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CUMULANTS OF NET-CHARGE DISTRIBUTION FROM PARTICLE–ANTIPARTICLE SOURCES*

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It is shown how high-order cumulants of net-charge distribution in hadronic collisions at the LHC energies can be expressed via lower-order terms under the assumption that particle–antiparticle pairs are produced in the independent local processes. It is argued and tested with the HIJING model that this assumption is typically valid for net-proton fluctuations in the case when no critical behaviour is present in the system. Values estimated in such a way can be considered as baselines for direct measurements of high-order net-charge fluctuations in real data.

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1. Introduction

In heavy-ion collision experiments, measurements of high-order fluctuations of conserved quantities, such as net-charge, net-baryon, net-strangeness, are of great importance since they should increase in the vicinity of the critical point of the QCD diagram [1] and may serve as a signature of the transition between the hadronic and partonic phases. These expectations are confirmed also by the lattice QCD calculations [2]. At the LHC energies, a smooth crossover between a hadron gas and the QGP is expected [2, 3]. Studies of the net-particle cumulant ratios are of a special interest because of their direct connection to susceptibilities theoretically calculable in the lattice QCD. In particular, net-proton fluctuations have been extensively studied experimentally [4, 5].

The net-charge is defined as $\Delta N = N^+ - N^-$, where N^+ and N^- are the numbers of positively and negatively charged particles measured in an

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event within rapidity acceptance Y (*i.e.* $y \in -Y/2, Y/2$). For example, the second cumulant of net-charge distribution is given by

$$\kappa_2(\Delta N) = \left\langle (\Delta N - \langle \Delta N \rangle)^2 \right\rangle = \left\langle (\Delta N)^2 \right\rangle - \langle \Delta N \rangle^2 , \qquad (1)$$

where angular brackets denote averaging over events. Expressions for higher-order cumulants are increasingly more complicated.

If fluctuations of both N^+ and N^- are Poissonian, ΔN has the Skellam distribution, with cumulants $\kappa_r(\Delta N) = \langle N^+ \rangle + (-1)^r \langle N^- \rangle$, $r = 1, 2, \ldots$, such that, for instance, ratio $\kappa_4/\kappa_2 = 1$. The Poissonian particle production is usually considered as a baseline model. However, in reality the cumulants of particle distributions are very sensitive to two so-called *non-dynamical* contributions that are not related to criticality in the system. The first contribution comes from the fluctuations of a number of emitting sources — the so-called "volume fluctuations" (VF) [6, 7]. The second contribution is due to charge conservation laws, for example, from neutral resonances decaying into pairs of oppositely charged particles. These two effects make interpretation of experimental measurements of the cumulants highly non-trivial, especially for higher-order cumulants. Both of them should be taken into account when one tries to extract signals of critical behaviour from measured observables [6, 8, 9]. At the LHC energies, however, it is possible to construct a simple baseline model that includes both these effects, if one assumes production of oppositely charged pairs that are nearly uncorrelated in rapidity. This is demonstrated in Section 2, and tested with HIJING event generator in Section 3 for the case of net-proton fluctuations. More details can be found in [10].

Yet another caveat about ordinary cumulants κ_r is that they get trivial contributions to all orders due to self-correlations. It was shown in [11, 12] that self-correlations can be removed systematically by constructing *factorial* cumulants. This is briefly considered in Section 4.

2. Decomposition of cumulants for two-particle sources

Creation of oppositely charged particle pairs is governed by a local charge conservation. The simplest case of a pair production process is a two-body neutral resonance decay, where integer +1 and −1 charges are produced and net-charge contribution to cumulants from a resonance is determined solely by its decay kinematics and resonance spectra. Another process is string fragmentation that produces fractional charges at each breaking point (quarks, diquarks), which then combine with partons from next breaking points. That may lead to a correlation between hadrons coming from several adjacent parts of a string, and, therefore, influence net-charge fluctuations in a complicated way.

In the case of protons and antiprotons, however, there are no resonances that decay into p and $\bar{p}$ pair. Such p–$\bar{p}$ pairs are produced mainly in string breaking (each of them may be produced directly or via a decay of a short-lived resonance). Moreover, a probability of production of two or more baryon pairs from adjacent parts of the same string is low. Registration of p–$\bar{p}$ pairs from jets in a low transverse momentum (p_{T}) range (typically one takes $p_{\mathrm{T}} \lesssim 2$ GeV$/c$) should be low as well. Therefore, if there are no processes other than resonance decays and string fragmentation, the p–$\bar{p}$ pairs visible in an event may be considered as nearly independent. This allows one to write simplifying expressions for the cumulants of net-charge fluctuations as it is described below.

Decompositions of cumulants for a system of N_{S} independent sources up to the fourth order are provided in [6] and up to the eighth order — in [10]. At the LHC energies, where $\langle N^+\rangle = \langle N^-\rangle$, the second and the fourth cumulants of net-charge distribution decompose as

$$\kappa_2(\Delta N) \;=\; k_2(\Delta n)\langle N_{\mathrm{S}}\rangle\,, \tag{2}$$
$$\kappa_4(\Delta N) \;=\; k_4(\Delta n)\langle N_{\mathrm{S}}\rangle + 3k_2^2(\Delta n)K_2(N_{\mathrm{S}})\,, \tag{3}$$

where $\Delta n = n^+ - n^-$ is a net-charge of a single source, and different notations for cumulants κ, k and K serve only for better visual distinction which distribution they are referred to. The ratio of the fourth to the second cumulant reads as

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = \frac{k_4}{k_2}(\Delta n) + 3k_2(\Delta n)\frac{K_2(N_{\mathrm{S}})}{\langle N_{\mathrm{S}}\rangle}\,. \tag{4}$$

The VF enter this equation via the second term that is proportional to the variance of the number of sources. The formulae above are valid for any types of sources, in particular, if "wounded nucleons" are considered [6]. Instead, we may treat the sources as *particle–antiparticle pairs*, for instance, p–$\bar{p}$. Note that these sources may be correlated to a certain extent (for example, due to radial and azimuthal flow) provided that swapping of the charges in each produced pair does not affect the physics of the whole event. The fourth cumulant for a single two-particle source simplifies to

$$k_4(\Delta n) = k_2(\Delta n) - 3k_2^2(\Delta n)\,. \tag{5}$$

We may now recall the argument that p–$\bar{p}$ pairs are nearly uncorrelated in rapidity and the fact that the distribution of $p(\bar{p})$ is nearly flat at mid-rapidity $|y| \lesssim 1$ at the LHC energies. It turns out that in this case, it is possible to rewrite the cumulant ratio (4) using quantities that are measurable in an experiment [10]

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N)R_2\left(N^+\right)\,, \tag{6}$$

where $R_2(N^+) = \langle N^+(N^+-1)\rangle / \langle N^+\rangle^2 - 1$ is the so-called robust variance of a number of positive particles measured within acceptance Y (equivalently, $R_2(N^-)$ could be used instead). Values of the cumulant ratio calculated with (6) could be considered as baselines for experimental measurements of the ratios (instead of, for instance, the Skellam baseline). Possible signals from critical phenomena would be indicated by some deviations from these baselines.

3. Application to realistic model

Validity of the assumptions about charged pair production done above was put into test using the HIJING Monte Carlo generator, which simulates multiple jet production and fragmentation of quark–gluon strings [13]. For that, analysis of net-proton fluctuations in Pb–Pb collisions simulated in HIJING was performed [10]. Protons and antiprotons within $y \in (-2, 2)$ and transverse momentum range of 0.6–2 GeV/c were selected. Figure 1 (a) shows the dependence on rapidity acceptance Y of the κ_4/κ_2 ratios calculated directly (circles) and by expression (6) (lines) in several centrality classes. Centrality was determined using multiplicity distribution in two symmetric $3 < |\eta| < 5$ ranges, which emulates the way of centrality determination in real experiments. A good agreement between the calculations can be seen in all classes, indicating that the assumption about the p–$\bar{p}$ pairs as nearly independent sources is approximately valid in HIJING. Slopes of the lines for different centrality classes reflect changes in VF via the second term

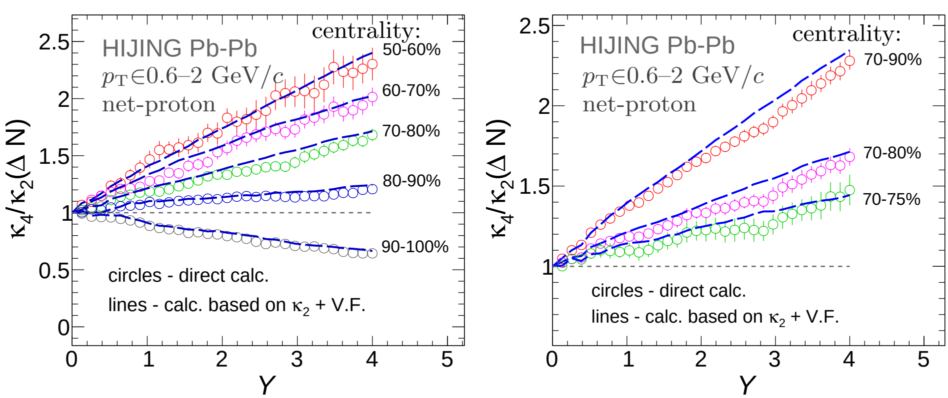

Fig. 1. Dependence of the net-proton κ_4/κ_2 ratio on the size of the rapidity acceptance Y in HIJING in Pb–Pb events at $\sqrt{s_{NN}} = 2.76$ TeV [10]. Direct calculations are shown by circles, analytical calculations with (6) — by dashed lines. Panel (a) — results in several centrality classes of the class width 10% are shown, (b) — dependence on the width of centrality class (20%, 10% and 5%) is demonstrated. Note that in each graph, there are point-by-point correlations as Y increases.

in (6). Panel (b) demonstrates a decrease of κ_4/κ_2 values with the width of a centrality class (when the width changes from 20% down to 5%), which is explained by a reduction of the volume fluctuations with the narrowing of the class. It was also checked that the robust variance $R_2(N^+)$ as a function of Y stays constant, which is essential for calculations with (6). More details of this study, in particular, a decomposition expression for the $\kappa_6/\kappa_2(\Delta N)$ ratio can be found in [10].

4. Factorial cumulants

The fourth-order factorial cumulant of net-charge distribution is given by

$$\begin{aligned} f_4 \; = \; & \kappa_4 - 6\left(\langle NQ^2\rangle - \langle N\rangle\langle Q^2\rangle - 2\langle NQ\rangle\langle Q\rangle + 2\langle N\rangle\langle Q\rangle^2\right) \\ & +8\left(\langle Q^2\rangle - \langle Q\rangle^2\right) + 3\left(\langle N^2\rangle - \langle N\rangle^2\right) - 6\langle N\rangle\,, \end{aligned} \tag{7}$$

where $N = N^+ + N^-$ and ΔN is denoted as Q for clarity [12]. It is interesting to check the behaviour of this observable in realistic models. As an example, Pb–Pb collisions from HIJING were analysed in the present work. Values of the factorial cumulant f_4 of net-proton distribution and the conventional κ_4/κ_2 ratios are shown in figure 2 as a function of the acceptance width Y in two centrality classes 70–80% and 80–90%[1]. The κ_4/κ_2 ratios are the

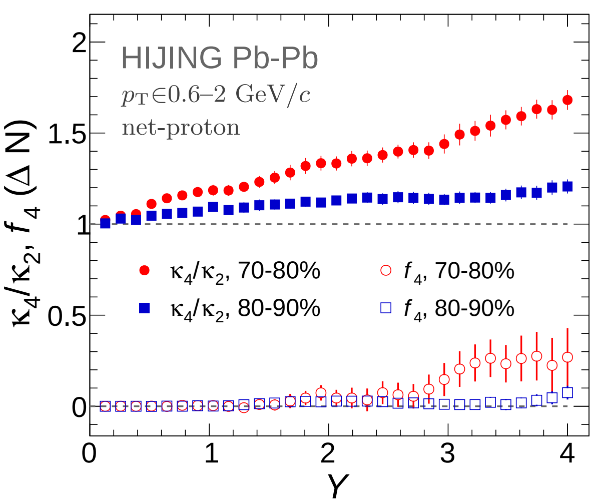

Fig. 2. Dependence net-proton cumulant ratio κ_4/κ_2 (closed markers) and factorial cumulant f_4 (open markers) on Y in HIJING. Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, centrality classes 70–80% and 80–90%.

[1] Results are shown for two peripheral centrality classes only, since for more central classes statistical uncertainties for f_4 are much larger.

same as in Fig. 1, and the values are above unity (*i.e.* above the Skellam baseline) due to the VF. Moreover, values in class of 70–80% are higher than in 80–90% since the VF in the former class are larger. In contrast, factorial cumulants f_4 are compatible with zero for both centralities. This is because factorial cumulants of the order of k remove contributions of lower orders $r < k$, which means, in particular, that net-proton f_4 should be suppressed in HIJING. Factorial cumulants are also much less sensitive to the VF than ordinary cumulants [12]. However, in distinction from ordinary cumulants, factorial cumulants cannot be directly compared with the lattice data [11, 12], therefore, their usefulness in studies of the QCD diagram is questionable.

5. Summary

It was shown that high-order cumulants of net-charge distribution can be decomposed into lower-order terms under the assumption of independent production of particle–antiparticle pairs. At the LHC energies, this should be a good approximation for net-proton fluctuations at mid-rapidity in the case if there is no critical behaviour in the system, as it was demonstrated for the κ_4/κ_2 ratio in HIJING. Such reduced expressions for high-order cumulants can be considered as baselines for direct experimental measurements. It was also shown that the fourth net-proton factorial cumulant in HIJING is compatible with zero, which also indicates that there are no sources of genuine high-order net-proton correlations in this generator.

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