STATISTICAL PROPERTIES OF TONAL AND ATONAL PIECES OF MUSIC* **

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Two pieces of music: *Allemende* from *Partita in A minor* by J.S. Bach and *Syrinx* by C. Debussy are studied as time series of pitches, intervals and note's duration times to reveal possible differences between tonal and atonal composition. Two statistical methods are used *(i)* occurrence frequency *versus* rank in the frequency table and *(ii)* time dependence of correlation functions. Deviations from the Zipf law towards a knee-like log–log plot instead of a straight line are noticed. A more white-noise tendency of the tonal piece results from the tune based on broken chords. The most significant signature of tonality turned out to be a deviation from the stationarity of the process. This indicates that the clear caesuras inherent to tonality are reflected in difference in the power spectrum of the process as compared to its autocorrelation function.

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1. Introduction

The existence of some mathematical rules underlying the human art was discovered at the beginning of the XX century. Jean-Baptiste Estoup [1] found that the occurrence frequency of words in meaningful texts is roughly inversely proportional to their rank in the frequency table, this rule is called Zipf's law [2]. Similar studies have been conducted for pieces of music since the 1970s. Voss and Clarke [3] found that the pitch and loudness fluctuation in music (classic, jazz, blues and rock radio stations recording 24 hours without a break) followed Zipf's law. In addition, the authors demonstrated that music created with random number generators resembled most closely the man made one if the correlation functions corresponded to $1/f$ (pink) noise. Further studies [4–7] allowed one, with the use of statistical methods, to distinguish some properties of style, genre, era and even author. A separate

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question is that of pleasure and beauty which are difficult to quantify as being functions of individual experience, cultural milieu, education, emotional state *etc.* The author of Ref. [8] suggests that emotional qualities in music (sadness, anger, happiness, *etc.*) are determined by statistical parameters such as dynamics, register, speed, and continuity. In the European circle, one of the most significant factors of musical esthetics is the development of tonality. Arnold Schönberg — the creator of dodecaphony defined tonality as: "The art of combining sounds into such sequences and such harmonies or harmonic sequences in a way to relate all sound events to the basic sound" [9]. Another definition given in *The New Grove Dictionary of Music and Musicians* reads: Tonality is a "system of relationships between sounds where the tonic or central sound is the most important element" [10]. In musical practice, a tonality amounts to a possibility of more or less unambiguous attribution of a chord to every note of a tune. The listener is or feels to be able to conclude a phrase by a cadence. The composers of baroque and classical era followed strict rules of chord sequence to be in accordance with tonality. In contrast to that, atonality is understood as a lack of anchoring at some sound that results in a sensation of unpredictability and disorder. In the present work we study two monophonic pieces. One of them composed in the baroque era was overwhelmed by *Basso Continuo* that defined regularly the chord for every note. The second piece was intentionally composed to break the rules of tonality.

1.1. Allemande from Partita in A minor for solo flute by J.S. Bach, BWV 1013

The piece was probably written in 1723 and was composed for a solo flute. The suite consists of four movements: *Allemande*, *Courante*, *Sarabanda* and *Gigue*. The first movement is *Allemande* — a German renaissance and baroque dance in duple metre often consisting of only sixteenths notes. Harmonic functions (chords) are clear what classifies the piece as a tonal music. The piece bases on minor and major triads, seventh and ninth dominants with individual chord components which determine the instantaneous harmonic function [11].

1.2. Syrinx by C. Debussy

Syrinx is a solo flute piece, written in 1913. Harmonic functions (chords) are not clear. The main components underlying this impressionist composition are colour, chord are timbre spots. Tonal tensions and 'resolutions', if present, do not follow from rules of harmony. The chromatic alterations prevent one from defining a dominating scale. The contrast of registers (low and high pitches), complicated rhythmic figures (quintolets or sixteenth triplets and octal triols), diverse and contrast dynamics are also visible in *Syrinx* [12].

2. Methods and results

The histograms of the occurrence frequency of pitches, intervals (pitch differences) and notes' duration times *versus* their ranks on the occurrence scale have been constructed. A computer software was used to fit straight lines in the log–log representation. A slope equal to -1 corresponds to the generic Zipf's law. The parameters of the fits for both pieces are gathered in Table I. The slopes for pitches are surprisingly close to each other and are inferior than -1 for pitches. The most pronounced difference is visible in the rhythm graph, where the *Syrinx* exhibits a tendency to a much more "fat tail" of rare note durations. Figures 1 and 2 illustrate the quality of fits

TABLE I

Fitted straight lines and their coefficients of determination R^2.

	J.S. Bach	C. Debussy
Intervals	$y = -1.96x + 2.87, R^2 = 0.82$	$y = -2.04x + 2.32, R^2 = 0.91$
Pitches	$y = -0.65x + 2.32, R^2 = 0.73$	$y = -0.6x + 1.75, \quad R^2 = 0.58$
Rhythm	$y = -4.19x + 2.51, R^2 = 0.72$	$y = -1.41x + 2.16, R^2 = 0.67$

for the occurrence frequency of intervals. Noteworthy is that the plots show knee-like shapes rather that linear form predicted by the Zipf law [2]. The power spectra of the processes of pitches (time series of pitches) for both pieces are given in figures 3 and 4. The power-law time dependencies are evident as well as the differences in the exponents.

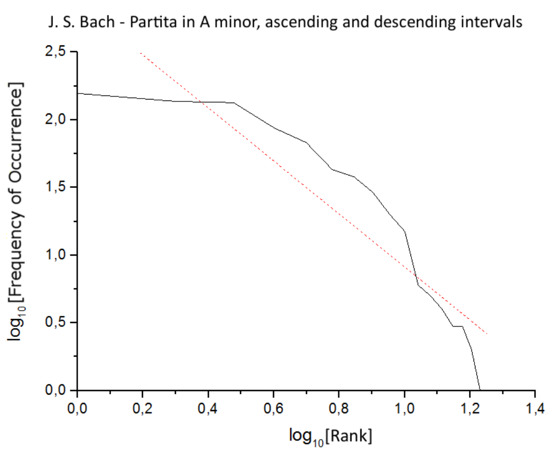

Fig. 1. Rank–Frequency distribution of intervals in *Allemande* from *Partita in A minor*, J.S. Bach, $y = -1.96x + 2.87$, $R^2 = 0.82$.

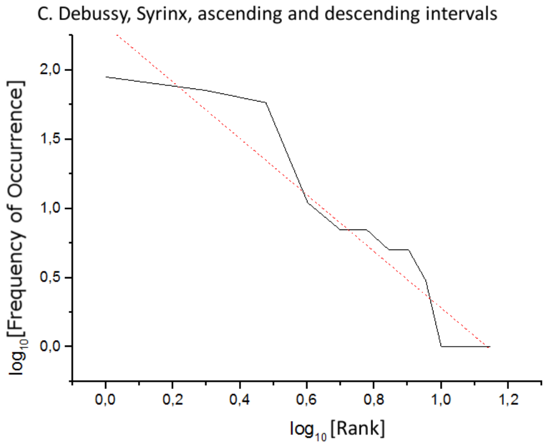

Fig. 2. Rank–Frequency distribution of intervals in *Syrinx* C. Debussy $y = -2.04x + 2.32$, $R^2 = 0.91$.

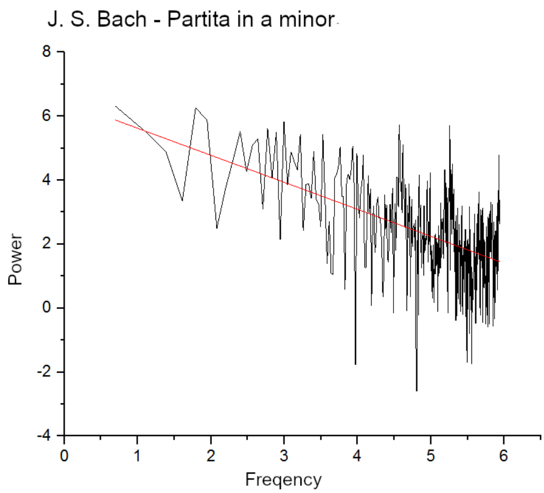

Fig. 3. Process power spectrum for *Allemande* from *Partita in A minor*, J.S. Bach, log–log scale, $\alpha = 0.84$.

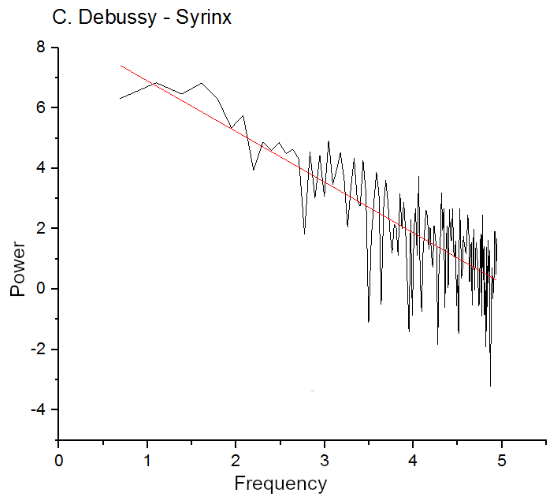

Fig. 4. Process power spectrum for *Syrinx*, C. Debussy, log–log scale, $\alpha = 1.67$.

3. Conclusions

The results obtained for the two contrasted pieces exhibit some common tendencies. Deviations from Zipf's law are discernable both in the values of the slopes of log–log fitted straight lines and in the forms of the log–log histograms. The latter form is rather knee-like than straight; the quality present but not discussed in refs [2]. The reason for that may lie in the smaller number of pitches/intervals usually present in music pieces compared with the number of words in the spoken and written language. Interesting is a much stronger slope of the histograms for intervals than for the pitches. The Vos and Clarke idea of a $1/f$ noise-like correlations in music pieces is only partly confirmed in the periodograms obtained. The power spectra of the pitch time series in *Allemande* and *Syrinx* show expontents 0.84 and 1.67, respectively, that means that Bach's tonal piece is deviated from the $1/f$ noise towards white noise and the atonal Debussy piece towards brown noise. This paradoxical, at first sight, result is understandable because in *Syrinx*, the consecutive notes follow neighboring steps of scale, whereas in the *Allemande*, they follow triads. The comparison of the power spectra of processes and their correlation functions (0.84 and 1.29 in *Allemande* and 1.67 and 1.70 in *Syrinx*) indicates that the pitch time series in the atonal piece is closer to a stationary process, according to the Wiener–Khinchin theorem, than the series in the tonal piece. It seems that the atonal piece would be essentially the same irrespective where it start, whereas the tonal one shows well-defined beginning and caesuras. This is, in fact, the only signature of tonality found in this study. More subtle tools, involving short- and medium-range sounds ordering, should be still worked out to better reveal/represent the sensation of logic and finality in the sound series characteristic of tonal music. The question of adequate signatures of tonality of pieces is still open.

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