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CONTRIBUTION OF LOW-LYING RESONANCES IN THE COULOMB BREAKUP OF ^{11}Be HALO NUCLEI*

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The aim of this work is to investigate the influence of low-lying resonances in the Coulomb breakup of ^{11}Be halo nuclei on a heavy target from intermediate (70 MeV/nucleon) to low energies (5 MeV/nucleon) within the non-perturbative time-dependent approach. The inclusion of the resonant states $5/2^+$, $3/2^-$, and $3/2^+$ of ^{11}Be into the computational scheme leads to a significant improvement of the theoretical model. The method can potentially be useful for interpretation of low-energy breakup experiments on different targets in studying the halo structure of nuclei.

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1. Introduction

Despite the fact that research on the exotic nuclei dates back to the 80s of the last century, this topic is still one of the most relevant. Breakup reactions have proven to be a useful tool in exploring halo nuclei, in which the loosely bound particles dissociate from the core through the interaction with a target. The breakup cross section provides useful information about the structure and properties of the halo systems [1].

A number of computational approaches [2–8] were developed to describe the breakup of one-nucleon halo nuclei, which are mainly applied to high and intermediate beam energies in the breakup reaction of ^{11}Be on a heavy target of ^{208}Pb. For intermediate beam energies (near 70 MeV/nucleon), there are rather accurate experimental data [9, 10]. However, for lower energies, only a few theoretical works have been performed so far [2, 11, 12]. Thus, the

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low beam energy region is of great interest, since this is the energy range (around 10 MeV/nucleon) of HIE-ISOLD at CERN and the future ReA12 at MSU that has hardly been investigated so far.

In the present work, we extend the theoretical model developed in [5, 6, 13] and successfully applied it to the breakup of halo nuclei ^{11}Be [5, 6], ^{15}C [6], and ^{17}F [13] at higher beam energies to the low-energy region. In this model, the time-dependent Schrödinger equation for a halo-nucleus is integrated with a non-perturbative algorithm on a three-dimensional spatial mesh. The use of the discrete-variable representation in 2D angular space and high-order finite differences for the radial part of the wave function allows avoiding the multipole expansion of the time-dependent Coulomb interaction between the projectile and the target [5, 14]. Another attractive feature of the method is its flexibility to the choice of the interactions between the halo-nucleus, the core, and the target [6], and in the definition of the projectile trajectory which can be classically treated simultaneously with the Schrödinger equation for the weakly-bound halo-nucleus of the projectile [15].

We also analyse in the frame of this model the influence of the ^{11}Be resonant states $5/2^+$, $3/2^-$, and $3/2^+$ [16–18] on the breakup processes. This analysis demonstrates the possibility of studying low-lying resonances in halo nuclei using their breakup reactions.

2. Theoretical framework and model inputs

The halo neutron is treated as a structureless particle weakly bound by the potential *V(r)* to the ^{10}Be core nucleus, where r is the relative variable between the neutron and the core. The dynamics of the halo neutron relative to the ^{10}Be core in the breakup reaction $^{11}\mathrm{Be}+^{208}\mathrm{Pb}\rightarrow^{10}\mathrm{Be}+n+^{208}\mathrm{Pb}$ is described by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\varPsi(\boldsymbol{r},t) = H(\boldsymbol{r},t)\varPsi(\boldsymbol{r},t) = [H_0(r) + V_{\mathrm{C}}(\boldsymbol{r},t)]\varPsi(\boldsymbol{r},t) \qquad (1)$$

in the projectile rest frame. In this expression,

$$H_0(r) = -\frac{\hbar^2}{2\mu}\varDelta_r + V(r) \qquad (2)$$

is the Hamiltonian describing a relative halo nucleon–core motion with reduced mass $\mu = m_n m_{\mathrm{c}}/M$, where m_n, m_{c} and $M = m_n + m_{\mathrm{c}}$ are the neutron, ^{10}Be core, and ^{11}Be masses, respectively. The potential $V(r)$ represents the sum of the l-dependent central potential $V_l(r)$ and the spin–orbit interaction $V_l^s(r)(\boldsymbol{ls})$. The interaction of the target nucleus with the projectile corresponds to the time-dependent Coulomb potential $V_{\mathrm{C}}(\boldsymbol{r},t)$, which is defined as

$$V_{\mathrm{C}}(\boldsymbol{r},t)=\frac{Z_{\mathrm{c}}Z_{\mathrm{t}}e^2}{|m_n\boldsymbol{r}/M+\boldsymbol{R}(t)|}-\frac{Z_{\mathrm{c}}Z_{\mathrm{t}}e^2}{R(t)}\,, \tag{3}$$

where Z_{c} and Z_{t} are charge numbers of the core and target, respectively, and $\boldsymbol{R}(t)=\boldsymbol{b}+\boldsymbol{v}_0t$ is the relative coordinate between the projectile and the target, where $\boldsymbol{b}$ is the impact parameter orthogonal to the initial velocity of the projectile $\boldsymbol{v}_0$. Here, we follow the definition accepted in [3–5].

To solve the time-dependent four-dimensional Schrödinger equation (1), we use the non-perturbative approach suggested and successfully applied for a number of different few-body problems in [5, 6, 13–15].

Our construction of the interaction $V(r)$ between the neutron and the ^{10}Be core is based on the parametrization used in [16] and represents the standard sum of the spherical Woods–Saxon potential $V_l(r)=-V_lf(r)$, where $f(r)=1/(1+\exp((r-R_0)/a))$ with the radius $R_0=2.585$ fm and the diffuseness $a=0.6$ fm, and the spin–orbit interaction

$$V_l^s(r)=V_{ls}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}f(r)(\boldsymbol{ls})\,. \tag{4}$$

The regular value $V_{ls}=21$ MeV fm^2 is used for the depth of the spin–orbit coupling potential (4) for a p-shell nucleus [3]. The Woods–Saxon potential with $V_l=62.52$ MeV ($l=0$) and $V_l=39.74$ MeV ($l=1$) reproduces the $1/2^+$ ground state of ^{11}Be at -0.503 MeV, the $1/2^-$ excited state at -0.183 MeV, and two resonant states $5/2^+$ and $3/2^+$ with the resonant parameters corresponding to theoretical [17] and experimental [18] values. To fix the position of the $3/2^-$ resonance close to the resonant value $E_{\mathrm{th}}=2.789$ MeV from [17], we tuned the parameters of the Woods–Saxon potential ourselves as $V_l=6.80$ MeV ($l=1$), $a=0.35$ fm, $R_0=2.5$ fm.

3. Breakup cross section

At previous studies of the breakup reaction of halo nucleus of ^{11}Be at a heavy target of ^{208}Pb within the time-dependent approach [5, 6], the total breakup cross section was calculated as a function of the energy E of the relative motion between the emitted neutron and the core nucleus where the l-wave component of the neutron wave function in the continuum spectrum was approximated by the regular spherical Bessel function [5, 6] and the breakup component was obtained by eliminating the bound states from the calculated wave packet [3, 5].

Since one of the main objectives of this work is to study the influence of resonant states of ^{11}Be on the reaction of its breakup, it becomes necessary to take into account the resonant nuclear interaction of the neutron with the core in the continuum spectrum. Therefore, we use alternative formulas for the breakup cross section including neutron interaction with the core [6, 13]

$$\frac{\mathrm{d}\sigma_{bu}(E)}{\mathrm{d}E} = \frac{4\mu k}{\hbar^2} \int\limits_{b_{\min}}^{b_{\max}} \sum_{lm} \left| \int \phi_{ljm}(k,r) Y_{lm}(\hat{r}) \Psi(\boldsymbol{r}, T_{\mathrm{out}}) \mathrm{d}\boldsymbol{r} \right|^2 b\,\mathrm{d}b\,. \quad (5)$$

Here, $\phi_{ljm}(k,r)$ is the radial part of the eigenfunction of the Hamiltonian $H_0(r)$ (2) in the continuum spectrum ($E = k^2\hbar^2/(2\mu) > 0$), normalized to $j_l(kr)$ as $kr \to \infty$ if $V(r) = 0$. The details of the numerical solution of the SE are described in [5, 6].

Following the investigation performed in Refs. [5, 6], the boundaries of integration (5) over the impact parameter b were chosen as $b_{\min} = 12$ fm and $b_{\max} = 400$ fm in order to achieve the demanded accuracy of the order of one percent in calculating the breakup cross section. It should be noted that the inclusion of the region $[0,\ b_{\min}]$ makes sense if the nuclear interaction between the target and the projectile is taken into account [13].

In [5] and [6], the breakup reaction ^{11}Be+^{208}Pb→^{10}Be+n+^{208}Pb was successfully investigated at 69 MeV/nucleon with the non-perturbative time-dependent approach we use here. However, the resonant states of ^{11}Be were not included in these calculations. Here, we overcome this drawback of the model: the resonant states $5/2^+$, $3/2^-$, and $3/2^+$ are taken into consideration.

In Fig. 1, we demonstrate that the inclusion of the resonant states in the interaction between the neutron and the ^{10}Be core gives a considerable contribution to the breakup cross sections. Overall, it is shown that the inclusion of the resonant states improves the agreement of the calculated breakup cross section with experimental data at 69 MeV/nucleon, where

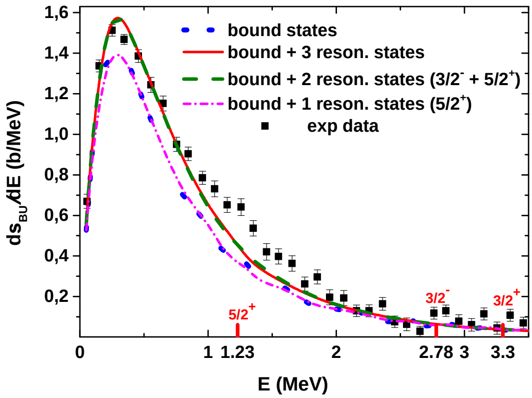

Fig. 1. The contribution of the resonant states $5/2^+$, $3/2^-$, $3/2^+$ to the breakup cross section $\mathrm{d}\sigma_{\mathrm{bu}}(E)/\mathrm{d}E$ in comparison with experimental data at 69 MeV/nucleon [10]. Convolution of the calculation with the experimental resolution was not performed.

most detailed and accurate experimental data are available [10]. To clarify the contribution of the dominant resonance to the breakup cross section, we performed the computation of the partial contribution of each resonance, which is illustrated in Fig. 1. It is shown that the resonances $3/2^+$ and $3/2^-$ make a slightly larger contribution to the cross section $\mathrm{d}\sigma_{\mathrm{bu}}(E)/\mathrm{d}E$ than $5/2^+$.

Then, we extend the approach for calculation of the breakup cross sections at low-energy beams up to 5 MeV/nucleon and investigate the contribution of the ^{11}Be resonance states in this region. Figure 2 (a) demonstrates the contribution to the breakup cross sections of the resonant states $5/2^+$, $3/2^+$, and $3/2^-$ at beam energies of 5 MeV/nucleon. It is shown that the inclusion of three resonant states ($5/2^+$, $3/2^+$, and $3/2^-$) of ^{11}Be into the breakup reaction considerably corrects the breakup cross sections, especially near the resonant energy 1.23 MeV of the $5/2^+$ resonance. In Fig. 2 (b), we present the results of the calculations, which take into account the influence of the resonant states ($5/2^+$, $3/2^-$, $3/2^+$) and the effect of the neutron–core interaction in the final state on the breakup cross section of the ^{11}Be nucleus. The analysis performed demonstrates a strong dependence of the calculated cross sections on the beam energy and an increase in their sensitivity to low-lying resonance $5/2^+$ with decreasing energy.

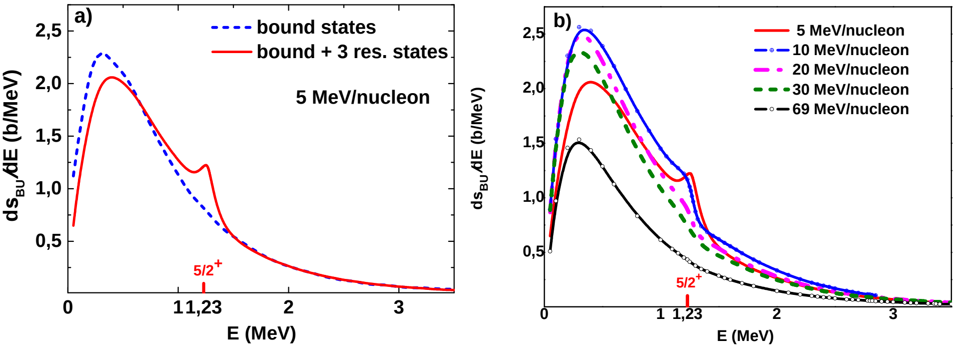

Fig. 2. The breakup cross sections calculated with only bound states of ^{11}Be in the computational scheme and with including three resonances ($5/2^+$, $3/2^-$, and $3/2^+$) of ^{11}Be at 5 MeV/nucleon (a). The breakup cross sections $\mathrm{d}\sigma_{\mathrm{bu}}(E)/\mathrm{d}E$ calculated for different beam energies with including the bound and three resonant states $5/2^+$, $3/2^-$, $3/2^+$ in the neutron–core interaction (b).

4. Summary

In this paper, the Coulomb breakup of one-neutron halo nuclei ^{11}Be on the heavy target ^{208}Pb has been studied within the non-perturbative time-dependent approach. The relative energy spectra of the fragments (neutron and core) were calculated for the Coulomb breakup of ^{11}Be on the ^{208}Pb target in the range of beam energies of 5–70 MeV/nucleon. In these calculations the influence of the resonant states $5/2^+$, $3/2^-$, and $3/2^+$ of ^{11}Be on the breakup cross section was taken into account. These results have been compared with experimental data [10] available at 69 MeV/nucleon. We have shown that the inclusion of the resonant states of ^{11}Be into the computational scheme leads to a significant improvement of the theoretical model, which gives a better agreement of the model description of the experimental data on the breakup cross sections [9, 10]. The developed computational scheme opens new possibilities in investigation of the Coulomb, as well as the nuclear, breakup of other halo nuclei on heavy and light targets.

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