THE $X(3872)$ AS A MASS DISTRIBUTION* **

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All existing experimental evidence of the bound state nature of the $X(3872)$ relies on considering its decay products with a finite experimental spectral mass resolution which is typically $\Delta m \geq 2$ MeV and much larger than its alleged binding energy, $B_X = 0.00(18)$ MeV. On the other hand, there is a neat cancellation in the 1^{++} channel for the invariant $D\bar{D}^*$ mass around the threshold between the continuum and bound state contribution. We discuss the impact of this effect for $X(3872)$ at finite temperature, in prompt production in pp collisions data with a finite $p_{\rm T}$ or the lineshapes of specific production experiments of exotic states involving triangle singularities.

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1. Introduction

While the QCD spectrum is expected to describe the experimentally observed hadronic states, the precise manner how this is supposed to happen is not at all clear in the real world where most states belong to the continuum. Besides, a quantitative measure of the "distance" between two different spectra is never used. Moreover, with the exception of low-lying hadronic states, which are truly stable particles at the strong interactions level, most

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of the remaining reported states are resonances with a finite lifetime and hence undergo strong decays. Besides, the very definition of the hadronic density of states includes the corresponding resonance background, which has no universal definition and its phenomenological determination depends on the particular process where the resonance shows up.

Two extremely different and complementary alternatives to this issue rely on either the individual level-by-level analysis based on hadronic reactions or on the collective approach addressing bulk properties such as level density of states or thermodynamic properties. While the Particle Data Booklet [1] summarizes all relevant information concerning established states with a given confidence level, there always arises the question which are the *a priori* pre-requisites making a possible observed peak or bump in a hadronic reaction qualify as an eligible hadronic state, so that a majority vote of the experts prevails. The thermodynamic approach does offer a global perspective based on the Hadron Resonance gas which works fairly well in the confined phase of a hot hadronic medium, and has been checked both in ultra-relativistic heavy-ions collisions as well as on lattice QCD. At the present moment, and after some long discussions on the nature of states, it is fair to say that the currently accepted PDG states are relatives to the naive quark model, either below or above the different two-heavy-light meson states, such as $D\bar{D}$, $D\bar{D}^*$, *etc.* This also suggests that the Hadron Resonance Gas corresponds to blindly implemented *all* the listed PDG states assuming their existence is established (see *e.g.* Ref. [2] for a pedagogical presentation and references therein).

The proliferation of X, Y, Z states in the heavy sector above the charmonium in the last decade at different experimental facilities makes this pertinent question more acute in view of the fact that many of them are of molecular nature and hence weakly bound: should these states be reported as genuine contributions to the hadronic level density? In this article, we share our views on the subject taking the prominent $X(3872)$ state as an example in the $J^{PC} = 1^{++}$ channel [3–5].

The question was raised long ago by Dashen and Kane [6] where they pointed out that the mere counting of hadrons depends on the energy or mass resolution ΔM with which we decide to bin the states. If we take ΔM to be the typical SU(3)-splitting, some states may not count, due to a cancellation of the bound state and the continuum contributions in the level density. As an example, they provided the uncontroversial deuteron state, a $J^{PC} = 1^{++}$ weakly bound state of a proton and a neutron with a total mass of $M_d = 1.87561$ GeV, where the binding energy $B = 2.2$ MeV is about one per mille the total mass, and their mean separation is much larger than the size of the nucleons (see also Ref. [7]). According to this observation, the deuteron should not be listed in the PDG. Thus, what can be said in this regard about the $X(3872)$?

2. Occupation number in the continuum

The best way to illustrate the impact of the Dashen–Kane cancellation is by looking at occupation numbers at finite temperature. For a single mass state, the occupation number is given by

$$\bar{n} = \frac{\langle N \rangle_T}{V} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{g}{\mathrm{e}^{\sqrt{k^2+m^2}/T} + \eta} = \frac{T^3}{2\pi^2} \sum_{n=1}^{\infty} g \frac{(-\eta)^{n+1}}{n} \left(\frac{m}{T}\right)^2 K_2\left(\frac{nm}{T}\right), \tag{1}$$

where $\eta = \pm 1$ for fermions or bosons, respectively, and K_2 are modified Bessel functions. This formula applies only to bound states, but according the quantum viral expansion [8], the contribution of binary collisions to the partition function should be accounted for as a modified occupation number in each J^{PC}channel

$$n(T) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{d}m \frac{g}{\mathrm{e}^{\sqrt{p^2+m^2}/T} + \eta} \rho(m)\,, \tag{2}$$

which takes into account all the two-particle states with their interaction characterized by their level density in the continuum which can be written in terms of two-particle bound states with masses m_n, and the scattering eigen phase-shifts in the channels sharing the same J^{PC} quantum numbers

$$\rho(m) = \sum_n \delta(m - m_n) + \frac{1}{\pi} \sum_{\alpha}^{n} \frac{\mathrm{d}\delta_\alpha}{\mathrm{d}m}\,, \qquad m = \sqrt{s}\ (\text{CM energy})\,. \tag{3}$$

Two interesting particular cases are worth to consider. For a (Breit–Wigner) resonance such as $\rho \to \pi\pi$ or $\Delta \to \pi N$, one has

$$\delta(m) = \tan^{-1}\left[\frac{m - m_\mathrm{R}}{\Gamma_\mathrm{R}}\right] \to \frac{1}{\pi}\delta'(m) = \frac{1}{\pi} \frac{\Gamma_\mathrm{R}}{(m - m_\mathrm{R})^2 + \Gamma_\mathrm{R}^2}\,. \tag{4}$$

The other case of interest is that of a weakly bound state close to the continuum $d \to pn$ or $X(3872) \to D\bar{D}^*$ is given by the effective range expansion (ERA)

$$p \cot\delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \ldots\,, \qquad m = \sqrt{p^2 + M^2}\,. \tag{5}$$

In Fig. 1, we show both $\delta(m)$ in the ERA approximation compared to coupled channel quark [9] and EFT models [10, 11], the modified version of Levinson's theorem with confined channels is implemented [12]. We also show $\rho(m)$ for different Gaussian smearings, where the positive and negative contributions illustrate the Dashen–Kane cancellation, with the consequence that the corresponding occupation number in the 1^{++} channel is largely reduced as compared to the elementary $X(3872)$, $n_{1^{++}}(T) \ll n_X(T)$ for moderate temperatures [3]. The previous formulas can be used in the possible production of $X(3872)$ in heavy-ion collisions (see references in [3]).

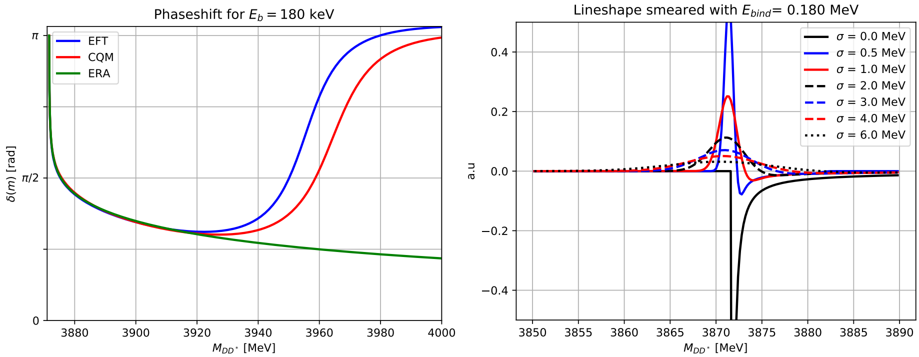

Fig. 1. Phase-shift for the $J^{PC} = 1^{++}$ comparing the effective range expansion with coupled channel quark or EFT models (left panel). Level density for different Gaussian smearings (right panel). We take $B_X = 180$ keV.

3. Production rates at RHIC and LHC

Surprisingly, both the deuteron and the $X(3872)$ have experimentally been produced in high-energy pp collisions, a somewhat puzzling result. The underlying reason of how such weakly heavy particles can be produced has not yet been found. Actually, the large production cross section has been interpreted as the signature of a compact object and possibly a tetraquark state [13]. However, unlike the deuteron which leaves a track in the calorimeter, the detected $X(3872)$ is through its $X(3872) \to \rho J/\psi, \omega J/\psi$ decay channels, which implies a mass distribution. Thus, any state within the resolution $\pm\Delta m/2$ with $J^{PC} = 1^{++}$ will be recorded. Quite generally, for an observable $O(m)$, we get

$$O_{\Delta m} \equiv \int_{m-\Delta m/2}^{m+\Delta m/2} \mathrm{d}M \rho(M) O(M) \,. \tag{6}$$

In the case of $\Delta m \gg |B| \equiv |M_B - M_{\rm tr}|$,

$$O|_{M^B \pm \Delta m} = O\left(M^B\right) + \frac{1}{\pi} \int_{M_{\rm tr}}^{M_{\rm tr}+\Delta m/2} \mathrm{d}M \delta'_\alpha(M) O(M) \,. \tag{7}$$

The thermodynamic arguments can be extended to $p_{\rm T}$ distributions assuming a Tsallis distribution [5]. For a pure mass state, it reads

$$\frac{\mathrm{d}^3 N}{\mathrm{d}^3 p} = \frac{gV}{(2\pi)^3} \left(1 + (q-1)\frac{E(p)}{T}\right)^{-\frac{q}{q-1}} \xrightarrow{q\to 1} \frac{gV}{(2\pi)^3} \mathrm{e}^{-\frac{E(p)}{T}} \,, \tag{8}$$

with $E(p) = \sqrt{m^2 + p^2}$, so that the production cross section becomes

$$\frac{1}{2\pi p_{\mathrm{T}}}\frac{\mathrm{d}\sigma(m)}{\mathrm{d}p_{\mathrm{T}}} = \mathcal{N}\int \mathrm{d}y\; E(p_{\mathrm{T}},y)\left[1+\frac{q-1}{T}E(p_{\mathrm{T}},y)\right]^{\frac{q}{1-q}} \tag{9}$$

with $E(p_{\mathrm{T}},y) = \sqrt{p_{\mathrm{T}}^2+m^2}\cosh y$, $\mathrm{d}^3N/(\mathrm{d}^2p_{\mathrm{T}}\mathrm{d}y) \equiv E_p\mathrm{d}^3N/\mathrm{d}^3p$, and $y = \tanh^{-1}(p_z/E_p)$ the rapidity and $\mathcal{N}$ the normalization. We have found that deuteron d and $X(3872)$ accelerator production data are fully compatible with the *same parameters* [5] and that in fact $\mathcal{N}_X \sim \mathcal{N}_d$, so that there is nothing more special about the X than the deuteron, and so its large production rate at high p_{T}.

4. Detection methods and accurate mass measurements

Recent proposals to make accurate measurements of mass based on triangle singularities [14, 15] $e^+e^- \to X(3872)\gamma$ have been made since the lineshape is very sensitive to the binding energy, assuming no nearby continuum effects inherent in the detection problem. If we fold the level density $\rho(m)$ in the $J^{PC} = 1^{++}$ channel with the detector efficiency function

$$R_\sigma(m,M) = \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(m-M)^2}{2\sigma^2}}\,, \tag{10}$$

we get corresponding smeared lineshape profiles (see Fig. 1, right panel). A Monte Carlo simulation of the effects in the smeared profiles for different samples with $N = 100$ and $N = 1000$ is shown in Fig. 2. As we see, the large differences advocated in [14, 15], corresponding to the $\sigma \to 0$ case, are somewhat blurred, and set a standard on the necessary statistics discriminating different signals. More details will be presented elsewhere.

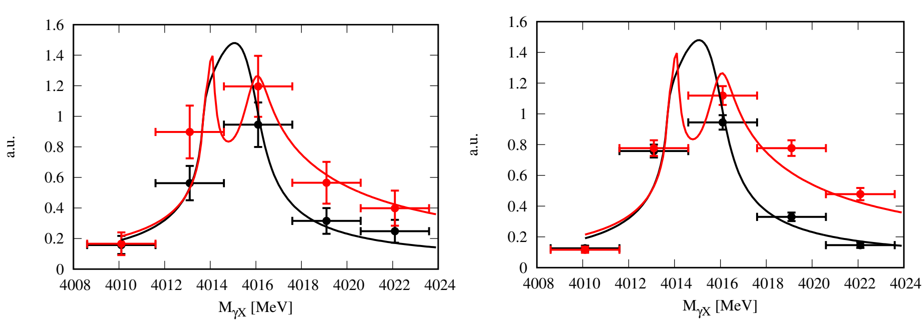

Fig. 2. (Color online) Binned smeared lineshapes of states for $\sigma = 1$ MeV, $E_w = 3$ MeV and $\Delta M = 20$ MeV for $E_b = 180$ keV (black points) and $E_b = -180$ keV (gray/red points). Full lines show the no-binned smeared lineshape (the same color code). The results are done with $N = 100$ (left), $N = 1000$ (right).

5. Conclusions

Regardless of its detailed molecular or quark–antiquark or mixed nature and composite structure, the weakly bound $X(3782)$ can be best regarded as a mass distribution as long as the operating resolution is much larger than the binding energy. The reason is that the signal of $X(3872)$ is by its decay products recorded within the detector resolution, and the mass distribution exhibits a cancellation between the bound state and the continuum. This is most clearly seen by analyzing the occupation number at finite temperature and in high-energy proton–proton collisions which show clear departures from the elementary limit. This cancellation is also of relevance in possible precision measurements based on a lineshape profile sensitive to the numerical value binding energy.

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