ELASTIC AND INELASTIC SCATTERING OF α-PARTICLES ON ^{11}B NUCLEI AT $E = 40$ AND 65 MeV AND V–W-AMBIGUITY OF THE CHOICE OF OPTICAL POTENTIALS*

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(Received November 23, 2021; accepted November 23, 2021)

In this paper, the experimental data of the elastic interactions of α-particles with ^{11}B nuclei at the energies of 40 and 65 MeV are measured. Registration and identification of scattered α-particles were carried out by a standard ΔE–E method implemented on the basis of a PC/AT personal computer. The angular distributions were analyzed using an optical model and the FRESCO and SPI-GENEO computer codes. It is shown that there is a strong correlation between the real and imaginary parts of the potential, which is expressed in the fact that changes in the real part can be compensated by the corresponding changes in the imaginary part (and *vice versa*), without changing the quality of the description of the experimental data. A good description of the experimental data has been achieved in the full range of angles with potentials having volume integrals in the range of 250–500 MeV fm^3.

DOI:10.5506/APhysPolBSupp.14.719

1. Introduction

The study of elastic and inelastic scattering of low- and medium-energy α-particles on nuclei is an important source of information for the dynamics of the nuclear reactions as well as the nuclear structure properties such as the deformation parameters of the distribution of masses, charges, and the structure of states in nuclei [1, 2]. Such direct processes occurring during

* Presented at III International Scientific Forum “Nuclear Science and Technologies”, Almaty, Kazakhstan, September 20–24, 2021.

collisions of α-particles with an energy greater than 10 MeV per nucleon allow us to obtain information about the properties of interaction of nuclear particles and the structure of specific nuclear states [2]. In these quasi-elastic processes, the direct mechanism dominates almost in its pure form without causing mixing of many excited states. At the same time, single-particle and quasi-linear, as well as various types of collective states are selectively excited. Knowledge of the intensity of nuclear interaction in various processes and their potentials will help to clarify the nature of an extensive class of nuclear transformations involving various nuclear particles in the input and output channels. There are a number of studies on the real and imaginary parts of nuclear potential (V–W)-correlations in the processes of elastic scattering of ions ^{3}He and alpha-particles on the nuclei of 1p-shells [3–5]. In these works, the analysis of the angular distribution of elastic scattering was carried out with potentials, imaginary part of which was included in itself as a volumetric, as well as superficial absorption. The presence of a strong correlation between V–W and disregard for the manifestation of the effect of nuclear radiation scattering did not allow to completely eliminate the discrete ambiguity of the choice of real potential. There were three families of potentials with volumetric integrals of 200, 300, and 400 MeV fm^3, which uniformly well describe the experimental differential divisions of elastic scattering. In the present study, the scattering of α-particles on the nucleus ^{11}B under the conditions of the manifestation of the phenomenon of nuclear rainbow [6, 7], which contributes to the limitation of the discrete ambiguity of the depth of the real part are presented. For a theoretical analysis of elastic scattering, in principle, the optical model of the nucleus (OM) [6, 8], which is the most developed, is used. The optimal parameters of the phenomenological optical potentials (OP) were obtained from the comparison of the results of the calculation with the experimental data. Most often, the potential is used with the Woods–Saxon parameterization, the form of which quite well reproduces the distribution of nuclear density.

2. Experimental part

Experimental angular distributions of elastic scattering of α-particles by ^{11}B nuclei were measured on the extracted beams of the U-150M isochronous cyclotron of the Institute of Nuclear Physics of the Republic of Kazakhstan at energies of 40 MeV. In the experiment, the metal foils made of boron isotopes were used as targets, the thickness of which was determined by weighing as well as from the energy losses of α-particles of the radioactive source ^{241}Am, ^{243}Am, ^{244}Cm, and ^{239}Pu with an accuracy of 6–7%. The working thickness of the targets varied from 0.1 to 0.2 mg/cm^2, depending on the energy of the incident particles. Registration and identification of scattered α-particles was carried out by the standard ΔE–E technique, implemented on the basis of a PC/AT personal computer. ORTEC silicon semiconductor detectors were used as counters. The magnitude of the current on the target varied depending on the angle in the range from several units to 200 nano-amperes. You can find all the information about the experiment in our previous article [6].

3. Theoretical analysis

The most developed method for extracting information about the interaction potentials of particles with atomic nuclei is the phenomenological analysis of experimental data on elastic scattering based on the OP of the nucleus, the substantiation and detailed mathematical formulation of which are presented in work [7]. Within the framework of this approach, the problem of scattering for a multinucleon system-nucleus is reduced to a simpler process — the scattering of particles in the mean field of a complex OP, the shape and depth of which are determined by optimizing the calculated values of potential parameters describing experimental data. The parameters of the potentials were chosen from the condition of the best fit of the theoretical cross sections to the experimental ones. One of the starting potentials was taken by us from [4], in which empirical expressions were proposed for the central potential with purely surface absorption, the values of the parameters of which depend on the ^{4}He energy and the mass of the target nucleus. This potential describes well the scattering of ^{4}He in the energy range from 10 to 220 MeV by nuclei from beryllium to lead. Other starting values of the parameters were taken from [5]. The results of the search with minimization in χ^2 are shown in Table I. For each energy, 7 sets of potentials were obtained, equally well describing the experimental data. Examples of descriptions are shown in figure 1. Volume integrals of the real parts of the potential (J_V), normalized to a pair of interacting particles, as seen from the table, have a significant scatter from 250 to 530 MeV fm^3.

For further discussion, it is useful to determine the areas of maximum sensitivity to the real and imaginary parts of the potentials. For this purpose, a series of calculations was carried out with potential 3 (Table I), the values of which were set on a grid of radii with a step of 0.1 fm. For a set of points in the range of r from 0 to 6.5 fm (for each of them separately), the upper and lower boundaries of $V(r)$ and $W(r)$ were sought, within which the changes in χ^2/N did not exceed 25%.

The obtained ratios V, W, and V/W depending on r are shown in Fig. 2. It can be seen that the region of maximum sensitivity to the real potential falls within the interval from 0.5 to 6 fm. It is somewhat narrower for the imaginary part and is in the range of 2.5–5.5 fm. These conclusions correlate

TABLE I

Parameters of potentials found from the analysis of elastic scattering of α-particles on ^{11}B nuclei at energies of 40 and 65 MeV.

$E_\alpha = 65$ MeV	Set1	Set2	Set3	Set4	Set5	Set6	Set7
V [MeV]	78	94.63	96.48	101.3	102.87	105.56	127.0
r_V [fm]	1.205	1.305	1.305	1.199	1.205	1.18	1.205
a_V [fm]	0.8	0.746	0.785	0.798	0.785	0.8	0.8
W [MeV]	13.41	20.72	28.41	18.11	16.096	19.98	19.41
r_W [fm]	1.65	1.55	1.55	1.57	1.65	1.55	1.35
a_W [fm]	0.761	0.749	0.578	0.814	0.792	0.65	0.761
J_V [MeV fm^3]	268.55	363.86	386.58	344.62	348.03	353.15	437.25
J_W [MeV fm^3]	89.85	118.42	141.51	112.7	110.5	115.2	130.04
χ^2/N	23.57	6.79	18.64	8.54	9.02	9.08	24.46
$E_\alpha = 40$ MeV	Set1	Set2	Set3	Set4	Set5	Set6	Set7
V [MeV]	74.71	76.65	81.20	125.73	138.14	147.11	151.84
r_V [fm]	1.25	1.215	1.215	1.245	1.285	1.23	1.2
a_V [fm]	0.848	0.869	0.86	0.743	0.768	0.74	0.75
W [MeV]	23.55	18.51	22.75	28.49	8.5	19.96	17.29
r_W [fm]	1.29	1.41	1.29	1.29	2.21	1.53	1.6
a_W [fm]	0.881	0.897	0.895	0.702	0.385	0.65	0.65
J_V [MeV fm^3]	293.2	291.2	289.8	434.5	525.65	493.3	486.9
J_W [MeV fm^3]	102.5	98.45	100.5	101.89	102.34	101.7	98.18
χ^2/N	9.0	8.67	9.01	16.3	15.39	16.36	3.5

with the results of [9, 10]. Calculations show that, in the sensitivity region, changes in $V(r)$ and $W(r)$ by about 10% only at one point lead to an increase in χ^2/N by 20%. On the contrary, in the central and peripheral regions, the same increase in 2 corresponds to a more than twofold change in the value of the potential.

Despite the fact that all the found potentials equally well reproduce the experimental data, the corresponding scattering matrices can differ greatly [11]. As an example, Fig. 3 shows their complex elements (S_j) related to potentials 2 and 7 of Table I for partial waves with the maximum contribution to the cross section. Potentials from these families are known to be approximately phase equivalent. The indicated potentials, as can be seen from the figure, are not phase equivalent and, therefore, do not belong to

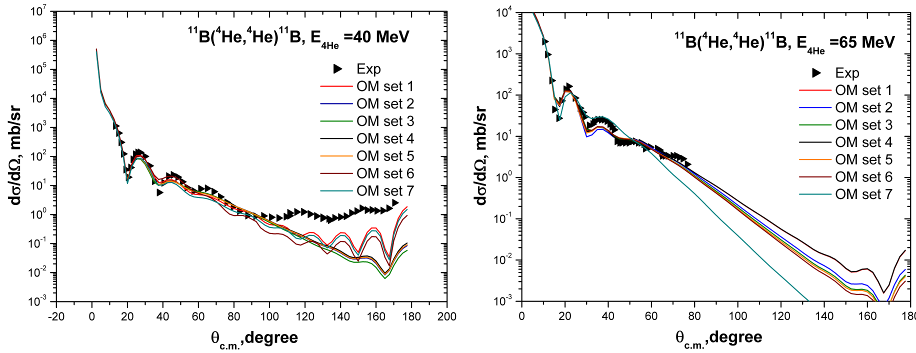

Fig. 1. Solid curves — calculations for the OM of the nucleus with potentials 1–7.

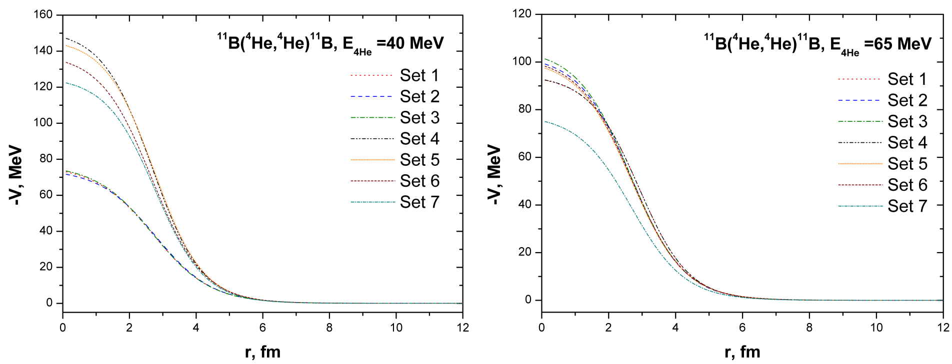

Fig. 2. Radial dependences of the real ($V(r)$) and imaginary ($W_D(r)$) parts of the potentials from Table I.

families associated with ordinary discrete and continuous ambiguities. To answer the question of how unambiguously the values of the volume integrals of the real part of the potential can be extracted from the scattering data, more detailed calculations were performed, in which the dependence of the quality of fit on J_V was investigated. Such calculations were carried out for a grid of fixed radii in the vicinity of their tabular values for each potential in Table I. The rest of the parameters, with the exception of the radii, were in the process of fitting the theoretical sections to the experimental ones on the basis of the χ^2/N criterion. The obtained dependence of χ^2/N on J_V is shown in Fig. 3. Naturally, it is not the only possible one and it is determined both by the starting values of the potentials and by the search procedure itself. Calculations indicate only that a satisfactory description of the experimental data can be obtained for practically any J_V values in the range of 250–530 MeV fm^3 and, thus, the volume integral of the real part cannot be determined from the scattering data without introducing addi-

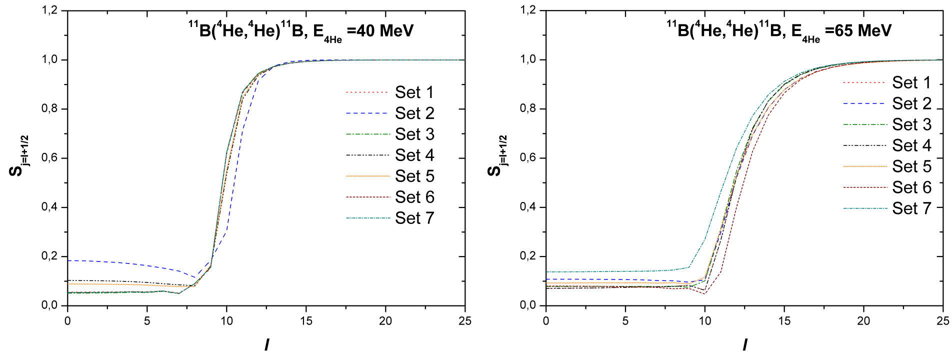

Fig. 3. The values of the moduli of the elements of the scattering matrix S_j and the dependence of the value χ^2/N on the volume integral of the real part for the potentials from Table I.

tional restrictions on parameters of potentials. Relatively higher values of χ^2/N for potentials with $J_V = 250$–300 MeV fm^3 do not mean that other J_V in this range that better describe the angular distributions cannot be found. In support of this, we note that for energies of 40 and 65 MeV, there are potentials with $J_V = 268$ and 344 MeV fm^3 (see Table I), which give a quite acceptable description. If we consider the continuous family of R_i and V_R which fit the data at a certain a_i, than $V_R \exp(R_i/a_i) = \mathrm{const}$.

We can find $V_R \exp(R_i/a_i) = 674.2 \pm 28(481.6 \pm 71.8)$ at energies of 40 and 65 MeV, respectively. In the same situation for the imaginary part by using all sets of the potential except the first set for 65 MeV and the second set of 40 MeV, we found $W_R \exp(R_i/a_i) = 184.9 \pm 15.4(133.87 \pm 20.3)$. However, the scattering is usually sensitive only to the tail of the potential *i.e.* at $r \gg R_i$, where

$$\frac{\hbar \exp(r-R_i)}{a_i} \gg 1\,; \qquad V_R\left(1+\frac{\hbar \exp(r-R_i)}{a_i}\right)^{-1} \cong V_R \frac{\hbar \exp(R_i)}{a_i}\,\frac{\hbar \exp(-r_i)}{a_i}\,. \tag{1}$$

As a result, for a given value of a_i, all combinations of V_R and R_i that satisfy the above relation define constant potential with the same tail for $r \gg R_i$, the same equation holds true for the imaginary part. This indicates that the Woods–Saxon is in a strong absorption situation. These findings are consistent with Igo ambiguity, which was found for α scattering at 40 MeV [12]. To judge the behavior of correlation between the real and imaginary parts of the optical potential, it is necessary to study a wide range of energies for this type of nuclear reaction.

To determine the correlation between the depth of the real and imaginary parts of the Woods–Saxon potentials, we analyzed the ^{11}B excitation level $5/2^-$ (4.445 MeV) using a deformation coefficient of 0.42. As a result, we came to the conclusion that taking into account the correlation between V and W, volume integrals can serve as additional criteria, they are sensitive precisely when the volume integral is above 420 MeV fm^3.

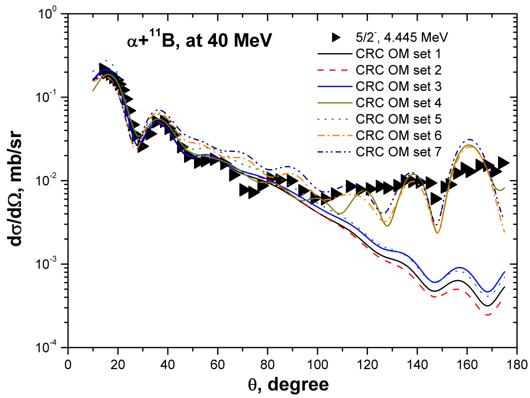

Fig. 4. Solid curves — calculations for the CRC of the nucleus with potentials 1–7.

4. Conclusions

At energies of 40 and 65 MeV, the experimental data of the elastic scattering of α-particles by ^{11}B nuclei are measured and investigated theoretically. Differential cross sections measured in a wide angular range (up to 170 degree) are analyzed within the framework of the OM using potentials in the Woods–Saxon parametrization, including both volumetric and surface absorption. The potential parameters were found phenomenologically by fitting the calculated cross sections to the experimental data. For each energy, seven potentials were found that equally well describe the experiment in the full angular range. Their volume integrals for the real part have a significant scatter from 250 to 530 MeV fm^3. The analysis showed that the values of both the real and imaginary parts of the potentials, even in the sensitivity region, can differ by more than two times. The observed scatter is explained by the strong correlation between V and W, so that even significant changes in the real part can be compensated for by corresponding changes in the imaginary (and *vice versa*), which indicates the existence of V–W ambiguity in the choice of the OP. This type of ambiguity, in spite of the clear manifestation of the NR effects in the scattering, makes it difficult to obtain information on the nuclear density distribution or relative material radii from the data of phenomenological analysis.

The work was supported by the program No. BR09158499 (Development of complex scientific research in nuclear and radiation physics on the basis of Kazakhstan's accelerator complexes) of the Ministry of Energy of the Republic of Kazakhstan.

REFERENCES

[1] N. Burtebayev *et al.*, *Int. J. Mod. Phys. E* **27**, 1850094 (2018).
[2] A.N. Danilov *et al.*, *Phys. Atom. Nuclei* **78**, 777 (2015).
[3] S.N. Ershov *et al.*, *Phys. Lett. B* **227**, 315 (1989).
[4] H.J. Trost, P. Lezoch, U. Strohbusch, *Nucl. Phys. A* **462**, 333 (1987).
[5] A.S. Dem'yanova *et al.*, *Phys. Scr.* **T32**, 89 (1990).
[6] N. Burtebaev *et al.*, *Phys. Atom. Nuclei* **68**, 1303 (2005).
[7] D.A. Goldberg, S.M. Smith, *Phys. Rev. Lett.* **29**, 500 (1972).
[8] D.A. Goldberg, S.M. Smith, *Phys. Rev. C* **10**, 1362 (1974).
[9] H. Yamaguchi *et al.*, *Phys. Rev. C* **83**, 034306 (2011).
[10] N. Burtebayev *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **11**, 99 (2018).
[11] N. Burtebayev *et al.*, *Eur. Phys. J. A* **55**, 38 (2019).
[12] G. Igo, *Phys. Rev.* **115**, 1665 (1959).