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INTERVIEWING THE WEAK WITH STRONG COUPLING REGIMES VIA THE BULK TO SHEAR VISCOSITY RATIO IN QCD*

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We analyze the thermal profiles of the bulk to shear viscosity ratio in a quasiparticle framework which describes the interaction in a deconfined medium through dynamical masses of its constituents. The temperature dependence of the effective masses is specified by a running coupling deduced from the lattice QCD thermodynamics. To study the impact of dynamical quarks on the transport properties of the hot medium, we confront the results in $N_{\rm f} = 2 + 1$ QCD with the observations in pure Yang–Mills theory. We show that dynamical quarks modify the behavior of the bulk to shear viscosity ratio and delay the restoration of conformal invariance. Around the (pseudo)critical temperature in both theories, the bulk to shear viscosity ratio behaves linearly in the squared speed of sound, as found in the AdS/CFT approach. At high temperature, the behavior of the ratio becomes quadratic, which corresponds to the perturbative QCD scaling. Thus, we find that the quasiparticle model is capable of describing the transport properties of the QCD in the weak and strong coupling regimes.

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1. Introduction

Transport parameters of strongly interacting matter are essential quantities which reveal the information not only about its transport properties but also about the dynamical evolution of the system. The transport properties of the hot QCD medium are usually quantified by the dimensionless ratio of the shear viscosity to entropy density η/s and the specific bulk viscosity ζ/s. The shear viscosity characterizes the dissipation of the energy during the longitudinal motion of the fluid. The η/s ratio exhibits a minimum near the critical temperature $T_{\rm c}$ and increases with temperature in

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different thermodynamic systems. On the other hand, the bulk viscosity ζ/s reflects the reaction of the fluid to a change of its volume and is expected to have a maximum in the vicinity of the deconfinement phase transition. At high temperatures, the ζ/s vanishes, indicating that the system approaches the conformal limit. The specific shear and bulk viscosities have been extensively examined in various approaches, such as the first-principle computational techniques, perturbative methods, or a class of effective models. The recent overview of different studies has been reported in [1, 2].

Another interesting but rarely studied quantity is the dimensionless ratio of the bulk to shear viscosity ζ/η, which is anticipated to behave differently in strongly- and weakly-coupled systems [3]. In strongly-coupled theories, including QCD around T_{c}, the ratio is linearly quantified by the speed of sound squared c_{s}^2 [4], while in theories with a weak coupling [5, 6], including the perturbative QCD (pQCD) approach [7, 8], the ζ/η is expressed quadratically in terms of c_{s}^2.

In this write-up, we will explore the above dependencies in the effective kinetic approach, where the deconfined matter is described through the quasiparticle excitations with dynamically generated effective masses [1, 2]. We compute the ζ/η ratio for the pure gluon plasma and the quark–gluon plasma (QGP) with light and strange quarks, confronting the results with the available lattice data and the pQCD predictions. Parametrizing the ratio by the linear and quadratic ansantzes in the speed of sound squared, we show that the QPM provides an effective interpolation between the non-perturbative and perturbative QCD regimes.

2. Transport parameters in the quasiparticle model

In the kinetic theory under the relaxation time approximation, the QGP is considered as a nearly equilibrated diluted medium. The shear viscosity η and the bulk viscosity ζ are defined as [1–3, 9]

$$\eta = \frac{1}{15T} \sum_{i=l,\bar{l},s,\bar{s},g} \int \frac{\mathrm{d}^3p}{(2\pi)^3} d_i f_i^0 \left(1 \pm f_i^0\right) \frac{\tau_i}{E_i^2} p^4 \,, \tag{1}$$

$$\zeta = \frac{1}{T} \sum_{i=l,\bar{l},s,\bar{s},g} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \, d_i f_i^0 \left(1 \pm f_i^0\right) \frac{\tau_i}{E_i^2} \left\{ \left(E_i^2 - T^2 \frac{\partial m_i^2}{\partial T^2} \right) c_{\mathrm{s}}^2 - \frac{p^2}{3} \right\}^2 , \tag{2}$$

where the $\pm$ corresponds to bosons and fermions, respectively. In QCD with $N_{\mathrm{f}} = 2+1$, the total coefficients are given by a sum of the contributions coming from light (up + down) and strange (anti-)quarks, as well as from gluons, while in pure Yang–Mills theory, the sum is composed solely of the gluon component. The degeneracy factors d_i are given as $d_{l,\bar{l}} = 12$ for light (anti-)quarks, $d_{s,\bar{s}} = 6$ for strange (anti-)quarks, and $d_g = 2(N_c^2 - 1) = 16$ for gluons. For the equilibrium momentum-distribution function f_i^0, we take the Fermi–Dirac (Bose–Einstein) statistics, $f_i^0 = (\exp(E_i/T) \pm 1)^{-1}$. We assume that the particles propagate on-shell with the energies $E_i = \sqrt{p^2 + m_i^2}$. The remaining terms in Eqs. (1), (2), the relaxation time τ_i and the speed of sound squared c_s^2, are discussed later.

As one may notice, the bulk viscosity depends on the derivative of the mass $\partial m_i^2/\partial T^2$, indicating that the mass is temperature dependent. One of the essential building blocks of the QPM are the dynamical masses of the particles, which change as they propagate and interact with the medium. We consider the QGP as a system of the quasiparticle excitations, *i.e.* weakly interacting particles with the effective masses expressed as

$$m_i^2 = \left(m_i^0\right)^2 + \mathit{\Pi}_i\,, \tag{3}$$

with the bare masses $m_l^0 = 5\,\mathrm{MeV}$, $m_s^0 = 95\,\mathrm{MeV}$ and $m_g^0 = 0$. The temperature and coupling dependence is introduced by the dynamically generated self-energies $\mathit{\Pi}_i$ [10, 11]

$$\mathit{\Pi}_{l,s}(T) \;=\; 2\left(m_{l,s}^0\sqrt{\frac{G(T)^2}{6}T^2} + \frac{G(T)^2}{6}T^2\right)\,, \tag{4}$$

$$\mathit{\Pi}_g(T) \;=\; \left(3 + \frac{N_f}{2}\right)\frac{G(T)^2}{6}T^2\,. \tag{5}$$

The effective coupling $G(T)$ is extracted from the entropy density calculated in lattice gauge theory for pure Yang–Mills [12] and for QCD with 2+1 quark flavors [13]. The coupling not only reproduces the perturbative dynamics at high temperature, but also covers the non-perturbative QCD features in the vicinity of the (pseudo)critical temperature [1]. We discuss the impact of $G(T)$ on the transport parameters in Sec. 3.

Coming back to Eqs. (1), (2), the relaxation time τ_i appears as a parameter of the approximate solution to the Boltzmann equation. In general, each transport coefficient is related to a particular dissipative phenomenon occurring in the viscous fluid. Thus, the shear and bulk viscosities should be characterized by separate relaxation times. Instead, for this study, we assume that the viscosities share a common τ_i for a given quasiparticle species. This allows us to obtain the ζ/η ratio independent of the relaxation time, whose careful evaluation has been presented in [1].

2.1. Speed of sound

In order to confirm that the thermodynamics controlled by the effective coupling of the QPM is consistent with the original lattice data, we compute the speed of sound squared using

$$c_{\mathrm{s}}^2 = \frac{s}{T}\left(\frac{\partial s}{\partial T}\right)^{-1}, \tag{6}$$

where s is the total entropy density of the QGP calculated as a function of temperature [1].

Figure 1 shows numerical results for the c_{s}^2 in pure Yang–Mills theory (left) and in QCD with $N_{\mathrm{f}} = 2+1$ (right). We notice that the speed of sound squared obtained in the QPM for the pure gluon plasma is in excellent agreement with the corresponding lattice QCD data both in the confined and the deconfined phase [12]. Moreover, below T_{c}, the QPM and the lQCD are consistent with the c_{s}^2 in a glueball resonance gas with the Hagedorn density of states [14].

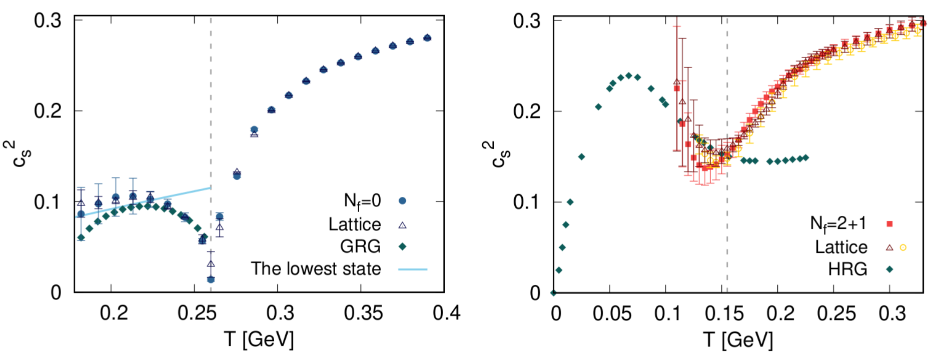

Fig. 1. Speed of sound squared as a function of temperature. Left: The result obtained in the quasiparticle model (full circles) for pure Yang–Mills theory with $T_{\mathrm{c}} = 0.26$ GeV (dashed line) is compared to the c_{s}^2 obtained from the lattice QCD calculations [12] (open triangles), and from the glueball resonance gas with the Hagedorn spectrum [12, 14] (diamonds). Right: The same quantity but for $N_{\mathrm{f}} = 2+1$ (squares) shown along with the corresponding lattice QCD data [13] (triangles) and [15] (circles) and the c_{s}^2 in the hadron resonance gas which considers the states with masses below 2.5 GeV [16] (diamonds). The pseudocritical temperature $T_{\mathrm{c}} = 0.155$ GeV is indicated by the dashed line.

In QCD with $2+1$ quark flavors, the QPM fairly reproduces the lattice QCD results [13, 15]. Below the pseudocritical temperature and within the considered uncertainties, the QPM also agrees with the c_{s}^2 in the hadron resonance gas (HRG) model [16]. However, right above T_{c}, the speed of sound squared in the HRG model exhibits a distinct deviation from the QPM and the lattice data, showing the boundary of the hadronic picture.

The non-monotonic behavior of the c_{s}^2 around T_{c} indicates the first-order phase transition in pure Yang–Mills theory and the crossover in QCD. This is a consequence of an apparent change of the lattice entropy density at T_{c} in both cases.

3. Bulk to shear viscosity ratio

Assuming equal relaxation times for the shear and bulk viscosities, we compute the ζ/η ratio using Eqs. (1), (2). In Eq. (2), the speed of sound squared is one of the main factors which quantifies the conformality of the system. As the c_{s}^2 reaches the Stefan–Boltzmann limit, $c_{\mathrm{s}}^2 = 1/3$, the bulk viscosity vanishes, indicating the restoration of conformal invariance. Therefore, finite ζ measures the deviation of the system from the conformal limit. This observation comes straightforwardly from the high-temperature limits of Eqs. (2) and (6).

For strongly-coupled theories and gauge/gravity duality, the behavior of the ratio is described as [4]

$$\frac{\zeta}{\eta} \propto \left(\frac{1}{3} - c_{\mathrm{s}}^2\right) . \tag{7}$$

In contrast, for weakly-coupled systems, such as an interacting photon gas [5], and in scalar field theories [6], the bulk to shear viscosity ratio is determined as

$$\frac{\zeta}{\eta} = 15\left(\frac{1}{3} - c_{\mathrm{s}}^2\right)^2 . \tag{8}$$

The perturbative QCD [7, 8] also suggests that at high temperatures, the bulk to shear viscosity ratio follows a tendency given by Eq. (8).

To identify the role of the quasiquarks in the restoration of conformal invariance, we compare the ζ/η ratio for the QGP to the corresponding result for the pure gluon plasma. Based on the observations in [3, 17, 18], we approximate our results by the conformality measure $\Delta c_{\mathrm{s}}^2 = 1/3 - c_{\mathrm{s}}^2$ with the fit parameters α, β, γ and δ

$$\text{Linear:} \quad \frac{\zeta}{\eta} = \alpha\left(\frac{1}{3} - c_{\mathrm{s}}^2\right) + \beta\,, \tag{9}$$

$$\text{Quadratic:} \quad \frac{\zeta}{\eta} = \gamma\left(\frac{1}{3} - c_{\mathrm{s}}^2\right)^2 + \delta\,. \tag{10}$$

Figure 2 illustrates the bulk to shear viscosity ratio in pure Yang–Mills (left) and in QCD with $N_{\rm f} = 2+1$ (right). Around $T_{\rm c}$ in pure SU(3) theory, the ratio ζ/η exhibits a linear dependence on $\Delta c_{\rm s}^2$, while at higher temperatures, the scaling becomes quadratic. A clear changeover is observed at $T \simeq 1.3\,T_{\rm c}$. This observation is in line with the overall behavior of ζ/η found earlier in a similar quasiparticle approach [3]. Near the first-order phase transition, our result is also consistent with ζ/η deduced from the available lattice data [19, 20].

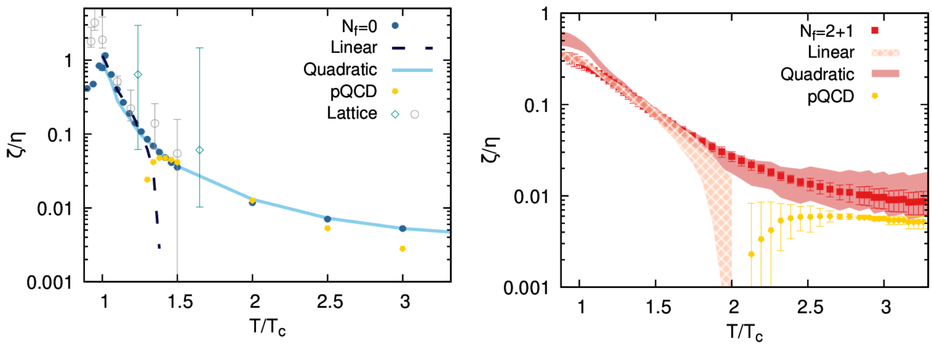

Fig. 2. The bulk to shear viscosity ratio as a function of $T/T_{\rm c}$ in pure Yang–Mills theory (left) and QCD with $N_{\rm f} = 2+1$ (right). In both figures, the linear and quadratic parameterizations are obtained, respectively, from Eqs. (9) and (10) with a set of fit parameters. Additionally, we present the results deduced from the perturbative QCD approach [7, 8] (full pentagons). Left: ζ/η in the QPM (full circles) shown along with the lattice data [19] (open diamonds), [20] (open circles) and the approximations for strong (dashed line) and weak (solid line) coupling regimes computed with $\alpha = 4.5, \beta = -0.3, \gamma = 12, \delta = 0.002$. Right: The ζ/η ratio in the QPM for $N_{\rm f} = 2+1$ QCD (full squares), parameterized linearly (checkered band) and quadratically (plain-colored band) in $\Delta c_{\rm s}^2$, using $\alpha = 2.15, \beta = -0.085, \gamma = 14, \delta = 0$.

Further, for temperatures $T \geq 1.4\,T_{\rm c}$, the QPM ratio corresponds to the pQCD estimation [7, 8], where the shear and bulk viscosities are expanded in the coupling up to the next-to-leading-log (NLL) order

$$\eta_{\rm NLL} = \frac{T^3}{g^4}\frac{\eta_1}{\ln(\mu_1^*/m_{\rm D})}\,, \qquad \zeta_{\rm NLL} = \frac{A\alpha_{\rm s}^2 T^3}{\ln(\mu_2^*/m_{\rm D})}\,. \tag{11}$$

Here, $\alpha_{\rm s} = g^2/4\pi$ is the strong coupling and $m_{\rm D}^2 = (1+N_{\rm f}/6)g^2T^2$ is the Debye mass squared. The set of parameters is given by $\eta_1 = 27.126$, $\mu_1^*/T = 2.765$, $A = 0.443$ and $\mu_2^*/T = 7.14$ for $N_{\rm f} = 0$, and $\eta_1 = 106.66$, $\mu_1^*/T = 2.957$, $A = 0.657$ and $\mu_2^*/T = 7.77$ for $N_{\rm f} = 3$.

In QCD with $N_\mathrm{f} = 2+1$, the ratio ζ/η scales as in pure Yang–Mills theory, but the changeover from linear to quadratic ansatz is shifted towards a higher temperature, $T \simeq 2\,T_\mathrm{c}$. Close to T_c, the ratio is well described by the linear parameterization in Δc_s^2, while the quadratic approximation apparently differs due to the presence of dynamical quarks. Near the crossover region, the effective coupling carries the non-perturbative QCD features, which result in rather large values of $G(T)$ [1] and influence the behavior of ζ/η ratio. The inclusion of the matter fields also generates a somewhat larger discrepancy between the QPM and the pQCD ratios.

For a deeper analysis of the connection between the bulk viscosity and conformality, we evaluate the bulk to shear viscosity ratio as a function of the measure Δc_s^2. We additionally consider the temperature profiles of the speed of sound squared for $T \geq T_\mathrm{c}$ in both theories. The numerical results are presented in Fig. 3. It is clear that as $T \to \infty$, the c_s^2 approaches the Stefan–Boltzmann limit. However, it happens much faster in pure Yang–Mills theory than in QCD with light and strange quarks. Consequently, the appearance of dynamical quarks delays the restoration of conformal invariance. The right panel of Fig. 3 additionally illustrates the changeover from linear to quadratic scaling of the ζ/η ratio.

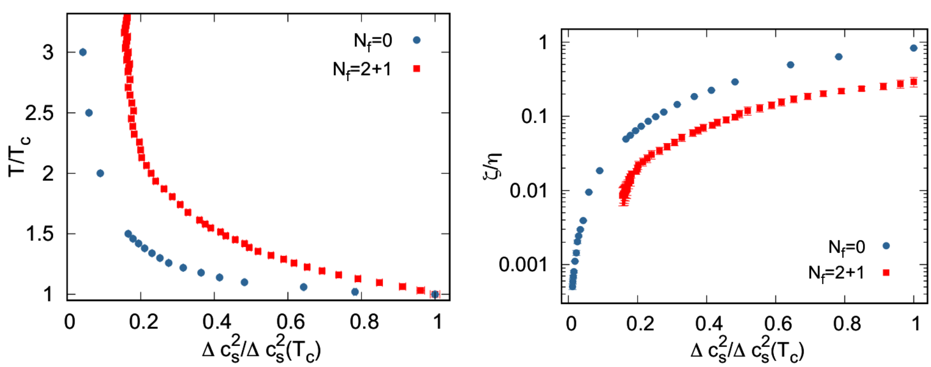

Fig. 3. The scaled temperature (left) and the bulk to shear viscosity ratio (right) as functions of the conformality measure, $\Delta c_\mathrm{s}^2 = 1/3 - c_\mathrm{s}^2$, normalized by its value at T_c in pure Yang–Mills theory (full circles) and in QCD (full squares).

4. Conclusions

Utilizing the successful quasiparticle model (QPM), we have assessed the influence of dynamical quarks on the behavior of the bulk to shear viscosity ratio. The ζ/η reflects the capability of the QPM to cover the weak and strong coupling regimes in pure Yang–Mills theory and QCD with $N_\mathrm{f} = 2+1$

at vanishing chemical potential. Comparing the results for both theories, we studied the flavor dependence of ζ/η ratio and quantified the role of dynamical quarks in the restoration of conformal invariance.

Based on kinetic theory under the relaxation time approximation, the QPM describes the deconfined matter in terms of weakly-interacting quasiparticles with dynamically generated masses depending on the effective running coupling. The temperature dependence of the coupling is defined from the entropy density computed on the lattice with a corresponding number of flavors.

We have verified the thermodynamics of the model and its effectiveness in the description of bulk parameters by computing the speed of sound squared. We observed that the QPM reproduces the $c_{\rm s}^2$ not only at high temperature but also close to $T_{\rm c}$. Moreover, it agrees with the result below but near the critical temperature, when a tower of hadronic resonances should be considered. Thus, the non-trivial QCD physics is correctly encoded in the effective coupling of the QPM.

Assuming equal relaxation times for the viscosity coefficients, we have computed the ratio of ζ/η and confronted it with the linear and quadratic parameterizations in terms of $\Delta c_{\rm s}^2 = 1/3 - c_{\rm s}^2$ which measures a deviation from conformal invariance. We noticed that the ratio scales linearly near $T_{\rm c}$, in accordance with the estimation given by the AdS/CFT approach for strongly-coupled theories [4]. As the temperature grows, the linear approximation switches to the quadratic one, which corresponds to the perturbative QCD expectation [7, 8]. Hence, the QPM effectively covers the non-perturbative and perturbative domains with a changeover depending on the number of flavors. While in pure Yang–Mills theory it emerges at $T \simeq 1.3\,T_{\rm c}$, in QCD with $N_{\rm f} = 2+1$, it appears at $T \simeq 2\,T_{\rm c}$. Therefore, the temperature range of the non-perturbative QCD is interestingly extended for a system with dynamical quarks. The quasiquarks also play an important role in the restoration of conformal invariance. In QCD, it is considerably delayed and takes place at a higher temperature in comparison to the pure Yang–Mills scenario.

To provide more reliable profiles of the transport parameters essential for the hydrodynamic simulations, one can modify the QPM introducing a finite chemical potential or separate relaxation times for the shear and bulk viscosities, which we leave as our future tasks.

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