

# BARYON FLUCTUATIONS IN EXTENDED LINEAR SIGMA MODEL\*

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The existence and the location of the critical end point (CEP) between the crossover and the first order part of the chiral phase transition in the phase diagram of strongly interacting matter is a heavily studied area of recent particle physics. The baryon number fluctuations and related quantities such as kurtosis and other susceptibility ratios, that are assumed to be good signatures of CEP, are calculated in an (axial)vector meson extended  $(2 + 1)$  flavor Polyakov linear sigma model (EL $\sigma$ M) at zero and finite  $\mu_B$ . It is compared with the results of lattice as well as other effective model calculations. Divergence of the kurtosis is found at the critical end point.

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## 1. Introduction

The existence and location of a possible critical end point (CEP) between crossover and first order chiral phase transition regions on the phase diagram of strongly interacting matter is conjectured by many theoretical models [1, 2]. If there is such a point, it should be located in a region, which is unreachable neither by perturbative quantum chromodynamics (QCD) nor with lattice calculations, therefore, we are only left with effective field theories. An effective, axial vector and vector meson extended  $(2 + 1)$  flavor Polyakov linear sigma model (EL $\sigma$ M) was built in [3, 4]. According to symmetry considerations, the Lagrangian of the model has the form of

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$$\begin{aligned}
\mathcal{L} = & \text{Tr} \left[ (D_\mu \phi)^\dagger (D^\mu \phi) \right] - m_0 \text{Tr} \left( \phi^\dagger \phi \right) - \lambda_1 \left[ \text{Tr} \left( \phi^\dagger \phi \right) \right]^2 - \lambda_2 \left[ \text{Tr} \left( \phi^\dagger \phi \right) \right]^3 \\
& + c_1 \left( \det \phi + \det \phi^\dagger \right) + \text{Tr} \left[ H \left( \phi + \phi^\dagger \right) \right] - \frac{1}{4} \text{Tr} \left[ L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} \right] \\
& + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L_\mu L^\mu + R_\mu R^\mu) \right] + \frac{h_1}{2} \text{Tr} \left( \phi^\dagger \phi \right) \text{Tr} \left[ L_\mu L^\mu + R_\mu R^\mu \right] \\
& + h_2 \text{Tr} \left[ (\phi R_\mu)^\dagger (\phi R^\mu) + (L_\mu \phi)^\dagger (L^\mu \phi) \right] + 2h_3 \text{Tr} \left[ R_\mu \psi^\dagger L^\mu \phi \right] \\
& - 2ig_2 \left( \text{Tr} \{ L_{\mu\nu} [L^\mu, L^\nu] \} + \text{Tr} \{ R_{\mu\nu} [R^\mu, R^\nu] \} \right) + \bar{\psi} \left[ i\gamma_\mu D^\mu - \mathcal{M} \right] \psi. \quad (1)
\end{aligned}$$

Here, the covariant derivatives can be written in terms of the electromagnetic fields  $A^\mu$ , and the gluon fields  $G^a = g_s G_a^\mu T^a$ , as

$$\begin{aligned}
D^\mu \phi &= \partial^\mu \phi - ig_1 (L_\mu \phi - \phi R_\mu) - ieA^\mu [T_3, \phi], \\
D^\mu \psi &= \partial^\mu \psi - iG^\mu \psi, \quad (2)
\end{aligned}$$

and the field strength tensors of the left- and right-handed (axial) vector fields are

$$\begin{aligned}
L^{\mu\nu} &= \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{ \partial^\nu L^\mu - ieA^\nu [T_3, L^\mu] \}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{ \partial^\nu R^\mu - ieA^\nu [T_3, R^\mu] \}. \quad (3)
\end{aligned}$$

The (pseudo)scalar and (axial) vector nonets are

$$\begin{aligned}
\phi &= \sum_{i=0}^8 (S_i + iP_i) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + K^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_s + i\eta_s \end{pmatrix}, \\
L^\mu &= \sum_{i=0}^8 (V_i^\mu + A_i^\mu) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 \\ K^{*-} + K_1^- & \bar{K}^{*0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu, \\
R^\mu &= \sum_{i=0}^8 (V_i^\mu - A_i^\mu) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 \\ K^{*-} - K_1^- & \bar{K}^{*0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu. \quad (4)
\end{aligned}$$

The  $\{T_i\}_{i=0}^8$  are the generators of  $U(3)$ , while  $S_i$ ,  $P_i$ ,  $V_i^\mu$  and  $A_i^\mu$  represent the scalar, pseudoscalar, vector and axial vector fields, respectively. Equation (4) also shows the assignment of the physical particles except in the 0–8

sector, for which we use the so-called nonstrange–strange basis defined as

$$\varphi_N = \frac{1}{\sqrt{3}} \left( \sqrt{2}\varphi_0 + \varphi_8 \right), \quad \varphi_S = \frac{1}{\sqrt{3}} \left( \varphi_0 - \sqrt{2}\varphi_8 \right), \quad \varphi \in (S_i, P_i, V_i, A_i). \quad (5)$$

In the Lagrangian of Eq. (1), two more constant external fields  $H$  and  $\Delta$  appear, which have the following forms:

$$H = H_0 T_0 + H_8 T_8 = \begin{pmatrix} \frac{h_{0N}}{2} & 0 & 0 \\ 0 & \frac{h_{0N}}{2} & 0 \\ 0 & 0 & \frac{h_{0S}}{\sqrt{2}} \end{pmatrix},$$

$$\Delta = \Delta_0 T_0 + \Delta_8 T_8 = \begin{pmatrix} \frac{\tilde{\delta}_N}{2} & 0 & 0 \\ 0 & \frac{\tilde{\delta}_N}{2} & 0 \\ 0 & 0 & \frac{\tilde{\delta}_S}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \delta_N & 0 & 0 \\ 0 & \delta_N & 0 \\ 0 & 0 & \delta_S \end{pmatrix}. \quad (6)$$

The model contains 15 parameters that are fitted with  $\chi^2$  method by using tree-level meson masses and decay widths. The fitting procedure and the parameters are detailed in [4].

To go to finite temperature, analytic continuation to imaginary time  $t \rightarrow -i\tau$  should be performed, thus the temporal part of the gluon field transformed as  $G_0(t, x) \rightarrow -iG_4(\tau, x)$ . Without going into the details, the Polyakov loop [5, 6] (path ordered Wilson loop in the temporal direction) can be defined as

$$L = \mathcal{P} \exp \left( i \int_0^\beta d\tau G_4(\tau, x) \right), \quad L^\dagger = (L)^\dagger. \quad (7)$$

The Polyakov loop variables are defined with the color traced Polyakov loops

$$\Phi(x) = \frac{1}{N_c} \text{Tr}_c L, \quad \bar{\Phi}(x) = \frac{1}{N_c} \text{Tr}_c L^\dagger \quad (8)$$

as being their thermal expectation values  $\langle \Phi \rangle$  and  $\langle \bar{\Phi} \rangle$  but, for simplicity, we from hereafter leave the  $\langle \rangle$  notation.

We want to study the thermodynamics of a symmetric quark matter ( $\mu_u = \mu_d = \mu_s = \mu_q = 1/3\mu_B$ ). For this, a grand potential  $\Omega(T, \mu_q)$  should be obtained. In the mean-field level approximation that is used for the evaluation of the grand potential, the vacuum and thermal fluctuations of the fermions are taken into account but those for the mesons are neglected. Thus, the meson potential is classical (tree-level), while the fermionic determinant is obtained after performing the functional integration over the quark field, which is evaluated for vanishing mesonic fluctuating fields. Finally, the grand potential reads

$$\Omega(T, \mu_q) = U(\langle M \rangle) + U(\Phi, \bar{\Phi}) + \Omega_{\bar{q}q}^{(0)}(T, \mu_q), \quad (9)$$

where  $U(\langle M \rangle)$  is the tree-level mesonic potential,  $\Omega_{\bar{q}q}^{(0)}(T, \mu_q)$  is the fermionic contribution calculated at nonvanishing scalar–isoscalar backgrounds and vanishing mesonic fluctuations. The  $U(\Phi, \bar{\Phi})$  term is the Polyakov loop potential, which can be simply added to the grand potential since it was treated at mean field level only, so there is no integration over the gluons. One can get the field equations by minimizing the grand potential with respect to the order parameters as  $\frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} = \frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} = 0$ .

## 2. Baryon number fluctuations

The baryon number fluctuations are characterized by the higher order cumulants of the net baryon number. These can be expressed with the generalized susceptibilities, the derivatives of the pressure, which is  $p = \Omega_{T=\mu_B=0} - \Omega$ , with respect to the (baryo)chemical potential

$$\chi_n^B = \left. \frac{\partial^n p / T^4}{\partial (\mu_B / T)^n} \right|_T = T^{n-4} \left. \frac{\partial^n p}{\partial \mu_B^n} \right|_T, \quad (10)$$

where dimensionless pressure and reduced chemical potential are used. Higher order cumulants (moments) diverges rapidly with the diverging correlation length, thus they can be good signatures of a critical end point [7].

To be able to compare the experimental results to the theoretically calculated ones, we should define quantities that are accessible from both sides. The ratios of the baryon number cumulants might be good candidates since the dependence on the finite volume cancels in these ratios. Note that from experimental point of view, this dependence or independence is not obvious, as it is investigated in [8]. One of these ratios is the (excess) kurtosis, that can be defined as the ratios of 4<sup>th</sup> ( $k_4$ ) and 2<sup>nd</sup> ( $k_2$ ) order cumulants as

$$\kappa = \frac{k_4}{k_2^2}. \quad (11)$$

Since

$$\begin{aligned} k_2 &= VT \frac{\partial^2 p}{\partial \mu_B^2} = VT \frac{T^4}{T^2} \chi_2^B = VT^3 \chi_2^B, \\ k_4 &= VT^3 \frac{\partial^4 p}{\partial \mu_B^4} = VT^3 \frac{T^4}{T^4} \chi_4^B = VT^3 \chi_4^B, \end{aligned} \quad (12)$$

and  $k_2 = \sigma^2$  is the variance, it can be rewritten as

$$\kappa \sigma^2 = \sigma^2 \frac{k_4}{k_2^2} = \frac{\chi_4^B}{\chi_2^B}. \quad (13)$$

This quantity,  $\kappa\sigma^2$ , we actually call kurtosis. We calculated the susceptibilities numerically with finite difference method.

### 3. Results

Now, we present our results in the EL $\sigma$ M both at zero and finite  $\mu_B$ . For zero chemical potential, the 2<sup>nd</sup> and 4<sup>th</sup> order susceptibilities and the kurtosis are shown in figure 1.

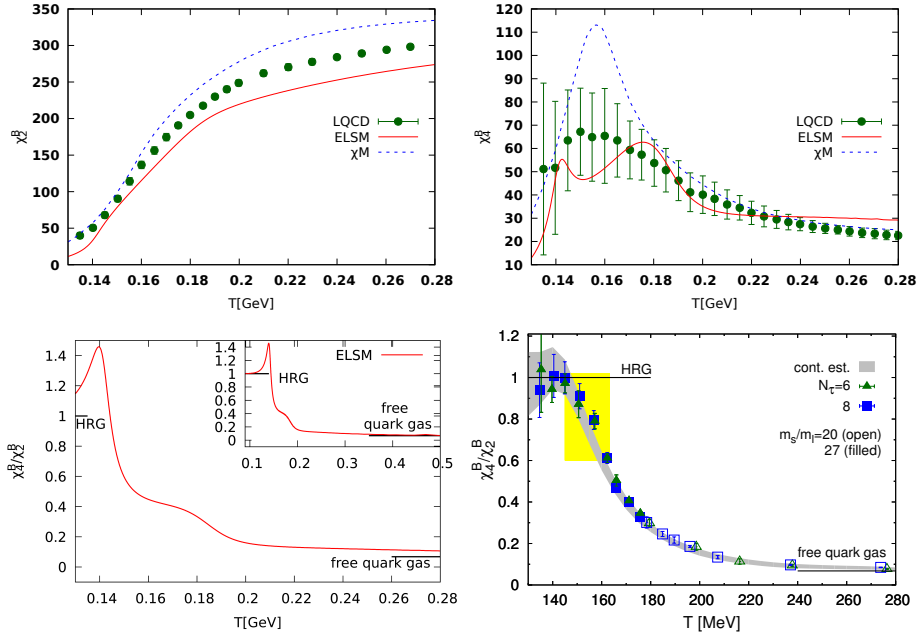


Fig. 1.  $\chi_2^B$  as a function of temperature in the EL $\sigma$ M, in the chiral matrix model, and on the lattice [9] (top left). Temperature dependence of  $\chi_4^B$  compared again to the chiral matrix model and lattice data (top right). The kurtosis in the ELSM (bottom left) and on the lattice [10] (bottom right). The inset of the bottom left figure contains the same curve depicted on a larger scale to show how the HRG and free quark gas limits are approached.

As it can be seen, our results are compatible with the lattice results. It is worth to note that in the case of the EL $\sigma$ M, there is a double peak structure around the phase transition (see *e.g.* top right of Fig. 1). The reason behind this behavior is that we have four order parameters. These, namely the Polyakov loop parameters  $\Phi$ ,  $\bar{\Phi}$  and the scalar–isoscalar vacuum expectation values  $\phi_N$ ,  $\phi_S$  change at slightly different temperatures, resulting in the separation of peaks for the different order parameters.

It is also worth to investigate the behavior of kurtosis at finite  $\mu_B$ , which is shown in Fig. 2. Going towards the predicted critical end point (CEP), the value of the kurtosis at the phase transition increases significantly and — as it is expected — even diverges at the CEP.

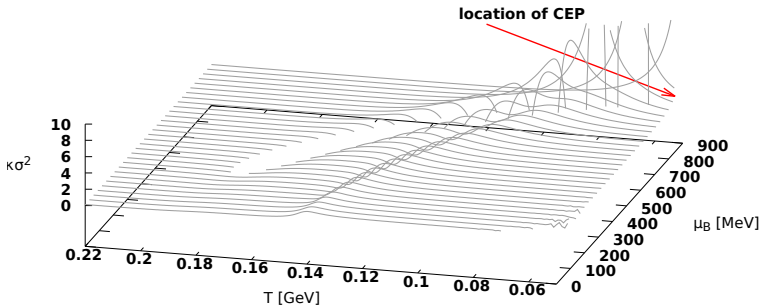


Fig. 2. The 3D plot of the kurtosis as a function of  $T$  and  $\mu_B$ . The arrow points toward our prediction of the CEP at  $\mu_B = 0.885$  GeV and  $T = 0.052$  GeV.

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