FORBIDDEN FREEZE-IN*

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We study and point out the importance of a frozen-in dark matter production regime via kinematically forbidden decays that arises from the development of thermal masses of plasma particles.

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1. Introduction

The existence of dark matter (DM) in the form of some kind of electrically neutral particle(s) is well-established by now. In order to explain the observed DM abundance, various production mechanisms have been devised, from which a particularly interesting one is the so-called "freeze-in" mechanism [1–3]. For the freeze-in, the assumption is that the DM particle is initially absent (*e.g.* negligible decay width of inflaton to DM) and cannot reach thermal equilibrium due to extremely weak interaction with the plasma. In contrast to freeze-out, the relic abundance "freezes-in" when the production of DM stops due to plasma particles falling out of equilibrium.

In these proceedings, we present our recent work [4], which focuses on a largely neglected case where a plasma particle S develops a substantial thermal mass and decays to DM (χ). The central point is that at high enough temperatures, the mass of S can become large thus opening kinematically forbidden decay to DM ($S \to \bar{\chi}\chi$)[1]. In the following sections, we take a closer look at the "forbidden freeze-in" regime and identify its main phenomenological features as general as possible.

2. Standard freeze-in

In this section, we summarize the standard freeze-in case, where the thermal mass of the mediator particle that decays to DM is neglected (*i.e.*

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[1] This production case was first noted in [5] for gravitino production.

the decay $S \to \bar{\chi}\chi$ is allowed in the vacuum). Assuming that the decays dominate the production, we can estimate that the DM relic abundance from the Boltzmann equation (BE) for the yield ($Y \equiv n/s$) takes the form of

$$-HsT\,\delta_h^{-1}\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}T} = \frac{\Gamma_\chi}{\pi^2}\,K_1\left(\frac{m_S}{T}\right)\,m_S^2\,T\,, \tag{1}$$

with Γ_χ the decay width to DM. The BE can be approximately solved by[2]

$$Y_{\mathrm{DM},0} \approx \left(\frac{\Gamma_\chi\,(m_S, m_\chi)}{1.6\times 10^{-36}\ \mathrm{GeV}}\right)\left(\frac{1\ \mathrm{GeV}}{m_S}\right)^2 m_\chi\left(\frac{1}{\sqrt{g}\,h}\right)\Bigg|_{x=\langle x\rangle}\,. \tag{2}$$

If the production of DM happens via non-renormalizable operators, *e.g.* DM production via a $2 \to 2$ process which occurs due to a dimension-d operator, the amplitude at high temperatures can be approximated as $|\mathcal{M}|^2 \approx \gamma_d\left(\frac{\sqrt{\hat{s}}}{\Lambda}\right)^{2n}$ (with $n = d - 4$). Following [3], we can show that the yield becomes

$$Y_{\mathrm{DM},0} \approx \frac{x_{\mathrm{RH}}^{1-2n} - x_0^{1-2n}}{2n-1}\left(\frac{4^n n!(n+1)!\gamma_d}{2.34\times 10^{-15}}\right)\left(\frac{m_S}{\Lambda}\right)^{2n}\left(\frac{1\,\mathrm{GeV}}{m_S}\right)\left(\frac{1}{\sqrt{g}\,h}\right)\Bigg|_{x\sim x_{\mathrm{RH}}}\,. \tag{3}$$

The two solutions of the relevant BEs show that the relic abundance freezes-in around the mass of the mediator (reheating temperature) for production due to renormalizable (non-renormalizable) operators.

3. Forbidden freeze-in

This section is dedicated to the production of DM via kinematically forbidden channels, *i.e.* forbidden freeze-in. First, we note that only decays can be kinematically forbidden, since for more that one initial state particles the center-of-mass energy increases with temperature. Besides, we note that the production via forbidden decays is generic in freeze-in, since the DM is produced by particles in thermal contact with the plasma which develop thermal mass corrections. Thus, it becomes apparent that there should be a region in the parameter space that is not accessible via the standard freeze-in.

In order to study this scenario as generally as possible, we assume that DM is a Dirac fermion and the mediator is a scalar singlet particle (S) that

[2] The relativistic degrees of freedom (g and h) are evaluated at the mean x defined by $\langle x\rangle \equiv \frac{\int_0^\infty \mathrm{d}x\, x^3 K_1(x)\times x}{\int_0^\infty \mathrm{d}x\, x^3 K_1(x)} \approx 3.4\,.$

interacts with the Standard Model (SM) via the Higgs. That is, the relevant Lagrangian terms are

$$\mathcal{L} \supset -y_\chi\, S\,\bar{\chi}\chi - \frac{\lambda_S}{4!}S^4 - \frac{1}{2}m_S^{0\,2}S^2 - m_\chi\bar{\chi}\chi\; + (SH\text{-terms})\,. \tag{4}$$

Without great loss of generality, we assume that the total mass of S is given by $m_{S,T}^2 \approx m_S^{0\,2} + \alpha^2 T^2$, where assuming that the self-interaction dominates the thermal mass correction, $\alpha^2 = \frac{\lambda_S}{24}$[3]. The BE for the production of DM is given by Eq. (1) with $m_S \to m_{S,T}$. However, if $m_\chi \gg m_S^0$, the production happens only at high temperature and the BE can be simplified to

$$\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}z} \approx \left(\frac{\Gamma_\chi}{5.93\times 10^{-19}\,\mathrm{GeV}}\right)\left(\frac{1\,\mathrm{GeV}}{2m_\chi}\right)^2 \frac{\alpha^4\; K_1(\alpha)}{\sqrt{g}\;h}\delta_h\; z\,, \tag{5}$$

where $z \equiv \frac{2m_\chi}{\alpha T}$ and $\Gamma_\chi \approx \frac{y_\chi^2}{4\pi}\frac{(1-z^2)^{3/2}}{z}m_\chi$. Since at $m_{S,T} = 2m_\chi$ the production stops, we integrate this BE from $z_{\mathrm{RH}} \to \infty$ to $z = 1$, which gives the solution[4]

$$Y_{\mathrm{DM},0} = \left(\frac{\alpha^2\; y_\chi}{5\times 10^{-9}}\right)^2 \left(\frac{1\;\mathrm{GeV}}{2m_\chi}\right)\; K_1(\alpha)\; \left(\frac{1}{\sqrt{g}\;h}\right)_{z=\langle z\rangle}\,. \tag{6}$$

We should point out that in this case, the freeze-in temperature is dictated by the DM mass, while in the standard treatment, it is typically around the mass of the mediator. We also note that the yield today is proportional to $\alpha^4 y_\chi^2$ which makes the forbidden freeze-in very inefficient for typical freeze-in couplings. So we expect the forbidden freeze-in regime to open-up new regions in the coupling y_χ along with the new mass ranges. These are two key features of the forbidden freeze-in scenario.

In analogy to the standard freeze-in case, we may define a decay width due to some non-renormalizable operator as $\Gamma_\chi \sim \frac{\gamma_{S\chi}}{16\pi}\;\alpha^{2n+1}(\frac{T}{\Lambda})^{2n}T$. Then, the solution to Eq. (5) becomes

$$Y_{\mathrm{DM},0} = \frac{z_{\mathrm{RH}}^{1-2n}-1}{2n-1}\left(\frac{\alpha^4\; K_1(\alpha)\;\gamma_{S\chi}}{2.96\times 10^{-17}}\right)\left(\frac{2m_\chi}{\Lambda}\right)^{2n}\left(\frac{1\,\mathrm{GeV}}{2m_\chi}\right)\left(\frac{1}{\sqrt{g}\;h}\right)\Big|_{z\sim z_{\mathrm{RH}}}\,, \tag{7}$$

where again we find that the forbidden freeze-in via non-renormalizable operators is dominated around the reheating temperature. The two solutions of Eq. (5) are shown in Fig. 1. The left panel shows the evolution of the yield assuming the renormalizable interactions, while the right one shows the production via non-renormalizable operators.

[3] If there are other interactions that affect the thermal mass, α gets simply shifted.

[4] The mean z is defined as $\langle z\rangle \equiv \frac{\int_0^1 \mathrm{d}z\;(1-z^2)^{3/2}\times z}{\int_0^1 \mathrm{d}z\;(1-z^2)^{3/2}} \approx 0.34\,.$

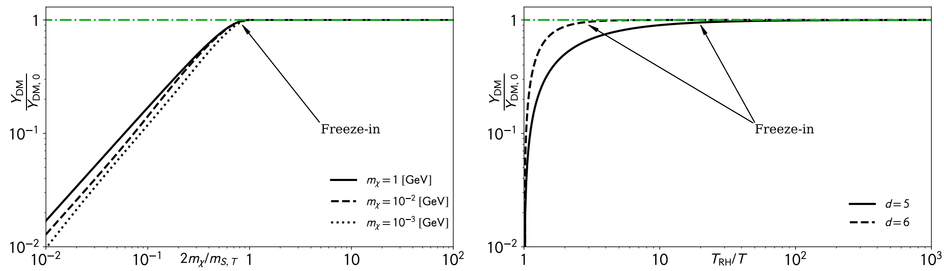

Fig. 1. A typical evolution of the DM yield for the forbidden freeze-in via renormalizable (left) and non-renormalizable (right) operators.

4. Standard versus forbidden freeze-in

Considering the Lagrangian terms of Eq. (4), we can now find the actual difference between the standard and forbidden freeze-in scenarios. Taking $m_{S,T}$ as the mass of S, we calculate the numerical solution of the Boltzmann equation (1). In Fig. 2, we show parameter space in the λ_S–y_χ plane (left) that produces the observed relic abundance [6] and the dependence of the relic abundance on m_χ (right). For the parameter space, we can now observe that the forbidden freeze-in case (delineated by dark grey/orange) produces for the most part a distinct region in the parameter space because, as mentioned before, the mass region is different but also the forbidden production tends to be inefficient for small self-interaction of S. The inefficiency of the production via the forbidden freeze-in can also be seen in the right panel of Fig. 2, where in the case for negligible m_S^0, the relic abundance is much smaller than the standard freeze-in one, with the same couplings. Furthermore, in the same panel we observe that at high enough m_χ, the relic abundance becomes independent of m_χ, which is something that does not happen in the standard case (at least for DM production via decays).

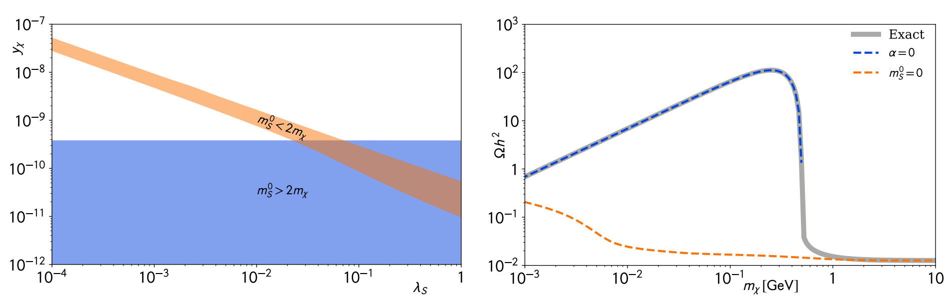

Fig. 2. (Colour on-line) The parameter space in the λ_S–y_χ plane (left) and the dependence of Ωh^2 on m_χ (right) given by the numerical solution of the BE (1), assuming thermal mass for S.

5. A portal model

So far, we have treated the dark sector as generally as possible. However, in a realistic scenario, the Higgs–S interactions can produce some complications, such as the early decoupling of S which may lead to a very different allowed parameter space than the one shown in Fig. 2. In this section, we show the allowed parameter space (in the λ_S–y_χ plane) forbidden freeze-in works in a "portal" model

$$\mathcal{L}^{\rm DM} = \bar{\chi}\left(i\gamma_\mu\partial^\mu - m_\chi\right)\chi + \frac{1}{2}(\partial^\mu S)(\partial_\mu S) - y_\chi S\bar{\chi}\chi - V_{HS}\,,$$

with the potential

$$V_{HS} = \frac{\mu_S^2}{2}\,S^2 + \frac{\lambda_S}{4!}\,S^4 + A\,S\,H^\dagger H + \lambda_{HS}\,S^2\,H^\dagger H\,.$$

Numerically solving the system of BEs describing the Higgs, S, and DM, we can find the parameter space that gives us the correct relic abundance shown in Fig. 3, where we note that this more realistic case produces a similar allowed parameter space to what is expected according to the discussion of the previous section. The inclusion of the Higgs–S interaction, however, introduces couplings with the SM (since Higgs and S mix at $T = 0$), resulting to a region of the parameter space (mainly the forbidden regime) that violates bounds from Big Bang nucleosynthesis [7], while another region may be probed by future experiments SHiP [8] and FASER [9].

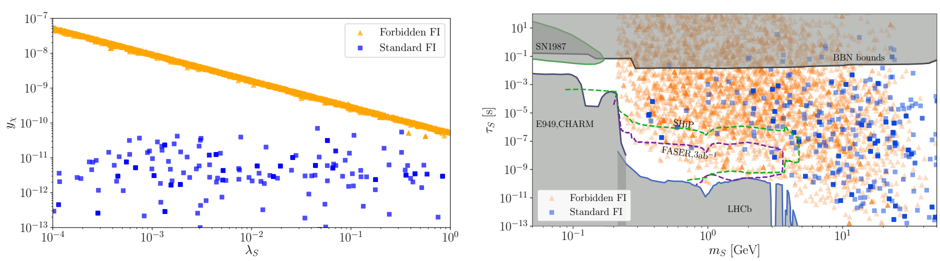

Fig. 3. The parameter space in the λ_S–y_χ plane (left) and the (vacuum) mass *versus* life-time of S (right).

6. Conclusions

Summing up, we have presented a general treatment of the DM production from kinematically forbidden channels, *i.e.* forbidden freeze-in scenario. We have considered both renormalizable and non-renormalizable cases, and have shown that we expect this regime to be in a (mostly) distinct parameter space in an (almost) model-independent fashion.

We have also considered a Higgs portal model, which shows how the forbidden freeze-in can be applied to a realistic scenario, which turned out to be also testable in future experiments.

Closing, we point out that this type of production appears to be generic in every freeze-in DM scenario, since typically thermal masses increase with the temperature and forbidden decays open-up in high enough temperatures. Since this type of production is largely neglected in the literature, more models need to be re-examined in order to identify their forbidden regime.

REFERENCES

[1] J.R. Ellis, J.E. Kim, D.V. Nanopoulos, *Phys. Lett. B* **145**, 181 (1984).

[2] L. Covi, H.-B. Kim, J.E. Kim, L. Roszkowski, *J. High Energy Phys.* **0105**, 033 (2001).

[3] L.J. Hall, K. Jedamzik, J. March-Russell, S.M. West, *J. High Energy Phys.* **1003**, 080 (2010).

[4] L. Darmé, A. Hryczuk, D. Karamitros, L. Roszkowski, *J. High Energy Phys.* **1911**, 159 (2019).

[5] V.S. Rychkov, A. Strumia, *Phys. Rev. D* **75**, 075011 (2007).

[6] N. Aghanim *et al.*, `arXiv:1807.06209 [astro-ph.CO]`.

[7] A. Fradette, M. Pospelov, *Phys. Rev. D* **96**, 075033 (2017).

[8] S. Alekhin *et al.*, *Rep. Prog. Phys.* **79**, 124201 (2016).

[9] J.L. Feng, I. Galon, F. Kling, S. Trojanowski, *Phys. Rev. D* **97**, 055034 (2018).