PROJECT OF THE TUNGSTEN COIL COOLED WITH LIQUID HELIUM FOR PRODUCING HOMOGENOUS HIGH MAGNETIC FIELDS*

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This article contains description of the coil for producing high magnetic fields which may be used in scientific research and technology. Two innovative solutions are applied in the project. The first innovation is a shape of the coil. In result, its effective magnetic field is produced in a cylindrical recess located eccentrically inside the coil. Winding of the coil consisted of wires parallel to the cylinder and axes protruding out of the device. The second innovation is the tungsten winding and cooling with liquid helium. The spatial distribution of the magnetic field induction produced by the coil is estimated. Electric power necessary for the coil to operate is also calculated. The advantages of the coil are possible production of a high and homogeneous magnetic field and reduction of supply power.

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1. Introduction

The high magnetic fields have many important applications, including among others scientific research, modern technologies and medical diagnostics [1, 2]. Generation of such fields requires the use of coils made of resistive conductors or superconductors which are cylindrical in shape. Wires in those coils form concentric circles or rings. The latter variant appears in so-called Bitter magnets [3, 4]. The effective magnetic field is generated inside a cylindrical space, the axis of which is directed through the centre of the scroll. Disadvantages of this solution are, among others: a decrease of the magnetic field induction when the distance from the centre of the opening increases and the appearance of a highly dispersed field outside the coil.

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2. Geometry of the coil

This article shows a project of the coil for which the effective magnetic field is generated within a cylindrical space with a radius of r_1 which is located inside a cylinder with a radius equal to r_2 [5]. The cross section of the device is shown schematically in Fig. 1. Axis of the cylinder 1 and the axis of the recess 2 are parallel but offset with regard to one another by a distance d. It has been shown that if a current of density j flows in parallel to its axis of such a recess, through a long cylinder, then a uniform magnetic field is produced within such a recess. The initial assumption is that the length of the cylinder l is much greater than its radius, which means that the conditions $l \gg r_1, r_2, d$ are met. The magnetic field induction B_c at any point A inside the recess is calculated using the superposition and Ampere's Law [6].

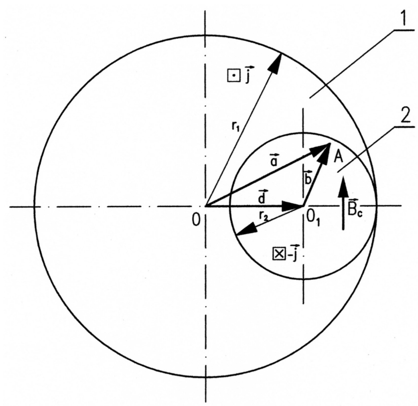

Fig. 1. Outline for magnetic field induction calculations inside a cylindrically shaped recess inside the cylindrical conductor; 1 — cylindrical conductor, 2 — cylindrical recess, $\boldsymbol{j}$ — current density, $\boldsymbol{B}_c$ — magnetic field induction inside the recess ($\boldsymbol{B}_c$ = const.), r_1 — conductor radius, r_2 — recess radius, d — distance between the cylinder and recess axes.

In such a case, the recess 2 is replaced with a cylinder, in which the electric current flows in a direction opposite to that as in cylinder 1. The resultant total current within the recess equals 0 which corresponds to the real situation. Using the designations accepted in Fig. 1, the magnetic field induction produced by a section of the cylinder and by the recess at point A, labelled respectively $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ are expressed by the formulas

$$\boldsymbol{B}_1 = \frac{\mu_0(\boldsymbol{j} \times \boldsymbol{a})}{2}, \quad (1)$$

$$\boldsymbol{B}_2 = \frac{\mu_0(\boldsymbol{j} \times \boldsymbol{b})}{2}, \quad (2)$$

where μ_0 represents the magnetic permeability of a vacuum. Moreover, for vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ indicating the position of point A, it is true that

$$\boldsymbol{d} = \boldsymbol{a} - \boldsymbol{b}\,. \tag{3}$$

Using the superposition principle and formulas (1)–(3), it follows that

$$\boldsymbol{B}_c = \boldsymbol{B}_1 + \boldsymbol{B}_2 = \frac{\mu_0(\boldsymbol{j} \times \boldsymbol{d})}{2} = \text{const.} \tag{4}$$

Formula (4) shows that the value of magnetic field induction is constant and directly proportional to the current density as well as the distance between the axes of the recess and the cylinder. It is due to the fact that in order to increase the magnetic field induction, the recess must be located as far as possible from the axis of the cylinder. It would be the best if the recess was adjacent to the cylinder surface and had a relatively law radius.

The results collected are true for an infinitely long cylinder as Ampere's Law is, in such a case, fulfilled completely. In practice, a monolithic cylinder with a recess could be substituted by a bundle of parallel wires filling up the space between the cylinder surface and outer recess surface. This concept along with the property of a magnetic field have been used within the coil proposed. Moreover, the cylinder with the recess was located in an external coaxial cylinder which is filled up by a bundle of wires. The wire bundle is composed of the same number of wires but the current inside them would flow in the opposite direction. As a result, the magnetic field outside the entire device has been reduced while the cylinder did not generate a magnetic field inside the recess.

The most appropriate material for the production of the winding is tungsten which does not reach the state of superconductivity but has, at the temperature of several Kelvin degrees, a very low resistivity of a magnitude of approximately 10^{-12}–10^{-13} Ωm [7]. Cryogenic temperatures are possible to be reached by means of cooling with the use of liquid helium. Since tungsten is at these temperatures still in a resistive state, the problem of exceeding the critical field does not exist. This problem does occur in the area of superconductors. It occurs when the intensity of flowing current becomes high enough to exceed the critical field which causes a rapid growth in the electrical resistance. A superconductor passes into the resistive state. Moreover, tungsten is characterized by a very high mechanical durability. This is a very positive feature as coils assigned to produce high magnetic fields are subjected to very strong tensions caused by the electrodynamic forces.

3. Spatial distribution of the magnetic field induction

It is obvious that a feasible coil must have a limited length which would in practice be comparable with its diameter. It is for this reason that more precise calculations must be performed for this case. In order to simplify the calculations, an initial condition was accepted as with a narrow straight-line conductor of limited length l which conducts a current with an intensity of dl (Fig. 2). The contribution dB to the magnetic field induction, as generated by the conductor at point P, located at the distance u from the conductor as measured along a line perpendicular to it and located at the distance z from its end ($0 \leq z \leq l$) has been described by formula [8]

$$\mathrm{d}B = \frac{\mu_0 \mathrm{d}I}{4\pi u} \left(\sin\alpha_1 + \sin\alpha_2\right) . \tag{5}$$

Sins of the angles α_1, α_2 in formula (5) are calculated by the formulas

$$\sin\alpha_1 = \frac{z}{\sqrt{u^2+z^2}}, \tag{6}$$

$$\sin\alpha_2 = \frac{l-z}{\sqrt{u^2+(l-z)^2}} . \tag{7}$$

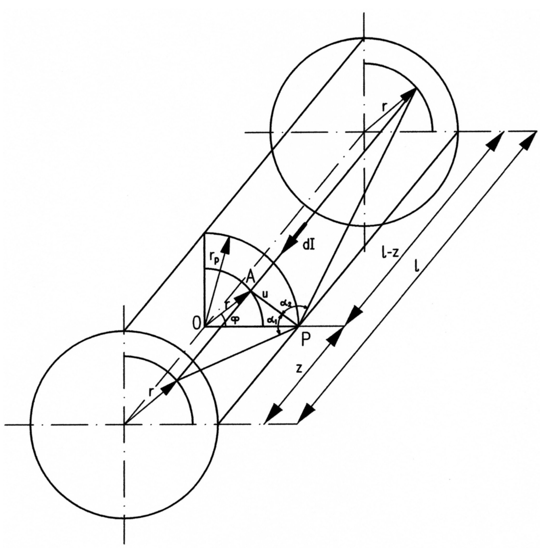

Fig. 2. Geometric dependences at longitudinal cross section used for calculating magnetic field induction of the short coil; dI — current intensity at the element of wire with a length of l.

Due to the shape of coil, a polar frame of reference has been accepted for the calculations (Fig. 3). Within such a frame, the point P is located at the distance r_p from the coil axis. Using the cosine theorem for an OPA triangle and designations assumed in Fig. 6, it is possible to calculate u

$$u = \sqrt{r_\mathrm{p}^2 + r^2 - 2rr_\mathrm{p}\cos\varphi}\,. \tag{8}$$

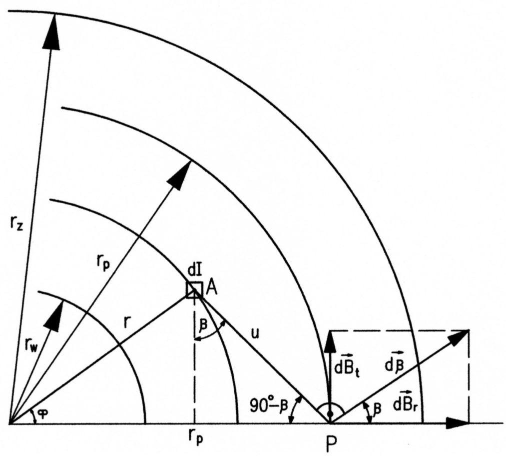

Fig. 3. Geometric dependences of transversal cross section used for calculating magnetic field induction of the short coil; $\mathrm{d}\boldsymbol{B}, \mathrm{d}\boldsymbol{B}_\mathrm{r}, \mathrm{d}\boldsymbol{B}_\mathrm{t}$ — contributions of magnetic field induction coming from the wire element with current intensity of $\mathrm{d}I$ respectively: total and its components — radial and transversal.

Using the polar frame of reference r, φ, the current intensity $\mathrm{d}I$ inside the conductor is expressed by means of the formula

$$\mathrm{d}I = j\mathrm{d}S = jr\mathrm{d}\varphi\mathrm{d}r\,. \tag{9}$$

The polar angle φ appearing in formula (9) fulfils the condition $0 \le \varphi \le 2\pi$. The contribution $\mathrm{d}B$ to the magnetic field induction is decomposed into the radial component $\mathrm{d}B_\mathrm{r}$ and the tangent component $\mathrm{d}B_\mathrm{t}$, the numerical values of which are expressed by the formulas

$$\mathrm{d}B_\mathrm{r} = \mathrm{d}B\cos\beta, \tag{10}$$

$$\mathrm{d}B_\mathrm{t} = \mathrm{d}B\sin\beta\,. \tag{11}$$

For the OPA triangle, the sinus theorem may be used which is represented by the formula

$$\frac{u}{\sin\varphi} = \frac{r}{\sin(90^\circ - \beta)} = \frac{r}{\cos\beta}\,. \tag{12}$$

Using formula (12), we get the following expressions for the trigonometric functions as represented in formulas (10), (11):

$$\cos\beta = \frac{r}{u}\sin\varphi\,, \tag{13}$$

$$\sin\beta = \sqrt{1-\left(\frac{r}{u}\sin\varphi\right)^2}\,. \tag{14}$$

After substitution of formulas (5), (9) and subsequently (13), (14) into formulas (10), (11), the following expressions are obtained for component contributions $\mathrm{d}B_{\mathrm{r}}, \mathrm{d}B_{\mathrm{t}}$ of the magnetic field induction at point P:

$$\mathrm{d}B_{\mathrm{r}} = \frac{\mu_0 j}{4\pi}\left(\sin\alpha_1+\sin\alpha_2\right)\left(\frac{r}{u}\right)^2\sin\varphi \mathrm{d}r\mathrm{d}\varphi\,, \tag{15}$$

$$\mathrm{d}B_{\mathrm{t}} = \frac{\mu_0 j}{4\pi}\left(\sin\alpha_1+\sin\alpha_2\right)\left(\frac{r}{u}\right)\sqrt{1-\left(\frac{r}{u}\sin\varphi\right)^2}\mathrm{d}r\mathrm{d}\varphi\,. \tag{16}$$

In order to calculate magnetic field induction values produced by a cylinder with a length l, the inner radius r_w and outer radius r_z at chosen point P at the coordinates z, r_{p}, the following double integrals may be calculated:

$$B_{\mathrm{r}}(z,r_{\mathrm{p}}) = \frac{\mu_0 j}{4\pi}\int\limits_{r=r_w}^{r=r_z}\int\limits_{\varphi=0}^{\varphi=2\pi}\left(\sin\alpha_1+\sin\alpha_2\right)\left(\frac{r}{u}\right)^2\sin\varphi \mathrm{d}r\mathrm{d}\varphi\,, \tag{17}$$

$$B_{\mathrm{r}}(z,r_{\mathrm{p}}) = \frac{\mu_0 j}{4\pi}\int\limits_{r=r_w}^{r=r_z}\int\limits_{\varphi=0}^{\varphi=2\pi}\left(\sin\alpha_1+\sin\alpha_2\right)\left(\frac{r}{u}\right)\sqrt{1-\left(\frac{r}{u}\sin\varphi\right)^2}\mathrm{d}r\mathrm{d}\varphi\,, \tag{18}$$

in which $u, \sin\alpha_1, \sin\alpha_2$ are expressed by the respective formulas (6), (7) and (5). It is easy to notice that because of the periodicity of the sine function, the result of calculation of (18) limited by $0 \leq \varphi \leq 2\pi$ gives a value of 0. As a result, the magnetic field has only the component B_{t} and is, therefore, directed perpendicularly to the surface passing through the cylinder axis.

By using formula (18), it is possible to calculate components coming from particular elements of the coil winding: a full internal cylinder, a recess and an external cylinder. It is assumed that dimensions of the coil possible to be realised will be the following: the radius of full internal cylinder $r_1 = 0.5$ m, the radius of recess $r_2 = 0.05$ m, the distance between the full cylinder's and the recess' axes $d = 0.45$ m, the radii of the external cylinder $r_3 = 0.51$ m and $r_4 = 0.71$ m, and the length of cylindrical parts of the winding $l = 2$ m. Current density inside the wire is accepted as $j = 200$ A/m^2.

The spatial distribution of the magnetic field produced by particular elements of the coil winding estimated with formula (18) and accepted values are presented in Figs. 4–6. Subsequently, the superposition principle was applied and the spatial distribution of the resultant magnetic field presented in Fig. 7 was obtained.

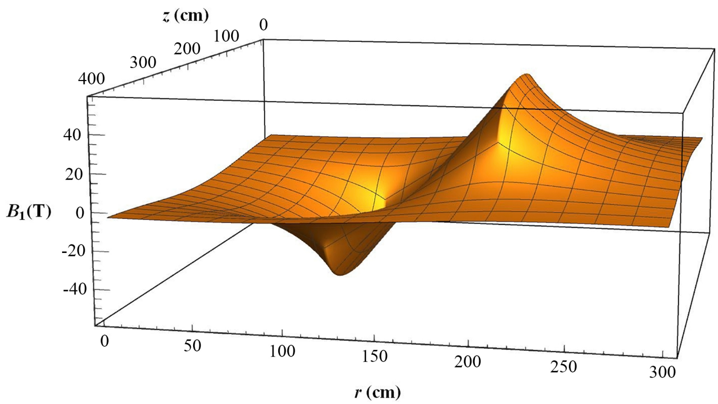

Fig. 4. Spatial distribution of magnetic field induction B_1, produced by the internal part of winding — full cylinder of radius $r_1 = 0.5$ m and length $l = 2$ m, by the effective current density $j = 200$ A/mm^2; the middle of the cylinder is at the point of coordinates $r = 150$ cm and $z = 200$ cm.

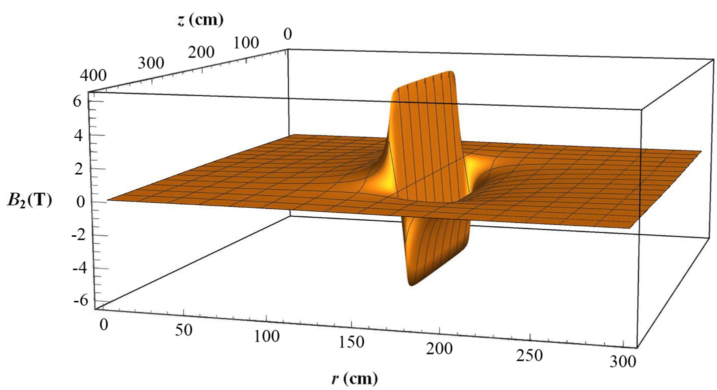

Fig. 5. Spatial distribution of magnetic field induction B_2 produced by the recess — full cylinder of radius $r_1 = 0.05$ m and length $l = 2$ m, by the effective current density $j = -200$ A/mm^2; cylinder axis passes at distance $d = 0.45$ m from the coil axis, and its middle is located at the point of coordinates $r = 195$ cm and $z = 200$ cm.

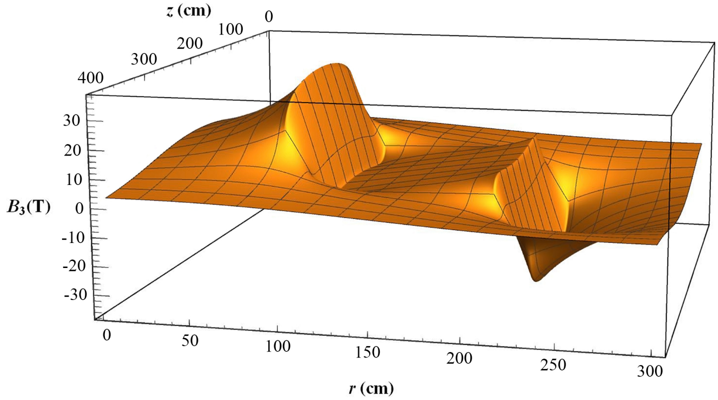

Fig. 6. Spatial distribution of magnetic field induction B_3, produced by the external part of winding — pipe with radii: internal $r_1 = 0.51$ m, external $r_2 = 0.71$ m and length $l = 2$ m by effective current density $j = -200$ A/mm^2, the middle of pipe is located at the point of coordinates $r = 150$ cm and $z = 200$ cm.

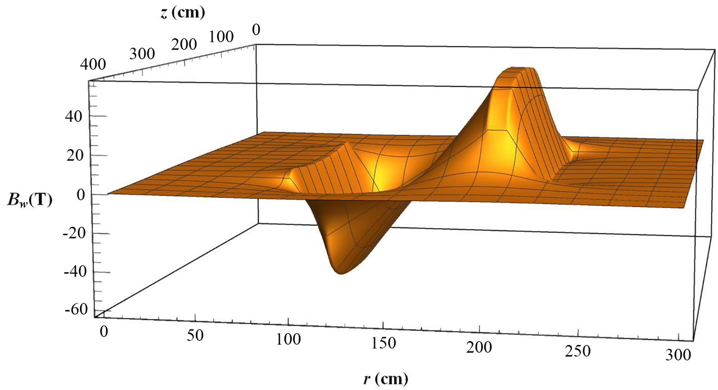

Fig. 7. Spatial distribution of the resultant magnetic field induction B_w produced by the coil consisting of elements shown in Figs. 4–6 (superposition of charts Figs. 4–6); the middle of the coil is located at the point of coordinates $r = 150$ cm and $z = 200$ cm.

In the conducted calculation, the contribution of the magnetic field induction coming from curved wire sections is neglected because they are located outside of the cylinders, and the current directions are opposite to each other, hence their fields cancel each other. The maximum induction in the middle of the coil is $B_{c\,\max} = 49.9$T and decreases with the distance from the coil centre.

4. Parameters of the supplying and cooling systems

Because the coil works in a resistive state, the power supply requires delivery of power P as expressed by the formula

$$P = \Delta U I\,, \tag{19}$$

where I represents the intensity of the current flowing through the winding and ΔU symbolizes the drop of voltage. Assuming that the wire has a square cross section of side $g = 10$ mm, the current intensity can be calculated using the formula

$$I = jg^2. \tag{20}$$

From formula (20) $I = 20$ kA is obtained. In accordance with Ohm's Law, the voltage drop on the winding is

$$\Delta U = I R_{\mathrm{p}}\,, \tag{21}$$

where R_{p} represents the total resistance of the wire used for the winding. The total resistance may be calculated from the formula

$$R_{\mathrm{p}} = \frac{\rho l_{\mathrm{p}}}{g^2}\,, \tag{22}$$

where l_{p} signifies the total length of wire, ρ — the resistivity of the material used. The winding should completely fill the space between the internal cylinder and the recess. With this assumption and previously used coil's dimensions, the length $l_{\mathrm{p}} = 3.85 \times 10^4$ m is obtained. Let the temperature of the winding be 1 K where the electrical resistivity of tungsten is $\rho = 1.6 \times 10^{-13}\ \Omega\,\mathrm{m}$ [7]. After applying this value and the earlier accepted values: $g = 10$ mm and $I = 20$ kA into formulas (19)–(21), one obtains the values: $R_{\mathrm{p}} = 1.6 \times 10^{-5}\ \Omega$, $\Delta U = 1.57$ V and $P = 30$ kW.

The heat transfer in systems cooled by liquid helium is realized by the evaporation of the liquid [9, 10]. The evaporation heat of liquid helium at a temperature of 1 K equals $c_{\mathrm{p}} = 1.5 \times 10^{-4}$ J/kg. Therefore, the mass of helium m_{h} which should flow during 1 s in order to remove the power P (a so-called flow intensity) is expressed by the formula

$$m_{\mathrm{h}} = \frac{P}{c_{\mathrm{p}}}\,, \tag{23}$$

while the volume V_{h} of liquid helium is expressed as

$$V_{\mathrm{h}} = \frac{m}{\rho_{\mathrm{h}}}\,, \tag{24}$$

where $\rho_\mathrm{h} = 145\ \mathrm{kg/m^3}$ is density of helium. After substitution of appropriate values into formulas (23), (24) we get $m_\mathrm{h} = 2.28\ \mathrm{kg/s}$ and $V_\mathrm{h} = 15.7\ \mathrm{dm^3/s}$. Production of calculated volume of liquid helium is possible by the present accessible liquefies (condensing units) [11].

Significant problems are mechanical stresses within coils producing high magnetic fields. These stresses are caused by interaction of coil elements in which currents flow in the same or opposite directions [12]. Value of those stresses s is equal to density of the magnetic field at a given point of the coil and can be calculated with the formula

$$\sigma = \frac{B^2}{2\mu_0}\,. \tag{25}$$

Therefore, after substitution of the previously obtained values $B_{c\,\mathrm{max}} = 49.9\,\mathrm{T}$ to formula (25), the stress value $\sigma = 0.99 \times 10^9\,\mathrm{N/m^2}$ is obtained, while the tensile strength of tungsten is $\sigma_r = 1.72 \times 10^9\,\mathrm{N/m^2}$. As a conclusion, the coil durability is sufficient.

5. Conclusions

The results collected indicated that the maximal value of magnetic field induction $B_{c\,\mathrm{max}} = 49.9$ T generated by the coil in question in approximately 20 T bigger than the induction produced by the most powerful resistive Bitter magnets [13]. This induction is comparable with field induction produced by hybrid magnets, consisting of Bitter magnets located inside large dimension superconducting magnet cooled by liquid helium. The tensile strength of windings made of tungsten is not exceeded even in the fields generated. For the coil dimensions assumed, an effective magnetic field obtained for testing, is several tens of times larger in volume than in large Bitter magnets. This does, however, mean that the dimensions of such a coil are several times greater than those of magnets.

Power necessary to supply the coil in the magnitude of approximately 30 kW is very low in comparison with power delivered to Bitter magnets producing fields of magnetic induction of 30 T. This represents approximately 10^{-3} of power supplying those magnets [14]. Current intensity of the power supply $I = 20$ kA has a comparable value as in the case of Bitter magnets. The flow intensity of liquid helium necessary to keep a steady temperature of the windings, despite its heat dispersion, has also a similar value as in the case of high magnetic field producing systems equipped with cryogenic cooling [15].

The comparisons conducted allow for the assumption of the hypothesis that the coil design considered here could constitute a useful source of high and steady magnetic fields of induction comparable to those generated by hybrid magnets. The advantage of such a coil would include: high homogeneity of effective field and its large volume combined with a reduced dispersed field outside of the winding. Verifying those hypotheses requires further research, the object of which would include solving detailed technical problems, such as the way of connecting winding sections or creation of the cooling channels. An interesting topic of future research could include calculation of what values of magnetic induction may be achieved in presently used Bitter magnets with plate sets made of tungsten and cooled by liquid helium, similarly to the coil considered here. Another important question is what power would need to be supplied in order to produce magnetic field with an induction matching presently in those magnets while using plates made of copper and being cooled by water.

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