.* **648**, 105 (1999); E.N.E. van Dalen, C. Fuchs, A. Faessler, *Phys. Rev. Lett.* **95**, 022302 (2005); C. Fuchs, *J. Phys. G: Nucl. Part. Phys.* **35**, 014049 (2008); D.P. Menezes *et al.*, *Phys. Rev. C* **76**, 064902 (2007).

[15] S. Kubis, M. Kutchera, *Phys. Lett. B* **399**, 191 (1997).

[16] J. Schaffner-Bielich *et al.*, *Phys. Rev. Lett.* **89**, 171101 (2002).

[17] J. Rożynek, «Symmetry Energy in Dense Nuclear Matter», in: M. Gaidarov, N. Minkov (Eds.) «Nuclear Theory, Vol. 38», *Heron Press*, Sofia 2019.

[18] B. Li, P. Krastev, D. Wen, N. Zhang, *Eur. Phys. J. A* **55**, 117 (2019).

[19] J.M. Lattimer, Y. Lim, *Astrophys. J.* **771**, 51 (2013).

[20] P.B. Demorest *et al.*, *Nature* **467**, 1081 (2010); J. Antoniadis *et al.*, *Science* **340**, 1233232 (2013).