A NEW EVALUATION OF a_μ^{SM} TO BE DEVIATED FROM THE WORLD AVERAGED a_μ^{exp} BY 1.6σ IS ACHIEVED BY A NOVEL APPROACH*

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The elaborated Unitary and Analytic models of pseudoscalar meson nonet electromagnetic structure, and to some extent also of nucleons, give more precise theoretical prediction for the hadronic contribution $\Delta\alpha_{\mathrm{had}}^{(5)}(t)$ to the running fine structure constant QED $\alpha(t)$ in the space-like region, which by the novel approach leads to the following complete SM muon anomalous magnetic moment value $a_\mu^{\mathrm{SM}} = (11659196.35 \pm 4.81) \times 10^{-10}$. This result deviates from the world average experimental value $a_\mu^{\mathrm{exp}} = (11659209 \pm 6) \times 10^{-10}$ by 12.65 ± 7.69, *i.e.* 1.6σ.

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1. Introduction

The SM muon anomalous magnetic moment consists in the contributions

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{(LO)had}} + a_\mu^{\mathrm{(NLO)had}} + a_\mu^{\mathrm{(NNLO)had}} + a_\mu^{\mathrm{(LbL)had}} + a_\mu^{\mathrm{EW}} \,. \quad (1)$$

The total uncertainties of a_μ^{SM} are given by the "Leading Order" hadronic contribution $a_\mu^{\mathrm{(LO)had}}$ which has been remarkably improved by recent evaluations in [1, 2], calculating the sum of three dispersion integrals

$$a_\mu^{\mathrm{(LO)had}} = \frac{\alpha^2(0)}{3\pi^2}\left(\int\limits_{m_{\pi^0}^2}^{s_{\mathrm{cut}}} \frac{\mathrm{d}s}{s}\frac{\sigma_{\mathrm{tot}}(e^+e^-\to \mathrm{had})}{\sigma_{\mathrm{tot}}(e^+e^-\to\mu^+\mu^-)}K(s)\right.$$

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$$+\int\limits_{s_{\rm cut}}^{s_{\rm pQCD}} \frac{{\rm d}s}{s} R^{\rm data}(s)K(s) + \int\limits_{s_{\rm pQCD}}^{\infty} \frac{{\rm d}s}{s} R^{\rm pQCD}(s)K(s)\Bigg) \,. \quad (2)$$

The $s_{\rm cut}$ is by various authors taken from the region 0.8–4 GeV^2, see Fig. 1, in which highly fluctuating $\sigma_{\rm tot}(e^+e^- \to {\rm had})$, due to hadronic resonances and threshold effects, in the first integral is changed to a smoother one, in the second integral to be dependent on inclusive $R^{\rm data}(s)$ bare data. The function $R^{\rm pQCD}(s)$ is calculated in the framework of the pQCD with active flavors $N_{\rm f} = 6$ as $R^{\rm data}(s)$ is given up to $s = 40\,000$ GeV^2.

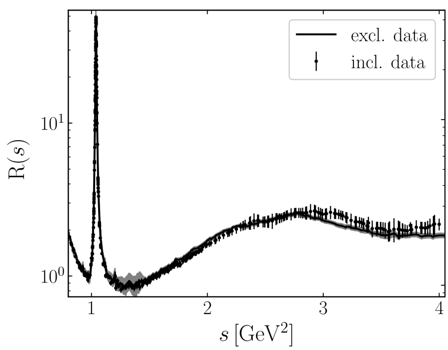

Fig. 1. A comparison of the sum of bare total cross sections $e^+e^- \to$ had (full line) and the inclusive R-data.

In the novel approach [3, 4], $a_\mu^{\rm (LO)had}$ is expressed through $\Delta\alpha_{\rm had}^{(5)}(t(x))$

$$a_\mu^{\rm (LO)had} = \frac{\alpha(0)}{\pi} \int\limits_0^1 {\rm d}x(1-x)\Delta\alpha_{\rm had}^{(5)}(t(x))\,, \quad (3)$$

and the 5 light quarks u, d, c, s, b contribution to $\Delta\alpha(t)$ in the running fine structure constant QED is

$$\alpha(t) = \frac{\alpha(0)}{1-\Delta\alpha(t)}\,. \quad (4)$$

This constant is suggested to be measured in space-like region [5] by the CERN North Area muon beam scattered on atomic electrons of Be and C.

2. Improved $a_\mu^{\rm SM}$ by theoretical evaluation of $\Delta\alpha_{\rm had}^{(5)}(t(x))$

The authors of papers [1, 2] improved the $a_\mu^{\rm (LO)had}$ value by using more precise data on $e^+e^- \to$ had processes. However, they used the same trapezoidal method of integration as in their previous papers. Here, another improvement of their results is achieved by novel approach (3), and optimally describing $e^+e^- \to$ had processes with two particles in the final state by pseudoscalar meson nonet Unitary and Analytic (U&A) electromagnetic (EM) structure models [6], which enable to calculate contributions by means of the continuous integration method. The trapezoidal method of integration is left only for evaluation of channels with a number of final particles more than two, which are giving mostly negligible contributions to $\Delta\alpha^{(5)}_{\rm had}(t(x))$. Simultaneously, a dependence of $a_\mu^{\rm (LO)had}$ on the chosen value of $s_{\rm cut}$ is investigated.

$\Delta\alpha^{(5)}_{\rm had}(t(x))$ is evaluated through the following three dispersion integrals:

$$\Delta\alpha^{(5)}_{\rm had}(t(x)) = -\frac{\alpha(0)t(x)}{3\pi}\left(\int\limits_{m^2_{\pi^0}}^{s_{\rm cut}} \frac{{\rm d}s'}{s'(s'-t(x))}\frac{\sigma^0_{\rm tot}(e^+e^-\to {\rm had})}{\sigma^0_{\rm tot}(e^+e^-\to \mu^+\mu^-)}\right.$$
$$\left. + \int\limits_{s_{\rm cut}}^{s_{\rm pQCD}} \frac{{\rm d}s'}{s'(s'-t(x))}R^{\rm data}(s') + \int\limits_{s_{\rm pQCD}}^{\infty} \frac{{\rm d}s'}{s'(s'-t(x))}R^{\rm pQCD}(s')\right). \quad (5)$$

The expression is similar to $a_\mu^{\rm (LO)had}$ in (2), however, now without the QED kernel $K(s)$.

Evaluations of the first and the second integral require the experimental data to be undressed of all "vacuum polarization" effects.

The bare single total cross sections in the first integral with sinking tendency of contributions beyond the process $e^+e^- \to \eta'\gamma$ are

$$\begin{aligned}
&\sigma^0_{\rm tot}\left(e^+e^-\to{\rm had}\right) = \sigma^0_{\rm tot}\left(e^+e^-\to\pi^+\pi^-\right) + \sigma^0_{\rm tot}\left(e^+e^-\to K^+K^-\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to K^0\bar{K}^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^0\gamma\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\eta\gamma\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\eta'\gamma\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^+\pi^-\pi^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^+\pi^-2\pi^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to 2\pi^+2\pi^-\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^0K^+K^-\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^0K^0\bar{K}^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^+K^-K^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^-K^+K^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^+\pi^-K^+K^-\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^0\pi^-K^+K^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^+\pi^0K^0K^-\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^0\pi^0K^0K^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to 2\pi^+2\pi^-2\pi^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^-K^+K^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^0\pi^0K^+K^-\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^0\pi^0K^0\bar{K}^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\pi^+\pi^0K^-K^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^-\pi^0K^+K^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to 2\pi^+2\pi^-2\pi^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\eta\pi^+\pi^-\right) + \sigma^0_{\rm tot}\left(e^+e^-\to 2\pi^+2\pi^-\pi^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\pi^0\omega\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\eta\pi^+\pi^-\pi^0\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\eta\phi\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\eta\omega\pi^0\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to\eta\omega\right) + \sigma^0_{\rm tot}\left(e^+e^-\to 3\pi^+3\pi^-\right) + \sigma^0_{\rm tot}\left(e^+e^-\to\eta 2\pi^+2\pi^-\right)\\
&+\sigma^0_{\rm tot}\left(e^+e^-\to p\bar{p}\right) + \sigma^0_{\rm tot}\left(e^+e^-\to n\bar{n}\right).
\end{aligned}$$

The first six of them are first expressed through the corresponding EM form factors (FFs) squared and then the free parameters of FFs are found in an optimal description of all existing bare FF data in space-like and time-like regions simultaneously [6].

In this approach, the pseudoscalar EM FFs are first split into isoscalar and isovector parts

$$\begin{aligned} F_{\pi^\pm}(s) &= F_\pi^{I=1}[W(s)]\,; F_{K^\pm}(s) = F_K^{I=0}[V(s)] + F_K^{I=1}[W(s)]\,; \\ F_{K^0}(s) &= F_K^{I=0}[V(s)] - F_K^{I=1}[W(s)]\,; F_{\pi^0\gamma}(s) = F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)]\,; \\ F_{\eta\gamma}(s) &= F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)]\,; F_{\eta'\gamma}(s) = F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)] \end{aligned} \tag{6}$$

then, all theoretical form factor properties are incorporated, whereby $F^{I=1}(s)$ are saturated by ρ, ρ', ρ'' and $F^{I=0}(s)$ by $\omega, \phi, \omega', \phi', \omega'', \phi''$. As a result, every $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ is defined on the four-sheeted Riemann surface and dependent on coupling constant ratios (f_{VMM}/f_V) and effective inelastic thresholds t_{in} as free parameters of the models. The U&A model [7] can be used also for estimation of the contributions of the last two total cross sections, $\sigma^0_{\text{tot}}(e^+e^- \to p\bar{p})$ and $\sigma^0_{\text{tot}}(e^+e^- \to n\bar{n})$, to $\Delta\alpha^{(5)}_{\text{had}}(t(x))$. Since s_{cut} are used from one author to another to be different, calculations are carried out one after the other with $s_{\text{cut}} = 0.8$ GeV^2 [8], $s_{\text{cut}} = 1.96$ GeV^2 [9], $s_{\text{cut}} = 2.0449$ GeV^2 [10], $s_{\text{cut}} = 3.0$ GeV^2 [11], $s_{\text{cut}} = 3.24$ GeV^2 [1], $s_{\text{cut}} = 3.75$ GeV^2 [2], $s_{\text{cut}} = 4.0$ GeV^2 [12]. Every contribution to $\Delta\alpha^{(5)}_{\text{had}}(t(x))$, calculated in the first integral, including also contributions from the second and third integral in (5), is represented by a summary curve in Fig. 2 in the logarithmic scale. However, curves dependent on s_{cut} are not distinguishable there.

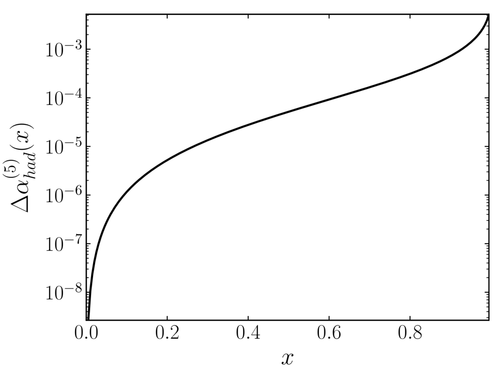

Fig. 2. Sum of all predicted curves in the logarithmic scale.

Therefore, we integrate every of them by means of relation (3) and the resultant values of $a_\mu^{\rm (LO)had}$ are presented in Table I from which one can see that they do not depend on the choice of $s_{\rm cut}$, besides the first one, by various authors. Its central value can be explained by the fact that $s_{\rm cut} = 0.8$ GeV^2 corresponds to 0.894 GeV, in this case the first integral in (5), evaluated prevailingly by means of the U&A models of the corresponding FFs, does not cover the contribution of the ϕ-meson peak from Fig. 3.

TABLE I

Values of $a_\mu^{\rm (LO)had}$.

$s_{\rm cut}$ [GeV^2]	$a_\mu^{\rm (LO)had} \times 10^{-10}$
0.80	(700.083 ± 2.866)
1.96	(706.666 ± 4.018)
2.0449	(707.005 ± 3.531)
3.0	(708.095 ± 4.616)
3.24	(707.591 ± 4.124)
3.752	(707.215 ± 4.226)
4.0	(706.820 ± 3.741)

It is taken into account by means of the trapezoidal integration in the second integral of (5) through the data in Fig. 3 from $R^{\rm data}(s')$ and these data are little bit sparse.

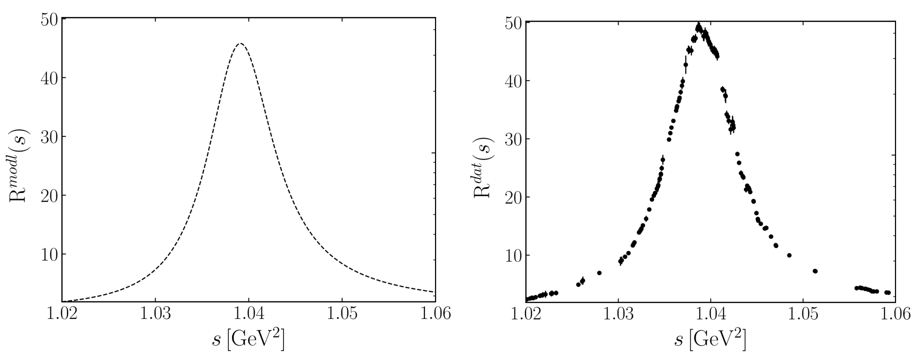

Fig. 3. ϕ-peak contribution by U&A-models (left figure) and by inclusive R (right figure).

The values in [1, 2], $(693.1 \pm 3.4) \times 10^{-10}$ and $(693.26 \pm 2.46) \times 10^{-10}$, respectively, are lower than ours in Table I and this effect can be explained by similar arguments as presented above. Evaluations of integrals from the lowest threshold $s = m^2_{\pi^0}$ up to the $s_{\rm pQCD}$ in [1, 2] have been carried out by the trapezoidal method.

Adding to our averaged value (the first value from Table I is not included) $\bar{a}_\mu^{(\mathrm{LO})\mathrm{had}} = (707.232 \pm 4.043) \times 10^{-10}$ the contributions from higher order hadronic loops, -9.87 ± 0.09 (NLO) and 1.24 ± 0.01 (NNLO), the hadronic light-by-light scattering 10.5 ± 2.6, as well as QED 11658471.895 ± 0.008 and electroweak effects 15.36 ± 0.10, one obtains the complete SM prediction to be $a_\mu^{\mathrm{SM}} = (11659196.35 \pm 4.81) \times 10^{-10}$. This result deviates from the world average experimental value $a_\mu^{\exp} = (11659209 \pm 6) \times 10^{-10}$ by $12.65 \pm 7.69 (1.6\sigma)$.

3. Conclusions

We have used a novel approach [3] and the elaborated Unitary and Analytic models of electromagnetic structure of pseudoscalar meson nonet [6] to improve the LO of hadronic contribution to muon $g-2$ anomaly achieved recently in [1] and [2]. Adding to our averaged value $\bar{a}_\mu^{(\mathrm{LO})\mathrm{had}} = (707.232 \pm 4.043) \times 10^{-10}$, the contributions from higher order hadronic loops, the hadronic light-by-light scattering contribution, as well as value from QED and electroweak effects, the obtained result $a_\mu^{\mathrm{SM}} = (11659196.35 \pm 4.81) \times 10^{-10}$ deviates from the world average experimental value $a_\mu^{\exp} = (11659209 \pm 6) \times 10^{-10}$ by 1.6σ.

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REFERENCES

[1] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, *Eur. Phys. J. C* **77**, 827 (2017).
[2] A. Keshavarzi, D. Nomura, Th. Teubner, *Phys. Rev. D* **97**, 114025 (2018).
[3] C.M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni, *Phys. Lett. B* **746**, 325 (2015).
[4] A.Z. Dubnickova, S. Dubnicka, A. Liptaj, *Acta Phys. Pol. B Proc. Suppl.* **13**, 17 (2020).
[5] G. Abbiendi *et al.*, *Eur. Phys. J. C* **77**, 139 (2017).
[6] S. Dubnicka, A.Z. Dubnickova, *Eur. Phys. J. Web Conf.* **37**, 01003 (2012).
[7] C. Adamuscin, E. Bartos, S. Dubnicka, A.Z. Dubnickova, *Phys. Rev. C* **93**, 055208 (2016).
[8] J.F. de Trocóniz, F.J. Ynduráin, *Phys. Rev. D* **71**, 073008 (2005).
[9] S. Eidelman, F. Jegerlehner, *Z. Phys. C* **67**, 585 (1995).
[10] K. Hagivara, A.D. Martin, D. Nomura, T. Teubner, *Phys. Lett. B* **649**, 173 (2007).
[11] A.Z. Dubnickova, S. Dubnicka, A. Liptaj, *Acta Phys. Pol. B Proc. Suppl.* **9**, 407 (2016).
[12] K. Hagivara *et al.*, *J. Phys. G* **38**, 085003 (2011).