

3-FLAVOR EXTENSION OF THE EXCLUDED VOLUME MODEL FOR THE HARD-CORE REPULSION*

DYANA C. DUARTE, SAUL HERNANDEZ-ORTIZ, KIE SANG JEONG

Institute for Nuclear Theory, University of Washington
Seattle, Washington 98195, USA

(Received July 23, 2020)

We construct a model for quarkyonic matter with an extended version of the excluded volume model to a three-flavor system, as an alternative to obtain a hard-soft behavior of the equation of state inferred from gravitational wave observations and analysis.

DOI:10.5506/APhysPolBSupp.14.73

1. Introduction

There are many motivations to study the phase structure of quantum chromodynamics (QCD) related to investigations of relativistic heavy-ion collisions, early universe, and compact stars. Moreover, the QCD phase diagram remains poorly understood, despite all the efforts dedicated to its description over the years, due to the difficulty of first principle calculations and the lack of experimental observation (for a good review in this subject see, for example, [1–4] and references therein). On the other hand, observation and analysis of gravitational waves have been providing important tools for the understanding of nuclear dense matter. In Ref. [5], there are some of the most important references in this topic.

Recent results of GW170817 strongly suggest that equation of state (EoS) has to be hard enough to support two solar masses at few times nuclear density but, at the same time, in the low-intermediate density regime, its hardness has to become moderated to satisfy a radius lesser than 13.5 km at 1.4 solar masses [5]. These constraints will also be reflected in the sound velocity squared, v_s^2 , that will increase fast with the density and exceeds its conformal value of $1/3$. This constraint for the radius was inferred by tidal deformability in the neutron star inspiral with 90% credence, and posterior analyses suggest even more compact neutron stars [6]. It is not an easy task

* Presented at *Excited QCD 2020*, Krynica Zdrój, Poland, February 2–8, 2020.

to conciliate an EoS hard at small densities with a moderate hardness at intermediate-high density. For example, allowing more degrees of freedom such as hyperons to the system usually makes the EoS softer, since they will take the low-momentum phase space and their decay channels may lead to a lower Fermi level. However, if the EoS becomes too soft, it would be necessary to consider some repulsive nuclear interaction and, even then, it could not be enough to support this small radius. Although many models predict a phase transition to a quark phase at high densities, this realization is also problematic, since there will be debates about signs of this hypothetical transition. As an alternative candidate it is worth to consider a model for the quarkyonic matter, which naturally generates the hard-soft evolution of EoS.

2. Quarkyonic matter for the description of neutron stars EoS

This novel phase of QCD is argued from large N_c construction, and was proposed in Ref. [7]. For low values of quark chemical potential μ_q , the deconfinement temperature T_d does not depend on μ_q , and hadronic phase is separated from deconfined phase. With increasing μ_q , a finite baryon density n_B appears and, when μ_q exceeds the quark mass, quarkyonic phase takes place. This change of baryon density is accompanied by a change in the pressure that is independent of N_c in hadron phase and of the order of N_c in quarkyonic phase. Description of quarkyonic matter as a weakly interacting system conflicts with the confinement feature; on the other hand, if we use a description in terms of a strongly interacting (baryonic) system, the pressure should be dominated by baryons interactions of $\mathcal{O}(N_c)$, instead of the kinetic pressure of $\mathcal{O}(1/N_c)$. To conciliate these apparently contradictory ideas, it was purposed that quarks deep inside Fermi sphere weakly interact due to the Pauli blocking, while quarks near the Fermi surface form baryons that interact strongly in a shell of width of Λ_{QCD} [7]. In this way, bulk quantities as pressure and entropy are dominated by quarks, while physical excitations are dominated by color singlet mesons and baryons.

In quarkyonic matter picture, nucleons and quarks degrees of freedom are quasiparticles: quarks inside a Fermi sphere are weakly interacting, while quarks near the Fermi surface strongly interact and are confined in baryon-like states. In the deep Fermi surface, the Pauli principle does not allow a strong interaction of quarks, but at the Fermi surface, with a shell width of $\sim \Lambda_{\text{QCD}}$, these quarks can interact strongly through infrared singular gluons. Recent results show that quarkyonic matter can take into account the hard-soft behavior of the EoS [8], and many works have been developed in this sense [9, 10].

3. Three-flavor excluded volume model

3.1. Nucleons in an excluded volume

We extended the excluded volume model for the 3-flavor (n , p , and Λ) system, where each nucleon has its own effective hard-core size n_0 from the repulsive interaction. The Fermi momentum and number density in this repulsive nature can be written as

$$n_{N_i}^{\text{ex}} = \frac{n_{N_i}}{1 - n_{\tilde{N}}/n_0} = \frac{2}{(2\pi)^3} \int_0^{k_F^{N_i}} d^3k, \quad (1)$$

$$n_{\tilde{N}} = n_n + n_p + (1 + \alpha)n_\Lambda, \quad (2)$$

where α determines the strength of hard-core repulsion of Λ hyperon with neutrons and protons, and is set to be $\{\alpha = 0.2\}$ ¹. $k_F^{N_i}$ represents the Fermi momentum in the reduced space. The energy density of the 3-flavor system can be written as [9]

$$\varepsilon_N = \left(1 - \frac{n_{\tilde{N}}}{n_0}\right) \sum_{i=n,p,\Lambda} \int_0^{k_F^{N_i}} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M_{N_i}^2} + \kappa_e n_e^{\frac{4}{3}}, \quad (3)$$

where the last term is included to take into account the electromagnetic charge, $n_p = n_e$, and we also have to consider the decay channels $\mu_n \rightarrow \mu_p + \mu_e$ and $\mu_\Lambda = \mu_n$. Furthermore, $\{M_\Lambda = 1.2M_{p,n}, \kappa_e = \frac{(3\pi^2)^{4/3}}{4\pi^2}, n_0 = 5\rho_0\}$ ² and chemical potentials are calculated as

$$\mu_f = \frac{\partial \varepsilon_N}{\partial n_f} \quad \text{and} \quad \mu_B = \sum_{f=n,p,\Lambda,e} \mu_f \frac{\partial n_f}{\partial n_B}. \quad (4)$$

In the left panel of Fig. 1, we show the density profile for the nucleonic system. In this case, hyperon Λ appears at around $n_B \sim 3.5\rho_0$, but is expelled at $n_B \sim n_0$ due to its large repulsion in comparison to other nucleons. Baryon chemical potential obtained for this model including only nucleons naturally diverges at $n_B = \rho_0$ as might be seen in the right panel of Fig. 1, and generates a singular sound velocity. This behavior can be understood using arguments similar to those of the Hagedorn model [11]: when the temperature exceeds its critical value and entropy diverges, the system prefers to create more degrees of freedom instead of populate higher energy states. For our particular case, when n_B exceeds hard-core density, the system will prefer to generate quark degrees of freedom.

¹ For more details about this and some other choices of α , see Ref. [10].

² Here, $n_0 = 5\rho_0$ is related to the effective hard-core size, $n_0 = 1/v_0$, while $\rho_0 \sim 0.16 \text{ fm}^{-3}$ is the nuclear saturation density.

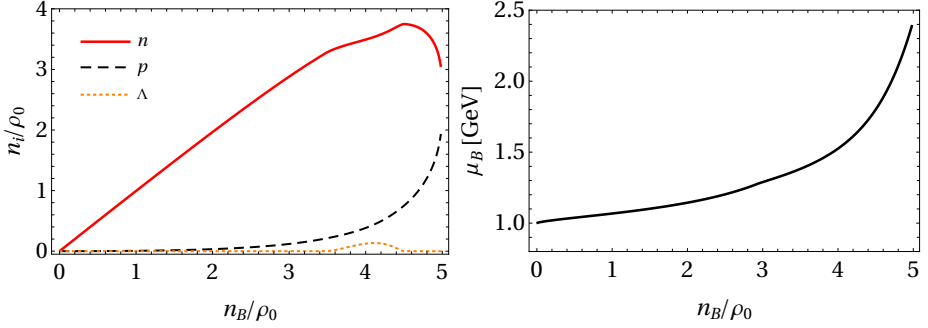


Fig. 1. Left: Density profile for neutrons (solid), protons (dashed) and Λ (dotted). Right: Baryon chemical potential given in Eq. (4). In both plots, we used $\alpha = 0.2$.

3.2. Mean-field mixture of nucleons and quarks

The energy density for the mean-field mixture of baryons and quarks can be written as

$$\begin{aligned} \varepsilon_{\text{mix}} = & \left(1 - \frac{n_{\tilde{N}}}{n_0}\right) \sum_{i=n,p,\Lambda} \int_0^{k_F^{N_i}} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M_{N_i}^2} \\ & + 2N_c \sum_{j=u,d,s} \int_0^{k_F^{Q_j}} \frac{d^3k}{2\pi^3} \sqrt{k^2 + M_{Q_j}^2} + \kappa_e n_e^{\frac{4}{3}}. \end{aligned} \quad (5)$$

Individual chemical potentials can also be evaluated through equations (4), now including the derivatives with respect to up, down and strange quarks. Charge neutrality is guaranteed by the condition $n_e = n_p + 2n_{\tilde{u}} - n_{\tilde{d}} - n_{\tilde{s}}$, and we also must consider the constraints $\mu_{\tilde{d}} = \mu_{\tilde{u}} + 3\mu_e$ and $\mu_{\tilde{d}} = \mu_{\tilde{s}}$ for β -equilibrium. Moreover, since the total baryon density is fixed, from the minimum of ε_{mix} , we also have the constraint $\mu_n = \mu_{\tilde{d}} - \mu_e$. In these expressions, tilde denotes units of baryon number.

In both panels of Fig. 2, one may see the density profile for the mean-field mixture. We keep fixed the total baryon density $n_B = n_N + n_{\tilde{Q}}$ and evaluate the configuration that minimizes energy density in Eq. (5). The results show that for $\alpha = 0.2$, s quark arise at $n_B \sim 6.2\rho_0$ and Λ hyperon at $n_B \sim 7.5\rho_0$. This behavior as well as the fractions of Λ and s are strongly dependent on the choice of α , as may be seen in Ref. [10].

One may see from the right panel of Fig. 3 that nucleonic system presents a very hard EoS, while the mean-field mixture shows a hard-soft behavior, in qualitative agreement to neutron star EoS obtained by gravitational waves data.

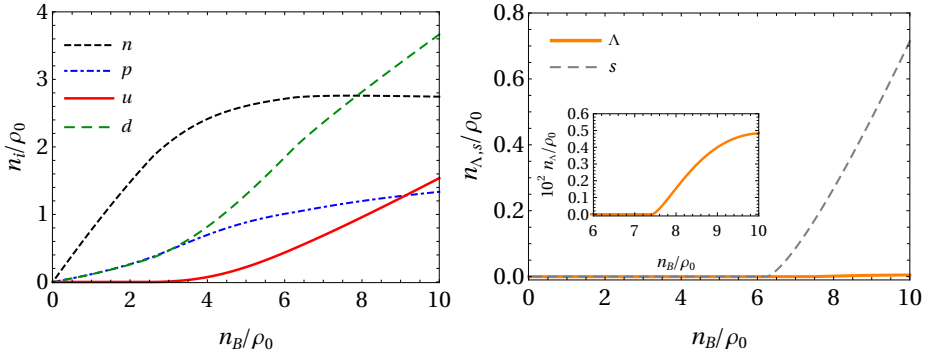


Fig. 2. Left: Density profile for neutron, proton, up and down quarks. Right: Density profile for hyperon Λ and strange quark. Both plots represent the results for the mean-field mixture.

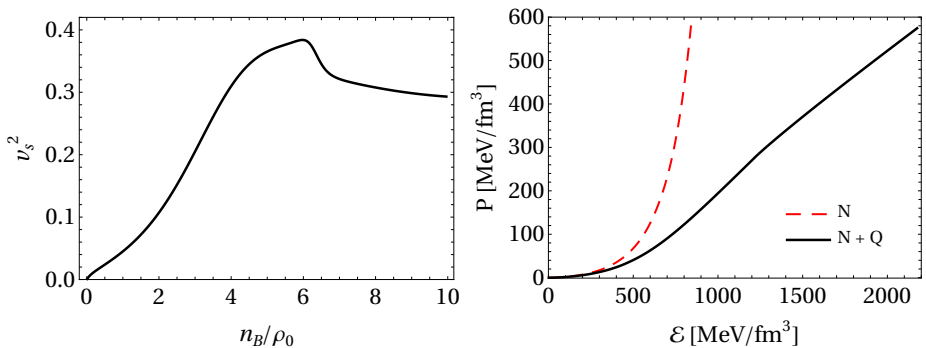


Fig. 3. Left: Sound velocity square. Right: Equation of state of nucleonic system (dashed) and mean-field mixture (solid).

4. Final remarks

We presented a model for the mixture of quarks and nucleons as an alternative to take into account the gravitational wave constraints for the EoS of neutron stars. The idea of making use of quarkyonic matter to this end was previously suggested in Ref. [8] where a formula was proposed for the thickness of Fermi surface of nucleons, and also in Ref. [9], for a single-flavor model where separated regions for nucleons and quarks were considered. In this first step, we still may not obtain realistic results for a direct comparison with EoS inferred from gravitational waves data, but the mean-field mixture already shows the expected hard-soft behavior. In our model, since we are not considering the Pauli exclusion principle, quark degrees of freedom are present even for very small baryon density, making the EoS harder in this

regime. The same analysis is being developed using a more realistic model for the quarkyonic picture, taking into account the Pauli principle where nucleons form a shell-like structure around a Fermi sphere of quarks.

Authors thanks Larry McLerran for insightful comments and discussions. Authors acknowledge the support of the Simons Foundation under the Multifarious Minds Program grant 557037. This work was supported by the U.S. Department of Energy under grant No. DE-FG02-00ER41132.

REFERENCES

- [1] M.A. Stephanov, *Prog. Theor. Phys. Suppl.* **153**, 139 (2004).
- [2] M. Stephanov, *PoS LATT2006*, 024 (2006).
- [3] K. Fukushima, T. Hatsuda, *Rep. Prog. Phys.* **74**, 014001 (2011).
- [4] S. Sharma, *PoS LATTICE2018*, 009 (2019).
- [5] LIGO Scientific and Virgo collaborations (B.P. Abbott *et al.*), *Phys. Rev. Lett.* **119**, 161101 (2017); *ibid.* **121**, 161101 (2018); F.J. Fattoyev, J. Piekarewicz, C.J. Horowitz, *Phys. Rev. Lett.* **120**, 172702 (2018); E. Annala, T. Gorda, A. Kurkela, A. Vuorinen, *Phys. Rev. Lett.* **120**, 172703 (2018); C. Raithel, F. Özel, D. Psaltis, *Astrophys. J.* **857**, L23 (2018); E.R. Most, L.R. Weih, L. Rezzolla, J. Schaffner-Bielich, *Phys. Rev. Lett.* **120**, 261103 (2018); I. Tews, J. Margueron, S. Reddy, *Eur. Phys. J. A* **55**, 97 (2019).
- [6] C.D. Capano *et al.*, *Nature Astron.* **4**, 625 (2020).
- [7] L. McLerran, R.D. Pisarski, *Nucl. Phys. A* **796**, 83 (2007).
- [8] L. McLerran, S. Reddy, *Phys. Rev. Lett.* **122**, 122701 (2019).
- [9] K.S. Jeong, L. McLerran, S. Sen, *Phys. Rev. C* **101**, 035201 (2020).
- [10] D.C. Duarte, S. Hernandez-Ortiz, K.S. Jeong, *Phys. Rev. C* **102**, 025203 (2020), [arXiv:2003.02362 \[nucl-th\]](#).
- [11] R. Hagedorn, *Nuovo Cim. Suppl.* **3**, 147 (1965).