BOOST-INVARIANT DESCRIPTION OF POLARIZATION WITHIN HYDRODYNAMICS WITH SPIN*

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We briefly review a recently proposed formalism of perfect-fluid hydrodynamics with spin, which is a generalization of the standard hydrodynamic framework and provides a natural tool for describing the evolution of spin-polarized systems of particles with spin 1/2. It is based on the de Groot–van Leeuwen–van Weert forms of energy-momentum and spin tensors, and conservation laws. Using Bjorken symmetry, we show how this formalism may be used to determine observables describing the polarization of particles measured in the experiment.

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1. Introduction

The spin polarization measurements of Λ hyperons recently made by the STAR Collaboration [1–4] prompted vast theoretical developments aiming at understanding the relation between the orbital angular momentum of the matter created in relativistic heavy-ion collisions and the average spin orientation of the particles emitted from such systems [5–28]. Lately, it has been seen that the thermal-based models which successfully describe the global spin polarization [29–32], unfortunately fail at explaining differential results [4]. These models assume that the spin polarization at the freeze-out is entirely determined by the so-called thermal vorticity [5, 33] and lack of the dynamical evolution of the spin polarization which takes place in the system's expansion. Following the ideas of Refs. [34–37], we investigate this possibility by extending the standard perfect-fluid hydrodynamic framework to include the dynamics of the spin degrees of freedom and analysing it in the Bjorken symmetry setup [38].

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2. Hydrodynamic equations

The perfect-fluid hydrodynamics for spin-1/2 particles is constructed based on the conservation laws for charge, energy, linear momentum and angular momentum with the de Groot– van Leeuwen–van Weert (GLW) [39] forms of the energy-momentum tensor, $T^{\alpha\beta}_{\rm GLW}$, and spin tensor, $S^{\alpha\beta\gamma}_{\rm GLW}$ [1], namely [34–36]

$$\partial_\mu N^\mu = 0\,, \qquad \partial_\mu T^{\mu\nu}_{\rm GLW} = 0\,, \qquad \partial_\lambda S^{\lambda,\alpha\beta}_{\rm GLW} = T^{\beta\alpha}_{\rm GLW} - T^{\alpha\beta}_{\rm GLW}\,, \tag{1}$$

with

$$N^\alpha = nU^\alpha\,, \qquad T^{\alpha\beta}_{\rm GLW} = (\varepsilon + P)U^\alpha U^\beta - Pg^{\alpha\beta}\,, \tag{2}$$

where N^α is the net baryon charge current, ε is the energy density, P is the pressure, n is the baryon density, and U^β is the time-like fluid flow four-vector. Since GLW energy-momentum tensor is symmetric in Eq. (1), the angular momentum conservation implies separate conservation of the spin part [37]. The spin current is given by $S^{\alpha,\beta\gamma}_{\rm GLW} = \mathcal{C}\left(n_{(0)}(T)U^\alpha\omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta\rm GLW}\right)$, where $\mathcal{C} = \cosh(\xi)$, and the auxiliary tensor $S^{\alpha,\beta\gamma}_{\Delta\rm GLW}$ is defined as [35]

$$\begin{aligned} S^{\alpha,\beta\gamma}_{\Delta\rm GLW} &= \mathcal{A}_{(0)}\, U^\alpha U^\delta U^{[\beta}\omega^{\gamma]}{}_\delta \\ &\quad + \mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_\delta + U^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}{}_\delta + U^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}{}_\delta\right)\,, \end{aligned} \tag{3}$$

with $\mathcal{B}_{(0)} = -\frac{2}{\hat{m}^2}s_{(0)}(T)$ and $\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T)$, where $n_{(0)}(T)$ and $s_{(0)}(T)$ are the number density and entropy density of spin-less and neutral massive Boltzmann particles, T is the temperature, $\Delta^{\alpha\beta}$ is the projector on the spatial direction to U, ξ is the ratio of baryon chemical potential, μ, and temperature, T, and $\hat{m}$ is the ratio of the particle mass and temperature.

3. Spin polarization tensor and boost-invariant flow

The polarization tensor $\omega_{\mu\nu}$ can be decomposed in the following way:

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta}U^\alpha\omega^\beta\,, \tag{4}$$

where κ and ω are four-vectors orthogonal to U. For boost-invariant and transversely homogeneous systems, we introduce the following basis:

$$\begin{aligned} U^\alpha &= \frac{1}{\tau}(t,0,0,z) = (\cosh(\eta),0,0,\sinh(\eta))\,, \\ X^\alpha &= (0,1,0,0)\,, \\ Y^\alpha &= (0,0,1,0)\,, \\ Z^\alpha &= \frac{1}{\tau}(z,0,0,t) = (\sinh(\eta),0,0,\cosh(\eta))\,, \end{aligned} \tag{5}$$

[1] Herein, we assume that spin polarization is small ($|\omega_{\mu\nu}| < 1$).

where $\tau = \sqrt{t^2 - z^2}$ is the longitudinal proper time and $\eta = \frac{1}{2}\ln((t+z)/(t-z))$ is the space-time rapidity.

Using above basis, one can decompose the vectors κ^μ and ω^μ as

$$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha \,, \tag{6}$$

$$\omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha \,, \tag{7}$$

where the coefficients $C_{\kappa X}$, $C_{\kappa Y}$, $C_{\kappa Z}$, $C_{\omega X}$, $C_{\omega Y}$, and $C_{\omega Z}$ are functions of τ only. Using the above forms of κ^α and ω^α in conservation law of spin tensor and projecting the resulting tensor equation on $U_\mu X_\nu$, $U_\mu Y_\nu$, $U_\mu Z_\nu$, $X_\mu Y_\nu$, $X_\mu Z_\nu$ and $Y_\mu Z_\nu$, we obtain the set of equations for the coefficients C. These coefficients turn out to evolve independently. The scalar functions $C_{\kappa X}$ and $C_{\kappa Y}$ (and similarly $C_{\omega X}$ and $C_{\omega Y}$) obey the same form of differential equations due to the rotational invariance in the transverse plane.

4. Information about spin polarization of particles at freeze-out

The knowledge about the evolution of spin polarization tensor allows us to calculate the average spin polarization per particle which is defined by $\langle \pi_\mu \rangle = E_p \frac{\mathrm{d}\Pi_\mu(p)}{\mathrm{d}^3 p} / E_p \frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3 p}$ [37] with

$$E_p \frac{\mathrm{d}\Pi_\mu(p)}{\mathrm{d}^3 p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda \, \mathrm{e}^{-\beta\cdot p} \, \tilde{\omega}_{\mu\beta} p^\beta \,, \tag{8}$$

and

$$E_p \frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3 p} = \frac{4\cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda \, \mathrm{e}^{-\beta\cdot p} \,, \tag{9}$$

where $E_p \frac{\mathrm{d}\Pi_\mu(p)}{\mathrm{d}^3 p}$ is the total value (integrated over freeze out hypersuface $\Delta\Sigma_\lambda$) of the Pauli–Lubański vector for particles with momentum p and $E_p \frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3 p}$ is the momentum density of all particles.

In the particle rest rame (PRF), using canonical boost, one can get the polarization vector $\langle \pi^\star_\mu \rangle$. Its longitudinal component is given as [38]

$$\langle \pi^\star_z \rangle = \frac{1}{8m} \left[\left(\frac{m\cosh(y_p) + m_\mathrm{T}}{m_\mathrm{T}\cosh(y_p) + m} \right) \left[\chi \left(C_{\kappa X} p_y - C_{\kappa Y} p_x \right) + 2 C_{\omega Z} m_\mathrm{T} \right] + \frac{\chi\, m \sinh(y_p) \left(C_{\omega X} p_x + C_{\omega Y} p_y \right)}{m_\mathrm{T}\cosh(y_p) + m} \right] , \tag{10}$$

where $\chi\left(\hat{m}_\mathrm{T}\right) = \left(K_0\left(\hat{m}_\mathrm{T}\right) + K_2\left(\hat{m}_\mathrm{T}\right)\right)/K_1\left(\hat{m}_\mathrm{T}\right)$, $\hat{m}_\mathrm{T}$ is the ratio of transverse mass (m_T), and temperature (T) and y_p is the rapidity.

5. Numerical results

Here, we present the numerical solutions of boost-invariant forms of the conservation laws. For the Bjorken geometry, conservation of charge current can be written as $\frac{\mathrm{d}n}{\mathrm{d}\tau} + \frac{n}{\tau} = 0$, and conservation of energy and linear momentum can be written as $\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + \frac{(\varepsilon+P)}{\tau} = 0$. In Fig. 1 (left), we show the proper-time dependence of temperature and baryon chemical potential obtained from boost-invariant forms of conservation laws. We have reproduced the established results that the temperature decreases with proper-time and the ratio of chemical potential over temperature increases with proper time. In Fig. 1 (right), we show the proper time dependence of the C coefficients that define the spin polarization.

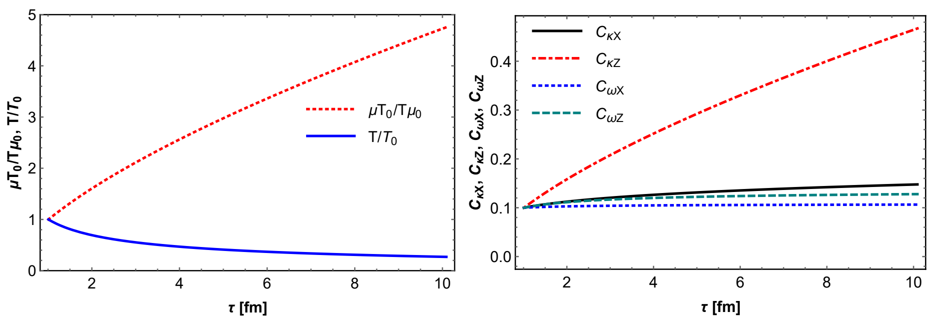

Fig. 1. Left: Proper-time dependence of temperature T divided by its initial value T_0 (solid line) and the ratio of μ (baryon chemical potential) and T (temperature) rescaled by the initial ratio μ_0/T_0 (dotted). Right: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$.

Using the values of thermodynamic parameters and C coefficients at freeze-out, we can get the different components of the PRF mean polarization vector $\langle \pi^{\star}_{\mu} \rangle$ as the functions of particle three-momentum, see Fig. 2.

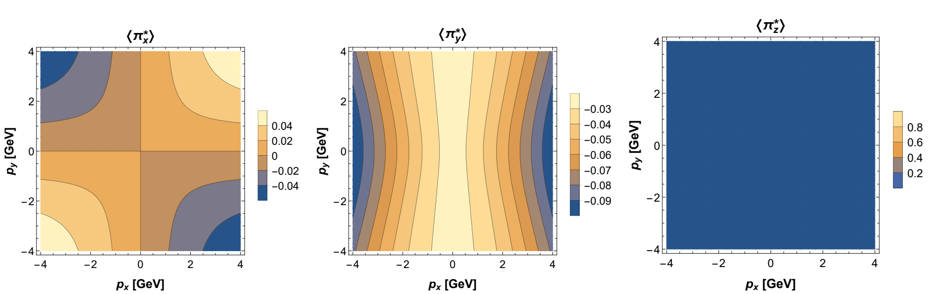

Fig. 2. Components of the PRF mean polarization three-vector of Λs with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $C_{\kappa,0} = (0,0,0)$ and $C_{\omega,0} = (0,0.1,0)$ for $y_p = 0$.

We observe that $\langle \pi^\star_y \rangle$ is negative, reflecting the initial spin angular momentum of the system (the original collision process has only orbital angular momentum perpendicular to the reaction plane and its direction is opposite to the y axis). Since the experiments are done at midrapidity, the longitudinal component ($\langle \pi^\star_z \rangle$) is zero and $\langle \pi^\star_x \rangle$ shows quadrupole structure. These results do not reproduce the observed experimental quadrupole structure of the longitudinal polarization due to the symmetries we have in our model.

6. Conclusion

Using the perfect-fluid hydrodynamics with spin we have presented the numerical results describing the space-time evolution of the spin polarization tensor for a Bjorken hydrodynamic background [40]. Our approach is based on the GLW forms of the energy-momentum and spin tensors assuming the spin polarization tensor in the leading order. It has been shown that six scalar functions C describing spin polarization evolve independently of each other and their proper-time dependence is weak. These results can also be used for the determination of the spin polarization of particles at the freeze-out hypersurface. We have also shown that the spin polarization of particles formed at freeze out reflects the initial direction of polarization.

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