HADRONIC RESONANCE GAS MODEL AND MULTIPLICITY DEPENDENCE IN p–p, p–Pb, Pb–Pb COLLISIONS: STRANGENESS ENHANCEMENT*

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Recently, the ALICE Collaboration has observed an interesting systematic behavior of ratios of identified particles to pions yields at the LHC, showing that they depend solely on the charged-particle multiplicity $dN_{ch}/d\eta$, in p–p, p–Pb and Pb–Pb collisions. In particular, the yields of (multi)strange particles, relative to pions increase with $dN_{ch}/d\eta$ and the enhancement becomes more pronounced with increasing strangeness content. We will argue that such a pattern of strangeness enhancement is arising naturally in the thermal model accounting for exact strangeness conservation. Furthermore, extending the thermal model by including hadron interactions within the S-matrix approach, the ALICE data can be well quantified by the thermal particle yields at the chiral-crossover temperature, as previously found in central Pb–Pb collisions.

1. Introduction

The thermal description of hadronic yields measured in nucleus–nucleus collisions by using the hadron resonance gas model (HRG) has been already

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very successful, see *e.g.* [1, 2]. The identifying feature of the HRG is that all stable hadrons and known hadronic resonances listed in the review of particle physics [3] are assumed to be in thermal and chemical equilibrium. This assumption drastically reduces the number of free parameters, as this stage is determined by just the chemical freeze-out temperature T, the various chemical potentials $\vec{\mu}$ determined by the conserved quantum numbers, and by the volume V of the system. It has been shown that this description is valid not only for a static, but also for an expanding fireball [4–6] that follows Bjorken's scaling expansion [7]. For an expanding system, and after integration over transverse momenta of produced hadrons, the effects of hydrodynamic flow cancel out in yields ratios, resulting in the same values as calculated in a fireball at rest. Furthermore, the HRG was also shown to provide a parameter-free description of the equation of state and some fluctuation observable calculated in lattice QCD (LQCD) in the hadronic phase [8, 9]. The above provides strong evidence that the HRG statistical operator is a very good approximation of the QCD partition function, and that particle yields produced in heavy-ion collisions are consistent with the expectation of first-principles-based LQCD calculations.

The yields produced in heavy-ion collisions have been the subject of intense discussions over the past few years and several proposals have been made to further improve the HRG model. This was particularly in view of the fact that in the HRG, the number of pions was underestimated, while the number of protons was overestimated. Several proposals to further improve or extend the HRG have been made recently:

— incomplete hadron mass-spectrum [9–12],

— chemical non-equilibrium at freeze-out [13–15],

— modification of hadron abundances in the hadronic phase [16–18],

— separate freeze-out for strange and non-strange hadrons [19–21],

— excluded volume interactions [1, 2, 22],

— energy-dependent Breit–Wigner widths [23],

— use the phase-shift analysis to take into account repulsive and attractive interactions [2, 24, 25],

— use the K-matrix formalism to take interactions into account [26].

We argue that in view of the recent ALICE Collaboration data, in particular, the observed dN_{ch}/dy dependence of particle yields of the (multi)strange hadrons in pp, pA and AA collisions at the LHC [27], essential improvements of the HRG model taking into account exact strangeness conservation and hadron interactions [28] can be made.

In the present analysis, we keep the basic structure of the HRG as determined in central Pb–Pb collisions with a single chemical freeze-out temperature, $T \approx 156.5$ MeV and with the off-equilibrium suppression factor $\gamma_{\rm s} = 1$, as well as with all chemical potentials set to zero [1]. The observed deviations at low multiplicities will be ascribed to imposing strangeness conservation via the canonical ensemble as explained in detail below. The deviations seen in the yields of protons and Λs are corrected by including interactions via the S-matrix formalism [29]. These lead to a 25% reduction in the proton yield and a 23% enhancement of the Λ yield [28].

We will show that a very good description is obtained for the variation of the strangeness content in the final state as a function of the number of charged hadrons at mid-rapidity by using the same fixed temperature value as in central Pb–Pb collisions, whereas the volume of the fireball at mid-rapidity is found to be linearly dependent on the number of charged hadrons. For small multiplicities, the canonical ensemble with global strangeness conservation is needed to quantify the observed suppression of (multi-)strange baryons. This is obtained by introducing a conservation of strangeness in the whole phase space which is parameterized by the canonical correlation volume that is larger than the fireball volume at mid-rapidity. The interactions introduced by the phase-shift analysis via the S-matrix formalism are essential for a quantitative description of the yields data. A more detailed description of the model and its comparison with ALICE data can be found in [28].

2. Including hadron interactions by using empirical phase-shifts

The change in the particle yield due to interactions is taken into account by introducing the kernel function $B(M)$, which is linked to scattering phase-shifts as follows [30]:

$$B(M) = 2\frac{\mathrm{d}}{\mathrm{d}M}\delta(M) \,, \tag{1}$$

where $\delta(M)$ is given by the empirical phase-shift analysis. For a well-defined resonance, this can be replaced by [30]

$$B(M) \approx 4\frac{M^2 \Gamma_{\rm R}}{\left(M^2 - M_{\rm R}^2\right)^2 + M^2 \Gamma_{\rm R}^2} \,, \tag{2}$$

where $\Gamma_{\rm R}$ is the width of the resonance and $M_{\rm R}$ is its mass. Finally, for a narrow resonance, this can be approximated by

$$B(M) \approx 4\pi M \delta \left(M^2 - M_{\rm R}^2\right) \,. \tag{3}$$

The particle yields are modified by an integral over the kernel function $B(M)$ as follows [25, 30, 31]:

$$N(T, M) = \int\limits_{m_{\text{th}}}^{\infty} \frac{\mathrm{d}M}{2\pi} B(M) N^{\text{id}}(T, M) \,, \tag{4}$$

where N^{id} is the particle density given by the ideal gas formula. From Eqs. (2) and (4), it is clear that for a system with purely attractive interactions which are dominated by the formation of resonances, the yields of particles are as in the mixture of ideal gases of stable hadrons and resonances, what constitutes the frequently used HRG model.

The extension of the HRG model as in Eq. (4), and an analysis using the available phase-shift data have been performed in [28, 30, 31]. In the following, we put particular attention to such a model extension that accounts for interactions of nucleons and hyperons to discuss the observed production of (multi)strange baryons in different colliding systems at the LHC [28].

3. Strangeness canonical ensemble

It is by now a well-established fact that the multiplicity of charged particles in the grand canonical and canonical ensembles, formulated with respect to conservation laws, can differ substantially. Indeed, it was pointed out by Hagedorn [32] that thermal models can overestimate the yield of charged particles when the grand canonical ensemble is used. Applying Hagedorn's argument to strangeness conservation, the reason for this is that when the number of particles as well as the interaction volume are small, one has to take into account that strange particles must be balanced by anti-strange particles to exactly conserve strangeness. Thus, the abundance of *e.g.* K^+ in the fireball of volume V_A at temperature T will not be proportional to the standard Boltzmann factor given by

$$N_{K^+} \approx V_A \exp\left(-\frac{m_{K^+}}{T}\right) \tag{5}$$

but instead by

$$N_{K^+} \approx V_A \mathrm{e}^{-\frac{m_{K^+}}{T}} V_{\text{C}} \left[g_{\bar{K}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{e}^{-\frac{E_{\bar{K}}}{T}} + g_\Lambda \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{e}^{-\frac{E_\Lambda}{T} + \frac{\mu_B}{T}} + \dots \right] . \tag{6}$$

This introduces one more exponential factor and implies a strong suppression of K^+ yields and leads to a quadratic volume dependence, where V_{C} is introduced as the canonical volume where exact strangeness conservation is fulfilled. In general, $V_A \neq V_{\text{C}}$.

The inclusion of constraints of exact strangeness conservation in this framework has been considered at a very early stage, see *e.g.* [33]. The partition function is modified by including a δ-function to enforce strangeness to be exactly zero

$$Z_{S=0} = \text{Tr}\left(e^{-(E-\mu)/T}\delta_{S,0}\right) . \tag{7}$$

This leads to replacing the standard grand canonical expression, *e.g.* for kaons

$$N_K = V_A e^{\frac{\mu}{T}} \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \tag{8}$$

by the following

$$N_K = V_A F_S \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} , \tag{9}$$

where

$$F_S = \frac{I_1(x)}{I_0(x)} \frac{S_1}{\sqrt{S_1 S_{-1}}} , \tag{10}$$

and $x \equiv 2\sqrt{S_1 S_{-1}}$ with $S_1 = Z_{\bar{K}} + Z_\Lambda + \ldots$ *i.e.*, $S_{s=\pm 1} = \sum_k Z_{k,s}$, where the sum is taken over all particles and resonances that carry strangeness s. The one-particle partition function, $Z_{k,s} = V_C n_{k,s}(T)$, with the particle density

$$n_{k,s}(T) = g_{k,s} \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E_{k,s}}{T}} . \tag{11}$$

In view of $\vec{\mu} = \vec{0}$, the thermal phase space of particles is the same as for anti-particles, *i.e.* $S_s = S_{-s}$. Consequently, from Eq. (9) the yields of kaons are obtained as

$$N_K = V_A \frac{I_1(x)}{I_0(x)} \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} . \tag{12}$$

For small values of x, this leads to the correction factors given in Eq. (6), whereas for large $x \gg 1$, it reproduces the grand canonical result.

The ratio of Bessel functions, $F_S = I_1/I_0$ is shown in Fig. 1. It is clear that F_S starts from zero and approaches one for large values of the argument. Thus, corrections due to exact strangeness conservation reduce particle yields, and disappear for large values of x, *i.e.* for large T and/or V_C, where the yields of particles reach their grand canonical ensemble values.

The canonical formalism summarized above can be also generalized to thermal systems which contain contributions from multistrange particles. In this case, the yield of particle carrying strange quantum number s, can be approximated by

$$\langle N_s \rangle_A \simeq V_A\, n_s \frac{I_s(S_1)}{I_0(S_1)} . \tag{13}$$

Thus, the ratio $I_s(S_1)/I_0(S_1)$, similarly to F_S in Eq. (9), is just a suppression factor which decreases in magnitude with increasing s of hadrons and with decreasing thermal phase space occupied by strange particles as described by the argument S_1 of the Bessel functions. Relevant ratios are shown in Fig. 1. A decrease of $S_1 = V_C \sum_k n_{k,s=1}$ is due to decreasing T, or decreasing V_C. These are the main properties of strangeness canonical suppression that have been introduced [35] to describe thermal production of multi(strange) hadrons in heavy-ion collisions.
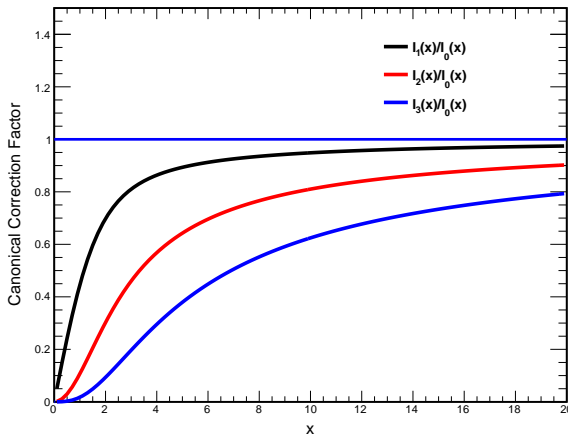

Fig. 1. Ratio of Bessel functions relevant for determining canonical corrections to particle multiplicities.

4. Strangeness production at the LHC

In the following, we will show that the thermal HRG model that includes S-matrix corrections for baryon interactions and accounts for exact strangeness conservation, provides a very good description and understanding of multistrange particle production systematic as observed by the ALICE Collaboration at the LHC in pp, pA and AA collisions.

In the application of the HRG model to particle production in heavy-ion collisions, the temperature and baryon-chemical potential at freezeout are linked to the collisions energy, whereas the volume parameters to charged particle multiplicities. At the LHC, the particles and anti-particles are produced with the same abundance, thus the thermal parameters characterizing yields of particles are only the temperature and the volume. Furthermore, due to the observed coincidence of chemical freezeout temperature in the most central AA collisions and the chiral-crossover temperature obtained in lattice QCD, we assume the same $T = 156.5$ MeV for all colliding systems at fixed $\sqrt{s}$. Consequently, to quantify strange particle yields and their dependence on $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y$ as observed by the ALICE Collaboration at the LHC,

within the above thermal model, there are only two volume parameters to be extracted from the data. The fireball volume V_A which is obtained from fits to measured yields of pions and protons at mid-rapidity, and the canonical volume V_C that is fitted to reproduce yields of strange and multistage hadrons.

The above thermal analysis for each multiplicity bin for p–p, p–Pb and Pb–Pb has been done using the latest version of the THERMUS code [34][1], that was extended to include S-matrix corrections for baryon interactions. The extracted volume parameters are shown in Fig. 2. As can be seen, the
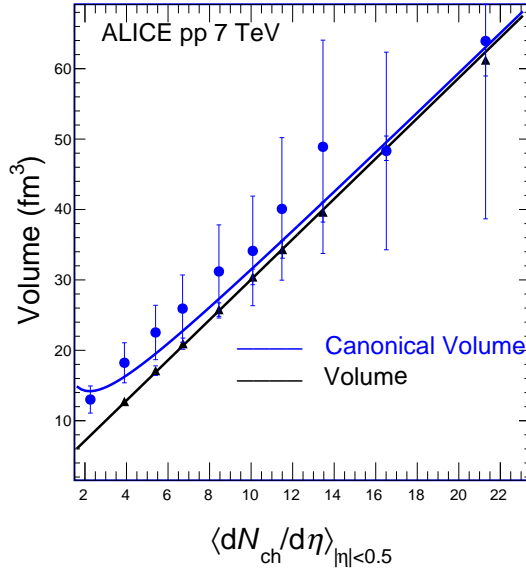

Fig. 2. Volume determined, using the HRG, from the ALICE Collaboration [27] for *pp* collisions at 7 TeV.

fireball volume can be determined very accurately with linear dependence on dN_{ch}/dy, whereas the canonical volume cannot, especially for large multiplicities where, within errors, it is consistent with being equal to the fireball volume. For low multiplicities, however, there is a clear difference indicating that $V_C > V_A$. The dependence of strange particle ratios as a function of multiplicity seen in Fig. 3 follows the behavior predicted in [35, 36]. Furthermore, data can be quantified, with the above formulation of the HRG model, quite satisfactorily. The model prediction on different particle yields agrees with the data up to two standard deviations for all $dN_{ch}/d\eta$. The data on pion yields are always slightly above the calculated points, while the kaons are always below. This has implications for the kaon-to-pion ratio

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[1] B. Hippolyte and Y. Schutz, https://github.com/thermus-project/THERMUS

which is seen in Fig. 3 to exhibit the largest deviations from the data, illustrating the pitfalls of comparing ratios in the thermal model. It is better to compare directly yields. A more complete description of the model setup and its comparison with the ALICE data can be found in [28].
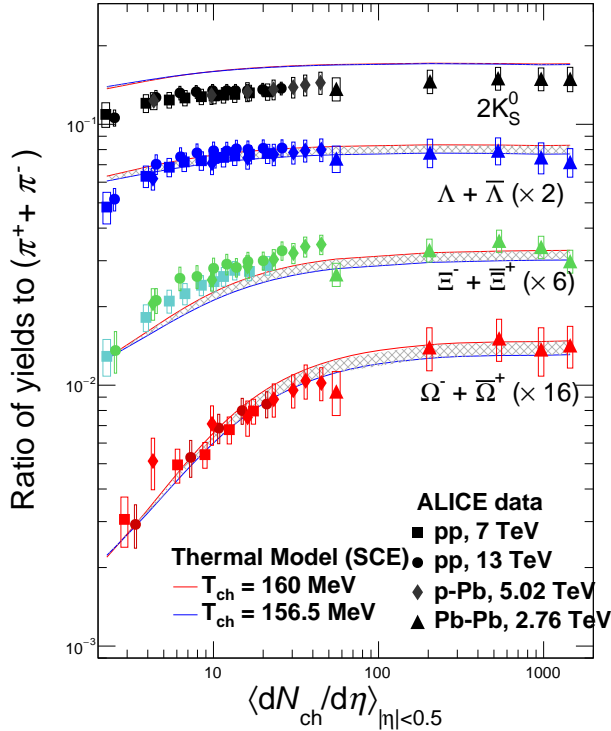

Fig. 3. Ratios of yields of strange particles to pions *versus* charged particle multiplicity.

5. Conclusions

We have shown that the observed behavior of (multi)strange hadron yields in *pp*, *pA* and *AA* collisions at the LHC with charged particle multiplicity can be explained naturally in the thermal model which accounts for exact strangeness conservation and S-matrix corrections to baryon interactions. The freezeout temperature T is linked to the collision energy and is independent of the colliding system. At the LHC, the value of T is well consistent with chiral crossover temperature calculated in LQCD. The fireball volume parameter at mid-rapidity scales linearly with $dN_{ch}/d\eta$, whereas the canonical volume parameter, where strangeness is exactly conserved, is larger than the fireball volume at mid-rapidity. Thus, exact conservation

of strangeness is to be imposed in the full phase-space rather than in the experimental acceptance at mid-rapidity. The inclusion of interactions via phase-shift data improves essentially the fits of the model to proton and hyperon yields data.

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