DETERMINATION OF THE ASYMPTOTIC NORMALIZATION COEFFICIENTS FOR ^{93}Zr+$n \to^{94}$Zr FROM THE NEUTRON TRANSFER REACTION ^{94}Zr(^{12}C,^{13}C)^{93}Zr*

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The main aim of this work is to determine values of the asymptotic normalization coefficients for ^{93}Zr+$n \to^{94}$Zr. For this aim, the recently measured experimental differential cross section of the neutron transfer reaction ^{94}Zr(^{12}C,^{13}C)^{93}Zr has been analysed. The modified distorted wave Born approximation has been used for determination of the values of the asymptotic normalization coefficients. It has been shown that this reaction is peripheral. The contribution of the three-body Coulomb dynamics of the main transfer mechanism has been taken into account. New values of the asymptotic normalization coefficients for ^{93}Zr+$n \to^{94}$Zr with their uncertainties have been obtained.

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1. Introduction

Neutron radiation captured by the radioactive isotope ^{93}Zr form the stable isotope ^{94}Zr in the path of the main s-process. At present, experimental data of the cross sections for the radiation capture ^{93}Zr$(n,\gamma)^{94}$Zr reaction are very scarce due to their smallness, as well as the radioactivity of the target [1–3]. In [1], the effective cross sections for radiative capture ^{93}Zr$(n,\gamma)^{94}$Zr reaction were measured at neutron energies from 2.6 to 300 keV. In [3], the ^{93}Zr$(n,\gamma)^{94}$Zr radiative capture cross sections were measured up to the neutron energy of 8 keV, and an analysis was performed within the framework of the R-matrix method. However, in [1, 3], the

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contribution of the direct (nonresonant) capture of the ^{93}Zr$(n,\gamma)^{94}$Zr reaction was not considered. The experimentally measured differential cross sections of one particle transfer reactions are usually used to determine values of spectroscopic factors or asymptotic normalization coefficients [4–7]. In [6], the neutron spectroscopic factor of ^{94}Zr nucleus was obtained from the measured differential cross section of the ^{94}Zr(^{12}C,^{13}C)^{93}Zr reaction and its value used to calculate the direct (nonresonant) capture cross section of the ^{93}Zr$(n,\gamma)^{94}$Zr reaction.

As noted in [8], in order to calculate the cross sections for radiative capture of a neutron by nuclei in the framework of the R-matrix method, it is also necessary to correctly estimate the contribution of nonresonant (direct) capture. However, considering the ^{93}Zr$(n,\gamma)^{94}$Zr reaction in the framework of the traditional R-matrix method [8], where the direct capture amplitude is parameterized through the spectroscopic factor of the ^{94}Zr nucleus in the (^{93}Zr$+n$) configuration, ambiguities appear in the estimates of the contribution of the direct capture associated with the determination of reliable values of the above quantity. For example, the spectroscopic factor calculated in [9] is 1.5 to 1.8 times greater than the empirical values recommended in [10, 11]. Nevertheless, as suggested in [12, 13], these ambiguities can be eliminated by parametrizing the direct capture amplitude through the asymptotic normalization coefficient (ANC) for the ^{94}Zr nucleus in the (^{93}Zr$+n$)-channel [14].

In this regard, it is of great interest to obtain information about the "indirectly determined" value of ANC for ^{93}Zr$+n \to ^{94}$Zr from the analysis of the peripheral nuclear neutron transfer reaction ^{94}Zr(^{12}C,^{13}C)^{93}Zr and its application to correctly estimate the cross section for direct radiative capture of a nuclear astrophysical reaction ^{93}Zr$(n,\gamma)^{94}$Zr.

2. Results and discussion

In the modified distorted wave Born approximation (MDWBA), the calculated differential cross section for the peripheral nuclear one-particle (a) transfer reaction $A(x,y)B$ ($x = y + a$ and $B = A + a$) is parameterized in terms of the product of squares ANC and has the form of [15, 16]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = C^2_{aA;l_Bj_B} R\left(E_i, \theta; b_{ya;l_xj_x}, b_{aA;l_Bj_B}\right), \tag{1}$$

$$R\left(E_i, \theta; b_{ya;l_xj_x}, b_{aA;l_Bj_B}\right) = N^{\mathrm{TBCE}} \left(\frac{C^2_{ya;l_xj_x}}{b^2_{ya;l_xj_x}}\right) \times \frac{\sigma^{\mathrm{DWBA}}_{l_xl_Bj_B}\left(E_i, \theta; b_{ya;l_xj_x}, b_{aA;l_Bj_B}\right)}{b^2_{aA;l_Bj_B}}, \tag{2}$$

where $\sigma^{\rm DWBA}_{l_x l_B j_B}$ is the single-particle DWBA cross section; $b_{ya;l_x j_x}$ and $b_{aA;l_B j_B}$ are the single-particle ANCs which determine the amplitudes of the tails of the shell model wave functions corresponding to bound states ($x = y + a$ and $B = A + a$); $C_{aA;l_B j_{B\max}}$ ($C_{ya;l_x j_x}$) is ANC for $A + a \to B$ ($y + a \to x$), which determine the amplitudes of the tail of the corresponding radial overlap functions [14]; $N^{\rm TBCE}$ is the Coulomb renormalization factor, arising due to correct taking into account the three-body Coulomb dynamics in the transfer mechanism [17, 18]; l_x and l_B are the orbital moments of particle a in the nuclei $x(= (y + a))$ and $B(= (A + a))$, respectively. j_x and j_B are the total angular momentum of the transferred particle a in nuclei x and B, respectively.

The peripheral character of the considered reaction in the region of the main peak of the angular distribution can be formulated by the condition [16, 19]

$$R\left(E_i, \theta; b_{ya;l_x j_x}, b_{aA;l_B j_B}\right) = f(E_i, \theta) \tag{3}$$

as a function of free parameters $b_{ya;l_x j_x} = b_{ya;l_x j_x}(r_0, a)$ and $b_{aA;l_B j_B} = b_{aA;l_B j_B}(r_0, a)$ which in turn are functions of the geometric parameters of the radius and diffusion for the Woods–Saxon potential, for all scattering angles and a fixed value of the energy E_i. If conditions (3) are satisfied, then the value of ANC for $A + a \to B$ can be determined from the condition

$$C^2_{aA;l_B j_B} = \frac{\left(\frac{{\rm d}\sigma^{\rm exp}}{{\rm d}\Omega}\right)_{\theta=\theta_{\rm peak}}}{R\left(E_i, \theta; b_{ya;l_x j_x}, b_{aA;l_B j_B}\right)} = {\rm const.}\,, \tag{4}$$

which must be fulfilled for each fixed energy E_i, scattering angle θ and the corresponding function $R_{l_x j_x l_B j_B}$ from Eq. (3).

The fulfillment of the conditions (3) and (4) (or their slight violation within the experimental errors of the differential cross section) makes it possible to obtain indirectly determined ANC values for $A + a \to B$, using the experimental differential cross sections ${\rm d}\sigma^{\rm exp}/{\rm d}\Omega$ in the main peak. For determining the value of ANC, the analysis of the differential cross section of the ^{94}Zr(^{12}C,^{13}C)^{93}Zr reaction, which was measured in [6, 20] at $E_{^{12}\rm C} = 66$ MeV, have been carried out within the framework of the MDWBA and on its basis the obtained ANC values for $^{93}{\rm Zr}+n \to {}^{94}{\rm Zr}$ are presented below.

The calculations were carried out using the LOLA code [21]. The optical potential (OP) parameters for the initial and final states were taken from Refs. [6, 20]. The value of ANC for $^{12}{\rm C}+n \to {}^{13}{\rm C}$ is equal to $C^2_{^{12}{\rm C}n;11/2} = 2.35 \pm 0.12$ fm^{-1} [22]. The value of single-particle ANC for $^{12}{\rm C}+n \to {}^{13}{\rm C}$ is equal to $b_{^{12}{\rm C}n;11/2} = 1.91$ fm^{-1}. For testing the peripheral character of the considered reaction, the parameters r_0 and a of the Woods–Saxon potential are varied in the intervals $1.15 \le r_0 \le 1.35$ fm and $0.59 \le a \le 0.71$ fm. This variation changes the single-particle ANC $b_{n^{93}{\rm Zr};25/2}$ in the range of $10.34 \leq b_{n^{93}\mathrm{Zr};25/2} \leq 19.07$. The single-particle DWBA cross section $\sigma^{\mathrm{DWBA}}_{1\,2\,5/2}(E_i,\theta)$ $(\theta \approx \theta_{\mathrm{peak}})$ is a rapidly changing function of the variable $b_{n^{93}\mathrm{Zr};25/2}$. Nevertheless, the calculated values of $R(E_i,\theta;b_{ya;l_xj_x},b_{aA;l_Bj_B})$ at a fixed value of $b_{n^{93}\mathrm{Zr};25/2}$ as a function of the free parameter $b_{n^{93}\mathrm{Zr};25/2}$ practically does not change within the aforementioned interval. The results of the calculation of the $R(E_i,\theta;b_{ya;l_xj_x},b_{aA;l_Bj_B})$ function and the ANC $C_{n^{93}\mathrm{Zr};25/2}$ of ^{94}Zr nucleus in the (^{93}Zr+n)-channel are shown in Fig. 1. As seen from the figure, the conditions (3) and (4) for $C^2_{n^{93}\mathrm{Zr};25/2}$ are satisfied with a high accuracy not exceeding the experimental error of the analyzed differential cross section [6, 20]. The calculations for other sets of OPs give similar results.

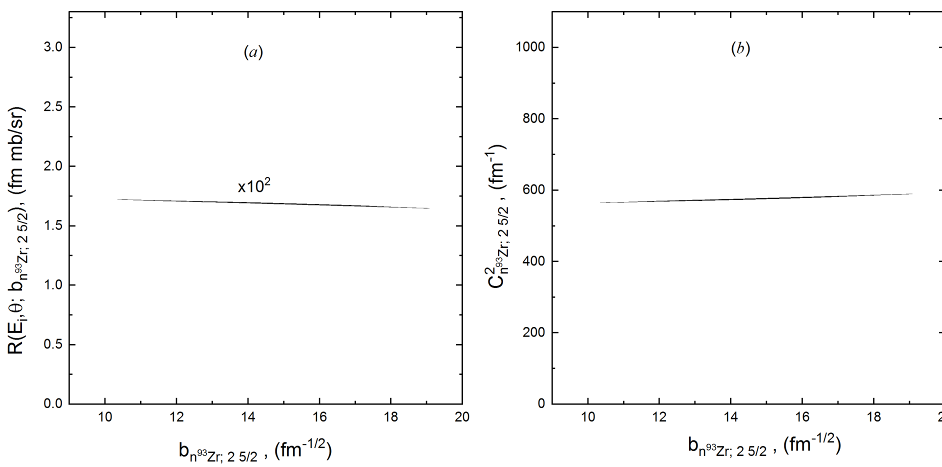

Fig. 1. The dependence of the $R(E_i,\theta;b_{ya;l_xj_x},b_{aA;l_Bj_B})$ function [in panel (a)] and the ANC $C_{n^{93}\mathrm{Zr};25/2}$ of ^{94}Zr nucleus in the ($^{93}\mathrm{Zr}+n$)-channel [in panel (b)] on the single-particle ANC $b_{n^{93}\mathrm{Zr};25/2}$ for the ^{94}Zr(^{12}C,^{13}C)^{93}Zr reaction at $E_{\mathrm{lab}}(^{12}\mathrm{C}) =$ 66 MeV for Set 1 at $\theta_{\mathrm{peak}} = 36.2^{\circ}$. The width of the band corresponds to variation of the parameters r_0 and a in the intervals of $1.15 \leq r_0 \leq 1.35$ fm and $0.59 \leq a \leq$ 0.71 fm.

The weighted values of the ANCs for ^{93}Zr+n $\rightarrow$^{94}Zr for each set of OPs are presented in Table I. The error of ANC is the mean square error associated with the uncertainty of the calculated values of the functions $R(E_i,\theta;b_{^{94}\mathrm{Zr}})$ depending on the free parameter $b_{n^{93}\mathrm{Zr};25/2}$ and the error of the experimental differential cross section.

The weighted average ANC value obtained from the weighted values of the ANCs corresponding to OP sets is equal to $C^2_{n^{93}\mathrm{Zr};25/2} = 573.9 \pm 54.1$ fm^{-1}. The weighted values of ANCs corresponding to sets of OPs were used to calculate the differential cross sections using relation (1). The results

TABLE I

The weighted values of ANCs $C^2_{n^{93}\mathrm{Zr};25/2}$ for $^{93}\mathrm{Zr}+n\rightarrow{}^{94}\mathrm{Zr}$, obtained from the analysis of the peripheral reaction $^{94}\mathrm{Zr}(^{12}\mathrm{C},^{13}\mathrm{C})^{93}\mathrm{Zr}$ for different sets of OPs.

θ [deg]	Set 1	Set 2	Set 3	Set 4
		$C^2_{n^{93}\mathrm{Zr};25/2}$		
29.5°	515.6 ± 56.6	582.8 ± 64.1	512.7 ± 56.3	503.7 ± 55.3
31.7°	606.0 ± 61.5	670.6 ± 68.0	593.6 ± 60.2	592.6 ± 60.1
33.9°	572.1 ± 56.9	623.3 ± 62.1	558.0 ± 56.5	568.8 ± 56.6
36.2°	571.1 ± 57.7	612.1 ± 61.8	561.4 ± 56.7	569.1 ± 57.5
		the weighted	mean values	
	562.3 ± 54.3	619.1 ± 58.7	555.7 ± 53.1	554.3 ± 53.9

of the differential cross section and its comparison with the experimental data from Refs. [6, 20] are shown in Fig. 2. As can be seen from the figure, the calculated cross sections within the framework of the MDWBA reproduce well the experimental data in the region of the main maximum of the angular distribution.

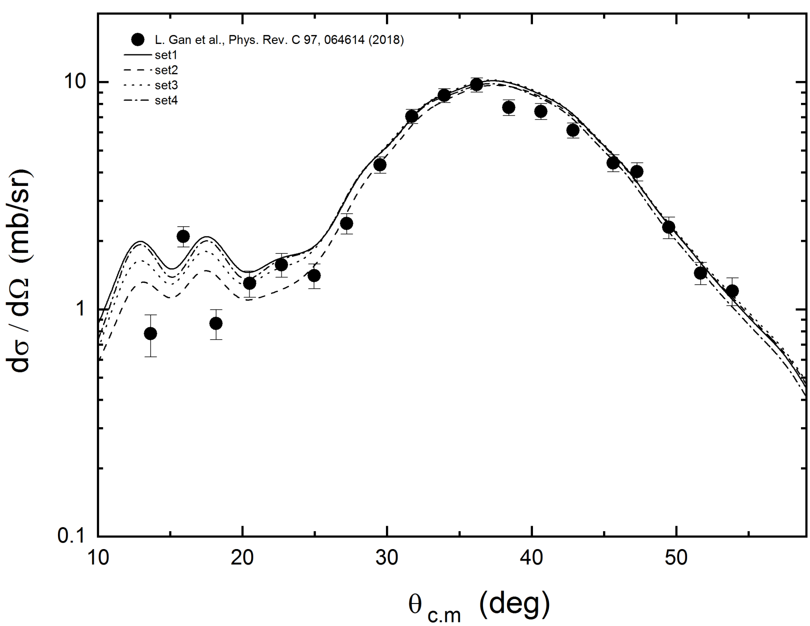

Fig. 2. The differential cross sections for the $^{94}\mathrm{Zr}(^{12}\mathrm{C},^{13}\mathrm{C})^{93}\mathrm{Zr}$ reaction at $E_{^{12}\mathrm{C}}=66$ MeV. The curves are the results of the calculation according to MDWBA, performed for various sets of OPs (Set 1–4) using the corresponding ANC from Table I. The experimental points are taken from Refs. [6, 20].

3. Conclusions

The results of the analysis of the experimental differential cross section of the neutron transfer reaction ^{94}Zr(^{12}C,^{13}C)^{93}Zr at $E_{^{12}\mathrm{C}} = 66$ MeV of four sets of optical potentials in the entrance and final states were carried out within the framework of the MDWBA. It was shown that this reaction is peripheral in the region of the main peak of the angular distribution. It has been demonstrated that the analyzed experimental data make it possible to determine in a model-independent way the ANC value for the ^{94}Zr nucleus in the (^{93}Zr$+n$) configuration. The new values of the asymptotic normalization coefficients for ^{93}Zr$+n \rightarrow ^{94}$Zr with their uncertainties have been extracted.

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