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# LIGHTEST STRANGE RESONANCE $\kappa/K_0^*(700)$ PRECISE DETERMINATION FROM A DISPERSIVE ANALYSIS OF DATA\*

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In this paper, we review our recent work where we determine with precision the parameters of the much debated lightest strange resonance  $\kappa/K_0^*(700)$  using as an input a constrained dispersive data analysis on  $\pi K \to \pi K$  and  $\pi \pi \to K \bar{K}$ . For this, we use forward and other sets of partial-wave dispersion relations obtained either from fixed-t or hyperbolic dispersion relations with different subtractions. Partial-wave hyperbolic dispersion relations are then used to extrapolate to the complex plane and establish in a model-independent way the existence of a pole associated to the  $\kappa/K_0^*(700)$ , and to obtain precise values for its mass, width and coupling to  $\pi K$ .

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#### 1. Introduction

The very existence of the lightest strange resonance, the so-called  $\kappa$  or  $K_0^*(700)$  meson, has been the matter of intense debate over almost four decades and, according to the Review of Particle Physics [1], still "Needs Confirmation". Most of the information on strange resonances below 2 GeV comes from  $\pi K$  scattering data. However, since this process is only observed indirectly as a sub-process in  $\pi N \to \pi K N'$ , it is affected by large

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systematic uncertainties, and the resulting data sets often show large inconsistencies between one another and even within the same set. For many years, this meant that semi-quantitative analysis in terms of simple models was considered good enough, yielding large model dependencies in the resonance parameters, or even questioning its existence.

Nevertheless, there is a mathematically rigorous definition of a resonance, which is process- and model-independent. It is given by the appearance of an associated pole sitting in the second Riemann sheet of the complex plane of any amplitude in which the resonance appears. The pole position is related to the resonance mass and width by  $\sqrt{s_{\text{pole}}} \simeq M_{\text{R}} - i\Gamma_{\text{R}}/2$ . For narrow resonances, their pole lies near the real or physical axis and, when the resonance is isolated from other resonances or analytic structures, it yields the characteristic peak seen in experiments. Only in such cases, simple models that describe data around that peak, like the familiar Breit–Wigner formula, provide good approximations to the pole position. However, for wide or overlapping resonances, or when they are close to threshold cuts or other analytic structures, simple models are unreliable to determine the existence and properties of a resonance.

Both these problems — conflicting data sets and model-dependent poles — are solved by dispersion relations. Here, we review our recent use of such relations as constrains to obtain precise and consistent parameterizations of  $\pi K \to \pi K$  and  $\pi \pi \to K \bar{K}$  data, which we next used as an input for partial-wave hyperbolic dispersion relations (PWHDR) to obtain a modelindependent precise determination of the  $\kappa/K_0^*(700)$  pole from data [2].

## 2. Our series of works

Over the last few years, we have published a series of works with the aim of obtaining reliable and precise  $\pi K$  and  $\pi \pi \to \bar{K} K$  data parameterizations from which to extract strange resonance poles. Our methods and many of our preliminary results have been reported in this conference series and now we present the final one for the  $\kappa/K_0^*$ (700).

In particular, in [3], we showed that simple unconstrained fits to  $\pi K \rightarrow \pi K$  data on S, P, D, F partial waves up to 1.8 GeV are inconsistent with Forward Dispersion Relations (FDR), even when carefully evaluating systematic uncertainties. Nevertheless, we provided a set of Constrained Fits to Data (CFD) satisfying a complete isospin set of FDR up to 1.6 GeV.

However, FDRs do not allow for a continuation to the complex plane for partial waves. As a substitute, we resorted to a powerful technique [4] that relies on the convergence on the complex plane of sequences of Padé approximants built from data. Since this method does not rely on a choice of resonance parameterization, it avoids such a kind of model dependence. In particular, there is no need for the assumption that the residue is fixed by the pole position as in a Breit–Wigner-like formula, which is customary in most strange resonance studies. However, since some truncation of the Padé is needed, the pole determination is affected by a systematic uncertainty, although smaller than the statistical one. Thus, the "Padé result" for the  $\kappa/K_0^*(700)$  pole we obtained in [5] with this method is shown in Fig. 1. Almost transparent gray symbols represent Breit-Wigner-like parameterizations, which are not appropriate for so wide resonances and also violate chiral symmetry constraints. More sound T-matrix pole determinations use analytic or dispersive methods, frequently with some sort of chiral symmetry constraints. Some of those listed in the RPP [1] are also shown in Fig. 1. It is worth mentioning that our "Padé result" confirmed, with analytic methods and data, that the  $\kappa$  pole was closer to 700 MeV and thus triggered the change of name in the RPP from  $K_0^*(800)$  to  $K_0^*(700)$ . As a final remark, we would like to emphasize that this method can be used both in the elastic and the inelastic region. Actually, we also provided results for other five strange resonances in the inelastic region, which were reported in the previous edition of this conference.



Fig. 1. (Color online)  $K_0^*(700)$  pole positions from the RPP [1]. We also show our results [2] using the Roy-Steiner equations, and as an input our UFD or CFD parameterizations. Red and blue points use for  $F^-$  a once-subtracted or an unsubtracted dispersion relation, respectively. This illustrates how unstable pole determinations are when using simple fits to data. Only once the Roy-Steiner equations are imposed as a constraint (CFD), both pole determinations fall on top of each other. We also provide the modulus of the g coupling to  $\pi K$ .

As a matter of fact, a simple continuation of our CFD parameterization constrained with FDRs yields a fairly reasonable pole, labeled "Conformal CFD" in Fig. 1, but since it is based on a particular parameterization, it is model-dependent. Incidentally, in [6] we calculated dispersively (not fit) the  $\kappa/K_0^*(700)$  Regge trajectory, using either our CFD or Padé pole position as the only input. The resulting  $\kappa/K_0^*(700)$  trajectory does not come out linear with respect to the mass squared and has a slope much smaller than that of ordinary mesons. This supports the non- $q\bar{q}$  dominant nature of the  $\kappa/K_0^*(700)$  and, therefore, of the light scalar meson nonet. This is a consequence not only of being wide, but of its residue (*i.e.* its coupling to  $\pi K$ ), being related to the mass and width differently than for ordinary resonances (like narrow Breit–Wigner-like resonances).

The best determination so far was the dispersive study of Descotes-Genon *et al.* [7]. There, PWHDR were used to continue to the complex plane a numerical *solution*, not a data fit, of the Roy–Steiner equations obtained from fixed-*t* dispersion relations. Actually, the pole in [7] could be considered a prediction, since the data is not used around the nominal  $\kappa/K_0^*(700)$  mass, but just at higher energies and from other partial waves. Despite this rigorous result obtained in 2006, the 2018 RPP still considers that the  $\kappa/K_0^*(700)$  "Needs Confirmation". Over the last years, we have been encouraged by RPP authors and other groups to provide such a confirmation.

Thus, in order to provide the needed confirmation, we are using data in the elastic region of the S, P, D, F waves together with 16 dispersion relations. Two of them are the FDRs already used in [3]. However, in order to reach the  $\kappa/K_0^*(700)$  pole, PWHDR are required. That their applicability range reaches the pole was shown in [7] but we have slightly modified the hyperbolae to enlarge the applicability in the real axis while still reaching the pole. The drawback of using partial-wave dispersion relations is that the input from  $\pi\pi \to K\bar{K}$  is needed. Thus, we consider four PWHDR for the  $\pi\pi \to K\bar{K}$  partial waves. Three are once-subtracted, for angular momentum and isospin (I, J) = (0, 0), (0, 2), (1, 1), and an additional unsubtracted one (1, 1). Moreover, we now consider the more dispersion relations for the Sand  $P \ \pi K$  partial waves: Four of them come from fixed-t and hyperbolic once-subtracted dispersion relations for  $F^+ = (F^{1/2} + 2F^{3/2})/3$ , whereas the other six are two from fixed-t dispersion relations and another four HDR for  $F^- \equiv (F^{1/2} - F^{3/2})/3$ , either unsubtracted or once-subtracted.

As a first step, we showed [8] that unconstrained fits to  $\pi\pi \to K\bar{K}$  fail to satisfy PWHDR, even considering systematic uncertainties. Nevertheless, we provided constrained fits to  $\pi\pi \to K\bar{K}$  Data (CFD), consistent with PWHDR and still describing data up to 1.47 GeV. Our fits extend up to 2 GeV, but 1.47 GeV is the applicability range of PWHDR in that channel.

In our latest work [2], we check that, once again, unconstrained fits to  $\pi K$  data (UFD) fail to satisfy partial-wave dispersion relations, even considering systematic uncertainties and the CFD  $\pi\pi \to K\bar{K}$  input. Thus, we have imposed the 16 dispersion relations described above to obtain our final constrained fits to data (CFD). The results for the (1/2,0) partial wave, where the  $\kappa/K_0^*(700)$  appears, are shown in Fig. 2. The disagreement of the UFD dispersive output is shown in the left panel. Note the big difference between the once-subtracted and unsubtracted curves, despite they come from the same apparently nice-looking UFD shown in Fig. 3. This illustrates that naive fits to  $\pi K$  data only are not reliable to obtain the  $\kappa/K_0^*(700)$  pole, since the same input yields the two very inconsistent "UFD" poles shown in Fig. 1. Only when we use the CFD, whose agreement between all dispersive representations of the  $f_0^{1/2}$  wave we show in the right panel of Fig. 2, we get consistent poles irrespective of using a once-subtracted or unsubtracted PWHDR. These are shown with solid red and blue symbols in Fig. 1, and constitute our final result. They are consistent with the pole in [7] obtained without data on the  $\kappa/K_0^*(700)$  region.



Fig. 2. (Color online) Dispersive outputs [2] for the  $f_0^{1/2}(s)$  partial wave versus the input from the data parameterization. Left: unconstrained fits to data (UFD). Note the huge discrepancies between the curves. Right: constrained fits to data (CFD). Now all curves agree within uncertainties. The CFD are also consistent for the other dispersion relations and partial waves. Namely, the average  $\chi^2/d.o.f.$  per dispersion relation is 0.7, whereas the average  $\chi^2/d.o.f.$  is 1.4 per fitted partial wave.

It may be surprising that imposing dispersion relations makes an almost imperceptible difference between the UFD and CFD for the  $f_0^{1/2}$ , as we show in Fig. 3. It is also not the effect of the vector  $f_1^{1/2}$  partial wave, which barely changes from our UFD to our CFD. Actually, in [7], their prediction for this wave was somewhat deviated from the scattering data, as shown in Fig. 3. The most relevant effect is a consistent description of the  $\pi\pi \to \bar{K}K$  (1,1) partial wave, particularly in the unphysical region. Simple models of  $\pi K$ scattering never pay attention to that contribution.



Fig. 3. Our [2] CFD versus UFD phase shifts  $\delta_{\ell}^{I}$  of  $f_{0}^{1/2}(s)$  (left) and  $f_{1}^{1/2}(s)$  (right). Data come from [9]. Descotes-Genon *et al.* is the numerical dispersive solution of [7].

Thus, we believe we have provided the required confirmation to establish the existence of the  $\kappa/K_0^*(700)$  resonance, as well as a precise and mathematically sound determination of its parameters.

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