CLARIFICATION OF THE SPECTROSCOPIC FACTOR FOR $^{25}\mathrm{Mg} - {}^{24}\mathrm{Mg} + n$ VIA THE "EXPERIMENTAL" ANC*

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(Received November 9, 2021; accepted November 9, 2021)

The results of the analysis of the $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ and $^{24}\mathrm{Mg}(d,p)^{25}\mathrm{Mg}$ reactions performed within the modified DWBA are presented. A possibility of clarifying the values of spectroscopic factors for the $B \to A + n/p$ configuration is demonstrated using the experimental data on the peripheral $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ and non-peripheral $^{24}\mathrm{Mg}(d,p)^{25}\mathrm{Mg}$ neutron transfer reactions.

DOI:10.5506/APhysPolBSupp.14.665

1. Introduction

Usually the differential cross sections (DCS) of nucleon transfer reactions are analyzed within the DWBA to determine spectroscopic factors (SF) (see, *e.g.*, [1] and references therein). It is well known that the uncertainties of such "indirectly determined" SF values resulting from the normalization of calculated DCS to experimental ones are often large even when the errors in the experimental differential cross section are small. One of the main reasons for this fault is a strong dependence of the extracted SF on the model single-particle potential parameters used in the DWBA

On the other hand, the values of asymptotic normalization coefficients (ANC) [2] that can be extracted from the analysis of peripheral reactions and are widely used for calculations of astrophysical S factors [3], have significantly lower model uncertainties.

However, it can be shown that in some cases, a joint analysis of experimental DCS of peripheral and non-peripheral nucleon transfer reactions within the framework of a modified DWBA [4] allows to significantly reduce the model dependence of the "indirectly determined" SF. In this paper, such a possibility is demonstrated by analyzing the $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ and $^{24}\mathrm{Mg}(d,p)^{25}\mathrm{Mg}$ reactions.

* Presented at III International Scientific Forum "Nuclear Science and Technologies", Almaty, Kazakhstan, September 20–24, 2021.

The analysis is fulfilled within the framework of the MDWBA method (see [4] and references therein). The experimental DCS $\mathrm{d}\sigma/\mathrm{d}\Omega$ for the peripheral transfer reaction $A(x,y)B$ (here, we assume a stripping process for certainty: $B = A + a$, $x = y + a$, a — transferred particle) in the MDWBA can be written in the form of

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = C_B^2 R_{lj}(E,\theta;\, b_{lj})\,, \tag{1}$$

$$R_{lj}(E,\theta;\, b_{lj}) = \left(\frac{C_x}{b_x}\right)^2 \frac{\sigma^{(\mathrm{DW})}(E,\theta;\, b_{lj})}{b_{lj}^2}\,, \tag{2}$$

where $l(j)$ is the orbital (total) angular momentum of the transferred particle, $C_{B/x}$ are the ANCs for $B \to A + a$ and $x \to y + a$ configurations; b_x, b_{lj} are the single-particle ANCs which determine the amplitude of the tails of the bound state wave functions of the transferred particle a in the nuclei x and B, respectively. The function $R(\ldots)$ is separated from the structure of the DCS formula to estimate the degree of peripherality of the particle a transfer and to evaluate the uncertainties of the theoretical approach in the extracted ANC value.

2. The analysis of the ^{25}Mg$(d,t)^{24}$Mg reaction and the ANC for ^{25}Mg $\to$ ^{24}Mg $+ n$

The $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ reaction was studied in a number of early published works at energies of 14.8 MeV [5], 18 MeV [6], 15.3 and 18 MeV in [7]. Here, the analysis of the experimental DCS of this reaction was carried out in the MDWBA at an energy of 14.8 MeV [5] and reanalysed at 14.5 MeV [8] at the same global potentials to specify the ANC for $^{25}\mathrm{Mg} \to {}^{24}\mathrm{Mg} + n$ configuration. In calculations, the value of the ANC for the bound state $t \to n + d$ was taken to be equal to $C_t^2 = 4.28$ fm^{-1} [9, 10]. In this case, the choice of the "standard" geometric parameters of the potential of the neutron bound state potential in triton: $r_0 = 1.25$ fm and $a = 0.65$ fm leads to the value of $b_t = 1.831$ fm$^{-1/2}$.

The optical potential (OP) parameters suitable for the analysis were selected according to the quality of description of the DCS of both the reaction under consideration and elastic scattering at the corresponding relative energy of interacting particles in the input and output channels. For the deuteron channel, the global parameters from Refs. [11, 12] and for the triton channel from Refs. [13, 14] were used (see Table I).

TABLE I

Parameters of the optical potentials for the entrance and exit channels for the ^{25}Mg$(d,t)^{24}$Mg and ^{24}Mg$(d,p)^{25}$Mg reactions. The letters D or V in column 6 denote the volume or surface terms, respectively.

Set	Channel	V_V [MeV]	r_V [fm]	a_V [fm]	$W_{D/V}$ [MeV]	r_W [fm]	a_W [fm]	V_{SO} [MeV]	a_{SO} [fm]	r_{SO} [fm]	R_C [fm]	Ref.
^{25}Mg$(d,t)^{24}$Mg												
set1	d	90.89	1.13	0.776	$10.387D$	1.386	0.712	3.557	0.972	1.011	1.303	[11]
	t	162.654	1.20	0.72	$41.446V$	1.4	0.84	2.50	1.20	0.72	1.40	[13]
set2	d	90.89	1.13	0.776	$10.387D$	1.386	0.712	3.557	0.972	1.011	1.303	[11]
	t	102.84	1.15	0.736	$16.520D$	1.378	0.80				1.40	[14]
set3	d	90.825	1.13	0.776	$10.377D$	1.386	0.712	3.557	0.972	1.011	1.303	[11]
	t	162.586	1.20	0.72	$41.314V$	1.4	0.84	2.50	1.20	0.72	1.40	[13]
set4	d	90.825	1.13	0.776	$10.377D$	1.386	0.712	3.557	0.972	1.011	1.303	[11]
	t	103.216	1.15	0.738	$16.730D$	1.374	0.80				1.40	[14]
^{24}Mg$(d,p)^{25}$Mg												
set5	d	91.156	1.13	0.755	$10.414D$	1.387	0.71	3.557	0.972	1.011	1.303	[11]
	p	42.82	1.26	0.67	$6.880D$	1.42	0.37	4.18	1.04	0.34	1.34	[15]
set6	d	80.48	1.25	0.741	$13.0D$	1.25	0.73	6.0	1.25	0.731	1.30	[12]
	p	42.82	1.26	0.67	$6.880D$	1.42	0.37	4.18	1.04	0.34	1.34	[15]
set7	d	91.935	1.13	0.755	$10.386D$	1.387	0.71	3.557	0.972	1.011	1.303	[11]
	p	42.82	1.26	0.67	$6.880D$	1.42	0.37	4.18	1.04	0.34	1.34	[15]
set8	d	80.165	1.25	0.741	$13.0D$	1.25	0.73	6.0	1.25	0.731	1.30	[12]
	p	42.82	1.26	0.67	$6.880D$	1.42	0.37	4.18	1.04	0.34	1.34	[15]

To check the degree of peripherality of the ^{25}Mg$(d,t)^{24}$Mg reaction, with formation of the ^{24}Mg nucleus in the ground state, we calculated the test function $R_{lj}(E,\theta;b_{lj}) = R(b_{n^{24}\mathrm{Mg}}) = R(b)$ varying the geometric parameters of the neutron bound state potential (in the form of Woods–Saxon) in the ranges of $1.1 \le r_0 \le 1.4$ fm and $0.5 \le a \le 0.8$ fm (see Fig. 1, left panel).

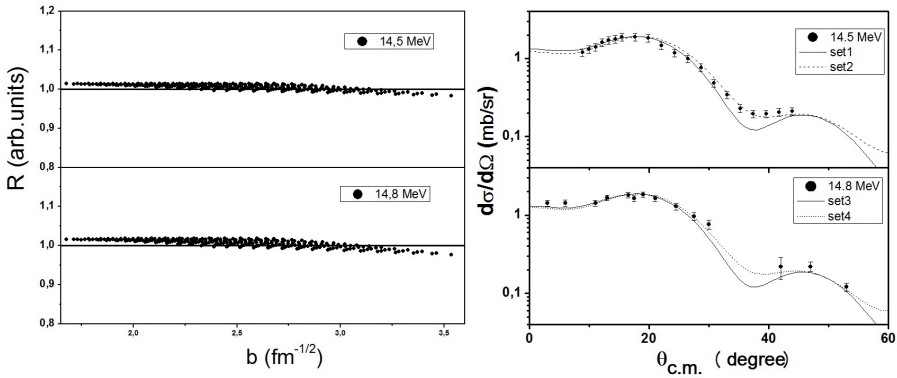

Fig. 1. Left panel: $R(b)$ dependence for the ^{25}Mg$(d,t)^{24}$Mg reaction at the angles θ corresponding to the main maximum of the DCS angular distributions; right panel: calculated and experimental angular distributions for the ^{25}Mg$(d,t)^{24}$Mg reaction. Experimental values for $E_d = 14.5$ MeV are from [8] and for $E_d = 14.8$ MeV from [5].

It can be seen that the function $R(b)$ is practically constant since it varies within 1.5–2.8% when the geometric parameters of the bound state potential change in the range of $1.1 \leq r_0 \leq 1.4$ fm and $0.5 \leq a \leq 0.8$ fm (note that the experimental errors of DCS are much larger). According to [16], this indicates the peripherality of the $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ reaction at the considered energies.

The right panel of figure 1 shows the calculated and experimental angular distributions for this reaction with the formation of the final ^{24}Mg nucleus in the ground state. It is seen that the calculated angular distributions are in satisfactory agreement with the experimental data.

The phenomenological values of the ANC C^2 for the $^{25}\mathrm{Mg}_{\mathrm{gs}} \to {}^{24}\mathrm{Mg} + n$ bound state, obtained from the analysis of the experimental data on the $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ reaction, are shown in Table II.

TABLE II

The ANC values from the analysis of the $^{25}\mathrm{Mg}(d,t)^{24}\mathrm{Mg}$ reaction for different sets of optical potentials. In brackets there are the experimental and theoretical uncertainties, respectively.

E [MeV]	Set	χ^2/n	$C^2_{n^{24}\mathrm{Mg}}$ [fm^{-1}]
14.5	set1	0.36	$1.70 \pm 0.21(0.187, 0.092)$
	set2	0.37	$1.67 \pm 0.20(0.184, 0.091)$
Average			$1.69 \pm 0.197(0.186, 0.065)$
14.8	set3	0.21	$1.69 \pm 0.21(0.203, 0.11)$
	set4	0.23	$1.67 \pm 0.20(0.191, 0.11)$
Average			$1.68 \pm 0.212(0.197, 0.078)$

The results show that the extracted squared ANC values for the configuration $^{25}\mathrm{Mg}_{\mathrm{gs}} \to {}^{24}\mathrm{Mg} + n$ at both energies coincide within the margin of error. The obtained ANC value within the error limits coincides with the value obtained by us at higher energies, $C^2 = 1.88 \pm 0.20$ [17], and the ANC square value recommended here for $^{25}\mathrm{Mg}_{\mathrm{gs}} \to {}^{24}\mathrm{Mg} + n$ is $C^2 = 1.75 \pm 0.22$. The presented errors in the values of the squared ANC include uncertainties associated with the dependence on the choice of the OP parameters, the deviations connected with small variations of the $R(b)$ function and the experimental errors.

3. The analysis of the ^{24}Mg$(d,p)^{25}$Mg reaction and spectroscopic factor for the $^{24}\mathbf{Mg} + n \to {}^{25}\mathbf{Mg}$

This reaction is analyzed at $E_d = 13.6$ [18] and 14.5 [8] MeV. In calculations, the ANC value for the bound state $d \to n + p$ was taken to be equal to $C_d^2 = 0.775$ fm^{-1} [19]. In this case, the choice of the "standard" geometric parameters of the potential of neutron bound state in deuteron: $r_0 = 1.25$ fm and $a = 0.65$ fm leads to the value of $b_d = 0.9392$ fm$^{-1/2}$ and to the value of the SF $Z_d = 0.878$. The OP parameters for the d channel were taken the same as in Section 2, whereas for the proton channel from [15] (see Table I).

The behavior of the test function $R(b)$ in the region of the main maximum of the angular distribution at both energies indicates a strong non-peripherality of the neutron transfer process in this reaction and, therefore, the incorrectness of the ANC extraction for the configuration $\{^{25}\text{Mg} \to {}^{24}\text{Mg} + n\}$ from the analysis. Figure 2 shows the areas of values of the test function $R(b)$ calculated at the main maximum of the angular distribution at both energies with the fixed pairs of OP in the input and output channels, which give the best description according to the χ^2 criterion. It is seen that the dependence of the R values on b differs significantly from the constant.

At the same time, owing to the found above value of the squared ANC for $^{25}\text{Mg} \to {}^{24}\text{Mg} + n$, from the analysis of the peripheral ^{25}Mg$(d,t)^{24}$Mg reaction, it is possible to find the corresponding value of the SF. At that, the uncertainty of its value connected with ambiguity of the choice of the geometric parameters of the nuclear potential of the $^{24}\text{Mg} + n$ bound state will be significantly minimized.

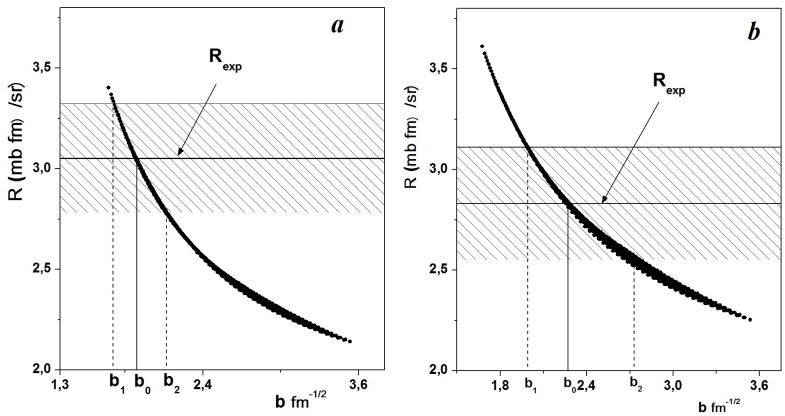

Fig. 2. Graphical determination of the value $b = b_0$ from the analysis of the ^{24}Mg$(d,p)^{25}$Mg reaction at $E_d = 13.6$ (a) and 14.5 MeV (b). The shaded area is a band determined by the error of the experimental DCS.

Figure 3 shows the calculated in the framework of MDWBA and the experimental angular distributions for the ^{24}Mg$(d,p)^{25}$Mg reaction at $E_d =$ 13.6 Mev [18] and 14.5 MeV [8]. The description in the region of the first (main) maximum of the angular distribution in both cases is satisfactory.

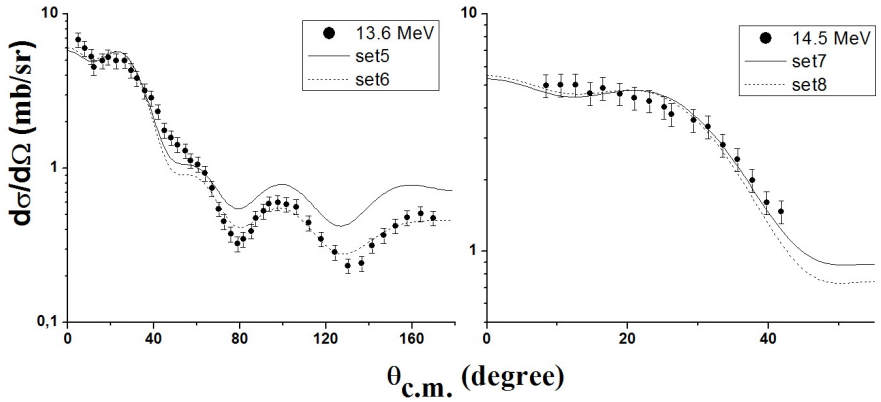

Fig. 3. Calculated and experimental angular distributions for the ^{24}Mg$(d,p)^{25}$Mg reaction.

As shown in [4], the square of ANC is uniquely related to the SF Z by the relation $C^2 = Zb^2$, and the SF value, $Z_{n^{24}\mathrm{Mg}}$, can be obtained if $b^2_{n^{24}\mathrm{Mg}}$ value is known. But, as mentioned earlier, the b value is not fixed because of the ambiguities in values of the geometry parameters of the neutron bound state potential, in our case for ^{24}Mg$+n$ configuration. However, it is possible to restrict the uncertainty in the values of b by knowing the value of ANC, which, in turn, allows one to fix the value of $R(b) = R_{\mathrm{exp}}$ from (1). Figure 2 shows a graphical method for finding the central value $b_{n^{24}\mathrm{Mg}} = b_0$ and the range of its uncertainties, determined by the errors of the experimental DCS. Then the SF is determined by the relation $Z = C^2_{n^{24}\mathrm{Mg}}/b^2_{n^{24}\mathrm{Mg}}$ with the errors defined by the errors of the numerator and denominator. The SF value for the $^{24}\mathrm{Mg} + n \to {}^{25}\mathrm{Mg}$ configuration found in this way is $Z = 0.42^{+0.13}_{-0.12}$.

REFERENCES

[1] W.T. Pinkston, G.R. Satchler, *Nucl. Phys.* **72**, 641 (1965).

[2] S.V. Artemov *et al.*, *Yad. Fiz.* **59**, 454 (1996) [*Phys. At. Nucl.* **59**, 428 (1996)].

[3] R.E. Tribble *et al.*, *Rep. Prog. Phys.* **77**, 106901 (2014).

[4] A.M. Mukhamedzhanov *et al.*, *Phys. Rev. C* **63**, 024612 (2001).

[5] A.I. Hamburger, *Phys. Rev. C* **118**, 1271 (1960).

[6] I.R. Gulamov *et al.*, in: «Proceedings of 40$^{\text{th}}$ All-Union Conference on Nuclear Spectroscopy and Nuclear Structure», Leningrad, USSR, 10–13 April 1990, p. 324.

[7] S.V. Artemov *et al.*, *Uzbek Phys. J.* **11**, 166 (2009) (in Russian).

[8] S.V. Artemov *et al.*, *Doklady Akademii Nauk Uzbekistan* **3**, 38 (2018) (in Russian).

[9] S.V. Artemov *et al.*, *Izv. RAN. Ser. Fiz.* **73**, 176 (2009) [*Bull. Russ. Acad. Sci. Phys.* **73**, 165 (2009)].

[10] N. Burtebayev *et al.*, *Acta Phys. Pol. B* **50**, 703 (2019).

[11] Haixia An, Chonghai Cai, *Phys. Rev. C* **73**, 054605 (2006).

[12] F. Hinterberger *et al.*, *Nucl. Phys. A* **111**, 265 (1968).

[13] H.J. Trost *et al.*, *Nucl. Phys. A* **462**, 333 (1987).

[14] F.D. Becchetti, Jr., G.W. Greenless, *Phys. Rev.* **182**, 1190 (1969).

[15] C.M. Perey, F.G. Perey, *At. Data Nucl. Data Tables* **17**, 1 (1976).

[16] R. Yarmukhamedov, L.D. Blokhintsev, *Phys. At. Nucl.* **81**, 616 (2018).

[17] S.V. Artemov *et al.*, *Int. J. Mod. Phys. E* **19**, 1102 (2010).

[18] B.B. Tokarevsky *et al.*, *Phys. At. Nucl.* **22**, 917 (1975).

[19] L.D. Blokhintsev *et al.*, *Fiz. Elem. Chast. At. Yad.* **8**, 1189 (1977) [*Sov. J. Part. Nucl.* **8**, 485 (1977)].