# DOUBLE PARTON DISTRIBUTIONS OF THE PION*

Wojciech Broniowski

Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland
and
H. Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences
31-342 Kraków, Poland
Wojciech.Broniowski@ifj.edu.pl

Enrique Ruiz Arriola

Departamento de Física Atómica, Molecular y Nuclear
and
Instituto Carlos I de Física Teórica y Computacional Universidad de Granada
18071 Granada, Spain
earriola@ugr.es

*(Received June 23, 2020)*

We present a calculation of valence double parton distributions of the pion in the framework of chiral quark models. The result obtained at the low-energy quark model scale is particularly simple, where in the chiral limit a factorized form follows, $D(x_1, x_2, \vec{q}) = \delta(1 - x_1 - x_2)F(\vec{q})$ with $x_{1,2}$ standing for the longitudinal momentum fractions of the valence quark and antiquark, and $\vec{q}$ denotes the relative transverse momentum. For $\vec{q} = \vec{0}$, the result satisfies the Gaunt–Sterling sum rules. The QCD evolution to higher scales is carried out within the dDGLAP framework. We argue that the ratios of the valence Mellin moments $\langle x_1^n x_2^m \rangle / \langle x_1^n \rangle \langle x_2^m \rangle$, which do not depend on the dDGLAP evolution, provide particularly convenient measures of the longitudinal correlations between the partons. Such ratios could be probed in future lattice QCD simulations.

DOI:10.5506/APhysPolBSupp.14.175

This contribution is based on our recent work on double parton distributions of the pion [1, 2], also explored by Courtoy *et al.* [3]. The old story of double parton distribution (dPDFs) [4], followed with early experimental

* Presented by W. Broniowski at *Excited QCD 2020*, Krynica Zdrój, Poland, February 2–8, 2020.

searches by the Axial Field Spectrometer Collaboration at the CERN ISR [5] and by the CDF Collaboration at Fermilab [6, 7], has recently picked up renewed interest [8–17] with a growing evidence from the LHC, *e.g.*, [18, 19]. Experimentally, the results concern mostly the structure of the proton which naturally provides a stable target. The proton, however, is much harder to model theoretically than the pion, for which there is a lack of experimental data. On the other hand, the pion is the simplest hadronic bound state appearing as a pseudo-Goldstone boson of the spontaneously broken chiral symmetry. Hence, we explore the pion, with the hope the result can be verified in the near future with lattice QCD studies, where the pion is readily available at the physical quark masses.

The field-theoretic definition of valence dPDFs generalizes the case of the single parton distribution functions (sPDFs), namely (see [20] and references therein)

$$\begin{aligned} D_{j_1 j_2}(x_1, x_2, \boldsymbol{b}) \; &= \; 2p^+ \int \mathrm{d}b^- \, \frac{\mathrm{d}z_1^-}{2\pi} \, \frac{dz_2^-}{2\pi} \, \mathrm{e}^{i\left(x_1 z_1^- + x_2 z_2^-\right)p^+} \\ &\quad \times \langle p| \, \mathcal{O}_{j_1}(b, z_1) \, \mathcal{O}_{j_2}(0, z_2) \, |p\rangle \big|_{z_1^+ = z_2^+ = b^+ = 0 \, , \boldsymbol{z}_1 = \boldsymbol{z}_2 = \boldsymbol{0}} \, , \end{aligned} \tag{1}$$

where $p$ is the momentum of the hadron, $j_{1,2}$ labels the quark species, $x_{1,2}$ are the fractions of the light-cone momentum of the hadron carried by the valence quark or antiquark, $z_{1,2}$ are the coordinates on the light front, $b$ is the spatial separation of the two bilocal operators for the valence quark and antiquark,

$$\begin{aligned} \mathcal{O}_q(y, z) \; &= \; \tfrac{1}{2} \, \bar{q} \left(y - \tfrac{z}{2}\right) \gamma^+ q \left(y + \tfrac{z}{2}\right) \, , \\ \mathcal{O}_{\bar{q}}(y, z) \; &= \; -\tfrac{1}{2} \, \bar{q} \left(y + \tfrac{z}{2}\right) \gamma^+ q \left(y - \tfrac{z}{2}\right) \, . \end{aligned} \tag{2}$$

Our convention for the light-cone variables is $a^\pm = (a^0 \pm a^3)/\sqrt{2}$, and $\boldsymbol{a} = (a_1, a_2)$. In the applied chiral quark model, the evaluation of (1) is carried out in the Fourier-conjugated space, as illustrated by the diagram in Fig. 1, where for definiteness we take the case of the charged pion, $\pi^+$. We note that the momentum $q$ flows between the two probing operators, bringing in the information on transverse structure of the pion. The integration over $q^-$ enforces the constraint $b^+ = 0$, whereas the transverse component $\vec{q}$ is the Fourier-conjugate variable corresponding to $\vec{b}$.

In the chiral limit, the result is particularly simple [1–3]

$$D_{u\bar{d}}\left(x_1, x_2, \boldsymbol{q}\right) = 1 \times \delta\left(1 - x_1 - x_2\right) \Theta F\left(\vec{q}\right) \, , \tag{3}$$

where the presence of the $\delta$ function reflects the conservation of the light-front momentum and $\Theta$ reflects the proper support $0 \leq x_1, x_2 \leq 1$, following

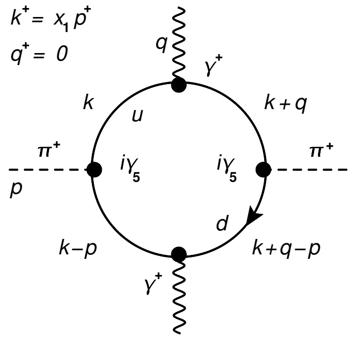


Fig. 1. Diagram for evaluation of the double valence quark distributions of $\pi^+$ at the leading-$N_c$ order (one-loop) in the NJL model. Note that $q^+ = 0$, whereas integration over $q^-$ is carried out.

from the Lorentz symmetry preserved in the calculation. The form factor in the transverse momentum, $F(\vec{q})$, depends on the adopted low-energy regularization scheme (see [2]), which is a necessary ingredient of the effective model. We note that the factorization of the light-front and transverse dynamics holds exactly only in the strict chiral limit. We note that, importantly, Eq. (3) satisfies the Gaunt–Stirling (GS) sum rules [21], holding in the case of $\vec{q} = 0$ and relating integrals of dPDFs with sPDFs. As the GS sum rules are preserved by the QCD evolution, they hold at any scale in the applied framework.

The QCD evolution is a necessary ingredient of the scheme, as first noticed in the study of sPDFs of the pion by Davidson and one of us (E.R.A.) [22]. Since then, the model, with evolution discussed in detail in [23], has been applied to numerous soft matrix elements relevant for high-energy processes, such as the parton distribution amplitude (PDA) [24], the generalized parton distributions (GPD) [23], and the quasi-parton distributions (QDF) [25, 26], with successful outcome. The dDGLAP evolution scheme [27, 28] is straightforward to implement in terms of the Mellin moments, similarly to sPDFs [2, 29]. Our results after evolution are shown in the left panels of Fig. 2. The right panels display the correlation $D_{u\bar{d}}(x_1, x_2)/D_u(x_1)D_{\bar{d}}(x_2)$. The regions in the plots marked with light gray/green have the correlation within 25% from the unity. We note that increasing the evolution scale brings the correlation closer to unity (*cf.* a similar behavior found for the gluon distributions of the nucleon in [30]).

A very simple measure of correlation is the ratio $\langle x_1^n x_2^m \rangle / \langle x_1^n \rangle \langle x_2^m \rangle$ which is independent of the evolution scale. In our model (in the chiral limit), we have

$$\frac{\langle x_1^n x_2^m \rangle}{\langle x_1^n \rangle \langle x_2^m \rangle} = \frac{(1+n)!(1+m)!}{(1+n+m)!} \,. \tag{4}$$

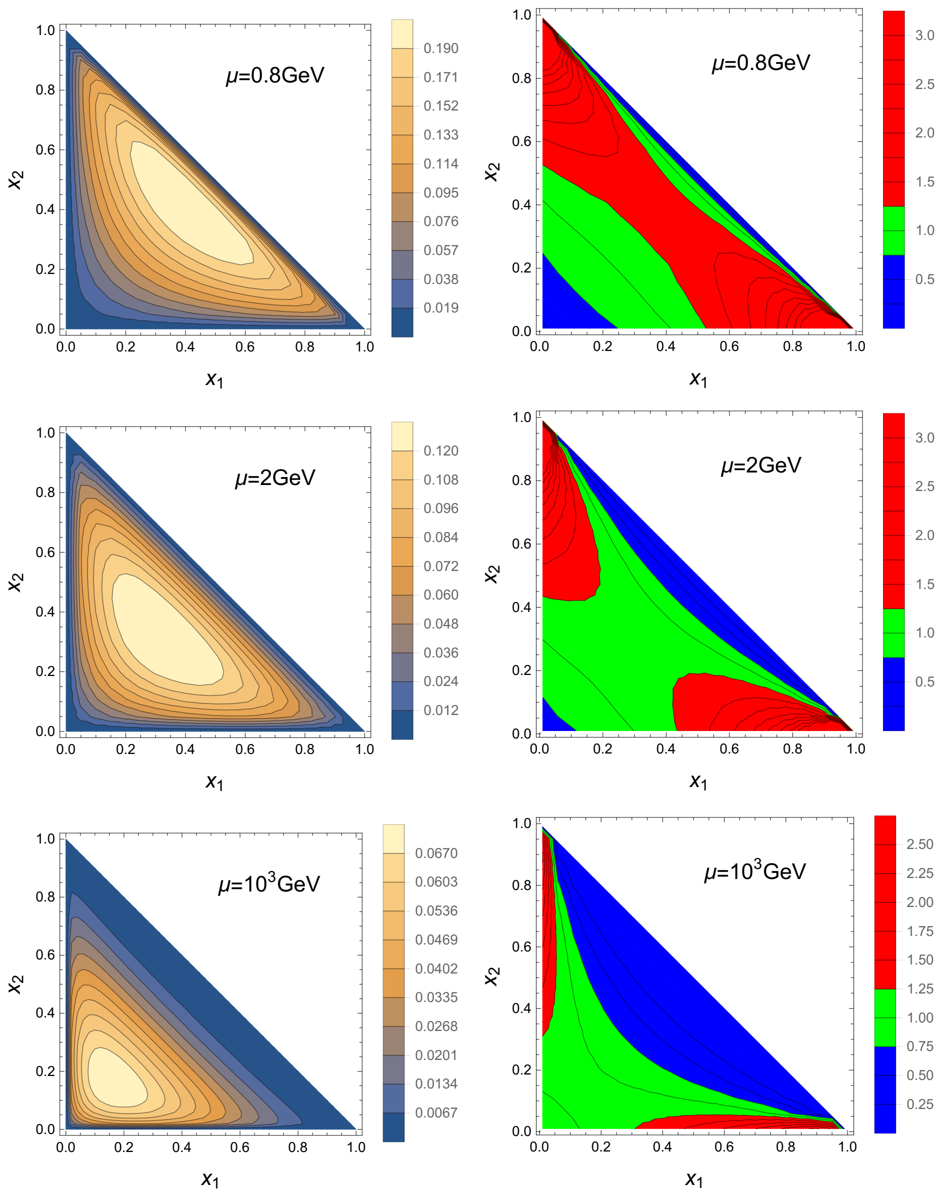


Fig. 2. (Color online) Left: Valence dPDF of the pion, $x_1 x_2 D_{u\bar{d}}(x_1, x_2)$, evolved with the dDGLAP equations to subsequent scales $\mu$. Right: Corresponding correlation $D_{u\bar{d}}(x_1, x_2)/D_u(x_1)D_{\bar{d}}(x_2)$.

The lowest correlation ratios can hopefully be probed in the future lattice QCD simulations, along the lines of determination of moments of sPDFs [31–34].

In summary, our results from a covariant framework combining chiral quark model evaluation with QCD evolution satisfy all formal requirements, in particular the GS sum rules. In the chiral limit, the light-front and transverse dynamics is factorized, whereas in the $x_1$ and $x_2$ spaces there is no factorization due to the momentum conservation. We have proposed simple scale-independent ratios of the Mellin moments as probes of the correlation. The lowest ratios could be obtained from future lattice simulations.

Supported by the National Science Centre, Poland (NCN) grant 2018/31/B/ST2/01022, the Spanish Ministerio de Economia y Competitividad and European FEDER funds grant FIS2017-85053-C2-1-P, and Junta de Andalucía grant FQM-225.

## REFERENCES

[1] W. Broniowski, E. Ruiz Arriola, «Double parton distributions of the pion in the NJL model», talk by W.B. at Light Cone 2019, Palaiseau, France, September 16—20, 2019, `https://indico.cern.ch/event/734913/contributions/3533707/attachments/1910076/3157511/LC19.pdf`

[2] W. Broniowski, E. Ruiz Arriola, *Phys. Rev. D* **101**, 014019 (2020), `arXiv:1910.03707 [hep-ph]`.

[3] A. Courtoy, S. Noguera, S. Scopetta, *J. High Energy Phys.* **1912**, 45 (2019), `arXiv:1909.09530 [hep-ph]`.

[4] J. Kuti, V.F. Weisskopf, *Phys. Rev. D* **4**, 3418 (1971).

[5] Axial Field Spectrometer Collaboration (T. Akesson *et al.*), *Z. Phys. C* **34**, 163 (1987).

[6] F. Abe *et al.*, *Phys. Rev. D* **47**, 4857 (1993).

[7] CDF Collaboration (F. Abe *et al.*), *Phys. Rev. D* **56**, 3811 (1997).

[8] B. Blok, Y. Dokshitzer, L. Frankfurt, M. Strikman, *Phys. Rev. D* **83**, 071501 (2011), `arXiv:1009.2714 [hep-ph]`.

[9] B. Blok, Yu. Dokshitser, L. Frankfurt, M. Strikman, *Eur. Phys. J. C* **72**, 1963 (2012), `arXiv:1106.5533 [hep-ph]`.

[10] P. Bartalini *et al.*, `arXiv:1111.0469 [hep-ph]`.

[11] A. Snigirev, *Phys. Atom. Nucl.* **74**, 158 (2011).

[12] M. Łuszczak, R. Maciuła, A. Szczurek, *Phys. Rev. D* **85**, 094034 (2012), `arXiv:1111.3255 [hep-ph]`.

[13] A.V. Manohar, W.J. Waalewijn, *Phys. Rev. D* **85**, 114009 (2012), `arXiv:1202.3794 [hep-ph]`.

[14] A.V. Manohar, W.J. Waalewijn, *Phys. Lett. B* **713**, 196 (2012), arXiv:1202.5034 [hep-ph].

[15] D. d'Enterria, A.M. Snigirev, *Phys. Lett. B* **718**, 1395 (2013), arXiv:1211.0197 [hep-ph].

[16] B. Blok, Yu. Dokshitzer, L. Frankfurt, M. Strikman, *Eur. Phys. J. C* **74**, 2926 (2014), arXiv:1306.3763 [hep-ph].

[17] P. Bartalini, J.R. Gaunt (Eds.) «Advanced Series on Directions in High Energy Physics, Chapter 1: Introduction», *World Scientific*, 2018, p. 1.

[18] ATLAS Collaboration (G. Aad *et al.*), *New J. Phys.* **15**, 033038 (2013), arXiv:1301.6872 [hep-ex].

[19] CMS Collaboration (A.M. Sirunyan *et al.*), *Eur. Phys. J. C* **80**, 41 (2020), arXiv:1909.06265 [hep-ex].

[20] M. Diehl, *PoS* **DIS2010**, 223 (2010), arXiv:1007.5477 [hep-ph].

[21] J.R. Gaunt, W.J. Stirling, *J. High Energy Phys.* **1003**, 5 (2010), arXiv:0910.4347 [hep-ph].

[22] R.M. Davidson, E. Ruiz Arriola, *Phys. Lett. B* **348**, 163 (1995).

[23] W. Broniowski, E. Ruiz Arriola, K. Golec-Biernat, *Phys. Rev. D* **77**, 034023 (2008), arXiv:0712.1012 [hep-ph].

[24] E. Ruiz Arriola, W. Broniowski, *Phys. Rev. D* **66**, 094016 (2002), arXiv:hep-ph/0207266.

[25] W. Broniowski, E. Ruiz Arriola, *Phys. Lett. B* **773**, 385 (2017), arXiv:1707.09588 [hep-ph].

[26] W. Broniowski, E. Ruiz Arriola, *Phys. Rev. D* **97**, 034031 (2018), arXiv:1711.03377 [hep-ph].

[27] R. Kirschner, *Phys. Lett. B* **84**, 266 (1979).

[28] V. Shelest, A. Snigirev, G. Zinovev, *Phys. Lett. B* **113**, 325 (1982).

[29] W. Broniowski, E. Ruiz Arriola, *Few-Body Syst.* **55**, 381 (2014), arXiv:1310.8419 [hep-ph].

[30] K. Golec-Biernat *et al.*, *Phys. Lett. B* **750**, 559 (2015), arXiv:1507.08583 [hep-ph].

[31] G. Martinelli, C.T. Sachrajda, *Phys. Lett. B* **196**, 184 (1987).

[32] A. Morelli, *Nucl. Phys. B* **392**, 518 (1993).

[33] C. Best *et al.*, *Phys. Rev. D* **56**, 2743 (1997), arXiv:hep-lat/9703014.

[34] W. Detmold, W. Melnitchouk, A.W. Thomas, *Phys. Rev. D* **68**, 034025 (2003), arXiv:hep-lat/0303015.