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THEORY AND PHENOMENOLOGY OF TRANSVERSE MOMENTUM DEPENDENT GLUON DISTRIBUTIONS IN HIGH ENERGY PROCESSES*

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We overview selected aspects of Transverse Momentum Dependent (TMD) gluon distributions in the high-energy limit. In particular, we discuss a suitable factorization formalism that allows to obtain phenomenological results for forward jet production processes at the LHC.

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1. Introduction

Unlike in Quantum Electrodynamics, where at present the field theoretical computations approach incredible multi-digit precision, the calculations in the fundamental theory of strong interactions — Quantum Chromodynamics (QCD) — are nowhere near. QCD is a nonlinear non-Abelian theory exhibiting the color confinement — *i.e.*, the physical states of the theory are not quarks and gluons carrying the color charge, but rather hadrons, which are color-neutral. Although complicated, the situation is not hopeless, thanks to the property of asymptotic freedom. When hadrons are probed at very small space-time volumes (*i.e.*, via highly energetic probes), quarks and gluons interact weakly and one can apply perturbation theory. More properly, thanks to *factorization theorems*, one can separate the perturbatively calculable component, a scattering amplitude, from the hadronic bound state, which, for certain processes, can be parameterized in terms of universal parton distribution function (PDF) (see *e.g.*, [1] for a review). The most standard is the collinear factorization theorem, where the transverse motion of the partons in a hadron is neglected in the hard scattering amplitude and integrated over in the PDF. However, in experiments conducted for instance at the LHC, there are few observables to which the collinear

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factorization is directly applicable. The reason is that it requires a single asymptotically large energy scale, such as transverse momenta of very energetic jets of particles. In practice, we often deal with at least two large but very different scales.

In addition, the center-of-mass energy of colliding systems successively grows: at the LHC Run 2, it has reached already 13 TeV. This introduces a potential need for a resummation to all orders in the strong coupling constant of the contributions enhanced at high energies — so-called small-x logarithms (see *e.g.*, [2] for a review of the high-energy QCD). This is a serious issue especially for jets produced in the forward rapidity region. The appropriate resummation is achieved via the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation in the moderate energy regime, or the Balitsky–JIMWLK (Balitsky–Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner) in the regime, where gluon density may become very large approaching the so-called saturated state, *e.g.*, in heavy-ion collisions. There exists an effective theory of QCD at high energy taking these phenomena into account: the so-called Color Glass Condensate (CGC) (see *e.g.*, [3]). An essential property of a hadron in the saturated state, according to the CGC theory, is that gluons have large transverse momenta, of the order of the so-called saturation scale Q_{s}. These momenta must be explicitly taken into account — not integrated over and hidden inside an unknown function, like in the collinear factorization. This feature resembles a more sophisticated and theoretically more challenging Transverse Momentum Dependent (TMD) factorization [1], where hadrons are parameterized in terms of transverse momentum dependent PDFs (TMD PDFs). In CGC, however, the notion of gluon distributions and factorization is not explicitly present.

In recent years, the connection of the CGC description of high-energy processes and the TMD PDFs (more specifically gluon PDFs that dominate at high energies) has been intensively studied. It turns out that both formalisms can be combined to construct a new factorization-like approach valid at high energies, which turns out to be very useful in carrying phenomenological calculations for the LHC processes. In the following, we shall review some aspects of TMD gluon distributions (Section 2). Next, in Section 3, we shall briefly summarize the new factorization formalism. Finally, in Section 4, we will demonstrate example applications to the LHC physics.

2. TMD gluon distributions at small-x

Field theoretical definition of unpolarized TMD gluon distribution involves hadronic matrix elements of bilocal gluon field operator

$$\mathcal{F}\left(x,k_{\mathrm{T}}\right)=2\int\frac{\mathrm{d}\xi^{-}\mathrm{d}^{2}\xi_{\mathrm{T}}}{(2\pi)^{3}P^{+}}\,\mathrm{e}^{ixP^{+}\xi^{-}-i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}}\left\langle P\right|F_{a}^{i+}\left(\xi\right)\,F_{a}^{i+}\left(0\right)\left|P\right\rangle\,, \tag{1}$$

where P is the momentum of a hadron and F is gluon field strength operator. We use the light-cone basis, $v^\mu = v^+ n_-^\mu + v^- n_+^\mu + v_{\mathrm{T}}^\mu$ for any four vector v, with $n_\pm = (1, 0, 0, \mp 1)/\sqrt{2}$. In (1) the gluon fields are separated on the light-cone along n_+ direction and also in the transverse direction. The Fourier transform with respect to the longitudinal displacement ξ^- gives the longitudinal fraction of hadron momentum carried by a gluon, x, while its transverse momentum is conjugate to the transverse displacement ξ_{T}. The above definition is not complete. We need to insert gauge links to make the bilocal operator gauge-invariant. While any gauge link path would ensure the gauge invariance, it turns out that the form of the gauge link is determined by the factorization. Namely, they resum gluons collinear to the hadron, emitted in the hard amplitude. A general method to find the gauge link structure was given in [4].

Unlike in the collinear factorization, where the gluon fields are displaced only along the light-cone direction, here, the transverse displacement renders several possible gauge link paths, including loops. In consequence, the TMD gluon distributions are not universal. However, a TMD gluon distribution for any process is given as a linear combination of the base distributions given in Table I, [5]. There, $\hat{F} = F_a t^a$ and the gauge links and loops are defined as: $\mathcal{U}^{[\pm]} = U(0, \pm\infty; 0_{\mathrm{T}}) U(\pm\infty, \xi^-; \xi_{\mathrm{T}})$, $\mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$, where $U(a, b; x_{\mathrm{T}}) = \mathcal{P} \exp\{ig \int_a^b \mathrm{d}x^- A_a^+(x^-, x_{\mathrm{T}}) t^a\}$.

So far, the discussion was general and not limited to the high-energy limit. Formally, such a limit is taken simply by setting $x \to 0$ in the operator definitions. It turns out that by trading the hadronic states to average over target color configurations used in the CGC theory, one can recover the CGC correlators of infinite Wilson lines [6]. Thanks to this identification, it is possible to use CGC methods to constraint the TMD gluon distributions

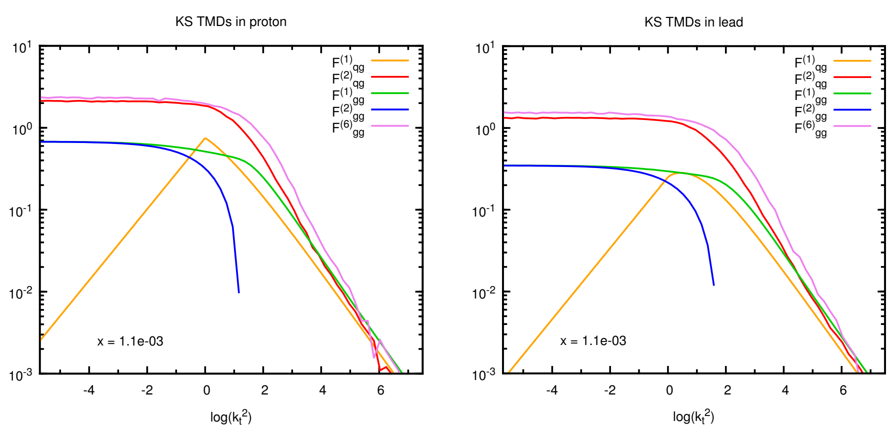

Fig. 1. TMD gluon distributions in the large N_{c} limit [7].

at small-x. In particular, in the large-N_{c} limit, all leading distributions can be calculated from $\mathcal{F}_{qg}^{(1)}$, which in turn can be fitted to experimental data. The result of such a procedure is shown in Fig. 1 [7].

TABLE I

Base TMD gluon distributions for arbitrary multiparticle process.

$\mathcal{F}_{qg}^{(1)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{qg}^{(2)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_{\mathrm{c}}} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{qg}^{(3)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(1)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_{\mathrm{c}}} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(2)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \frac{1}{N_{\mathrm{c}}} \langle P\| \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[\square]\dagger}\right] \mathrm{Tr}\left[\hat{F}^{i+}(0)\mathcal{U}^{[\square]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(3)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(4)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[-]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(5)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \langle P\| \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(6)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \times \langle P\| \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_{\mathrm{c}}} \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_{\mathrm{c}}} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\|P\rangle$
$\mathcal{F}_{gg}^{(7)}(x,k_{\mathrm{T}}) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3 P^+} \mathrm{e}^{ixP^+\xi^- - i\vec{k}_{\mathrm{T}}\cdot\vec{\xi}_{\mathrm{T}}} \times \langle P\| \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_{\mathrm{c}}} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\|P\rangle$

3. Small-x improved TMD factorization (ITMD)

Although the CGC theory can, in principle, give a complete description of the scattering processes in the high-energy (eikonal) limit, it becomes overwhelmingly complicated for multiparticle final states. Indeed, it contains both the genuine multi-parton interactions and power corrections to the leading twist operators, with saturation scale being comparable to other scales. However, in the context of the LHC jet phenomenology, the largest scale is definitely set by the jet transverse momenta: $\mu \gg Q_s$. In that regime, one can use a very elegant factorization formula resembling a generalized TMD factorization. For dijets, it was first proposed in [6] to leading power and later extended beyond that in [8]. Recently, it was proved that the last approach corresponds to exact resummation of all kinematic twists in the CGC theory [9]. The corresponding factorization formula for forward dijets in proton–proton or proton–nucleus collisions has the following form:

$$\frac{\mathrm{d}\sigma^{AB\to 2j+X}}{\mathrm{d}^2q_\mathrm{T}\mathrm{d}^2k_\mathrm{T}\mathrm{d}y_1\mathrm{d}y_2} \sim \sum_{a,c,d} x_B f_{a/p}\left(x_B,\mu\right)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}\Phi^{(i)}_{ag\to cd}\left(x_\mathrm{A},k_\mathrm{T},\mu\right) , \quad (2)$$

where A is a hadron probed at small-x (proton or nucleus), while B corresponds to a proton probed at rather large-x (this asymmetry follows from the requirement of forward jets). Above, $f_{a/B}$ is the collinear PDF for parton a inside a proton, $K^{(i)}_{ag^*\to cd}$ are off-shell hard factors for partonic sub-process $ag \to cd$, and $\Phi^{(i)}_{ag\to cd}$ are small-x TMD gluon distributions. Further, p_1, p_2 are the momenta of jets, y_1, y_2 their rapidity and $\vec{q}_\mathrm{T} = \vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}1}$ is the transverse momentum imbalance of the jets. The explicit expressions for the off-shell hard factors as well as the TMD gluon distributions (given as linear combinations of the base distributions) are given in [8].

4. Forward jet production at the LHC

Let us now look at a particular application of the ITMD formalism described in the preceding section. In Fig. 2, we show a comparison of theo-

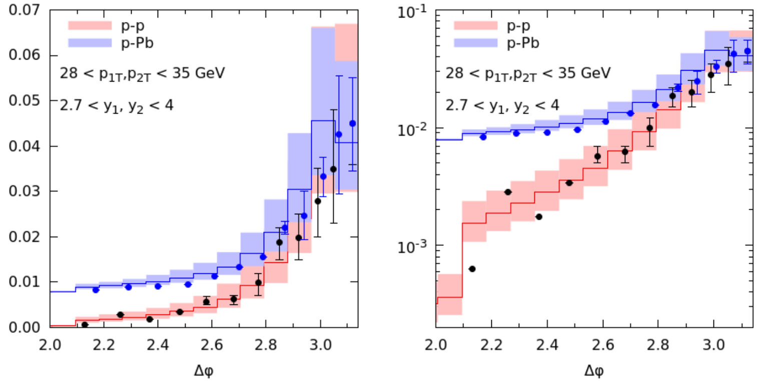

Fig. 2. (Color online) Comparison of theory calculations using ITMD formula and experimental ATLAS data [10] (shifted as described in [11]) for dijet azimuthal correlations. Left plot: linear scale (arbitrary units), right plot: log scale.

retical calculations (light gray/red and dark gray/blue bands) *versus* experimental ATLAS data [10] for dijet azimuthal correlation in p–p and p–Pb collisions. Since no cross-section measurement was performed, we focus on shapes of the distributions. To highlight the difference in tails of the distributions, the data and calculations have been shifted so that the p–p and p–Pb data match in the first bin. We observe nice description of the shapes, provided in addition to the factorization formula (2) we used the Sudakov resummation, necessary due to the presence of the hard scale give by the transverse momenta of jets. For further details, see [11].

5. Summary

Although a proliferation of TMD gluon distributions might be viewed as an issue by orthodox collinear factorization theorists, it is an inherent and extremely interesting property of QCD. While the universality is lost in traditional sense, there is only a finite number of base distributions that build up a distribution for any process in a predictable way. Moreover, these proliferated distributions are a necessary ingredient of the description of processes occurring at high energies, in consistence with the CGC theory.

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