PHENOMENOLOGICAL ANALYSIS OF MULTIPOLE MIXTURE COEFFICIENTS δ(E2/M1) OF ROTATIONAL BANDS IN ^{156}Gd*

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We have shown that the effects of nonadiabaticity, observed in the magnetic characteristics can be satisfactorily explained by the Coriolis mixing of the states of the rotational bands in ^{156}Gd.

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1. Introduction

Analyzing the available experimental data obtained in the $(\alpha, 2n)$, (n, γ), and $(n, n'\gamma)$ reactions on the ^{156}Gd nucleus, one may assume that almost all excited levels up to the excitation energy about 2 MeV are observed in this nucleus [1]. Now, it is well known that there are five rotational bands built on the bases with $K^\pi = 0^+$, two bands with $K^\pi = 2^+$, and fifteen dipole levels of positive parity. The energies of 1^+ levels and the probabilities of excitations have been determined in Ref. [2]. These data are very important for the classification of existing energy levels as well as to study similar levels in neighboring nuclei.

The reduced probabilities of electric quadrupole transitions from rotational states to the levels of the main band are known experimentally. The ratios of the transition probabilities, the multipole mixture coefficients, and the magnetic moments have been also measured [1–4].

It is interesting to note that these experimental data reveal deviations from the rules of the adiabatic theory (Alagi rules) [5]. For example, significant deviations from the rules are observed in the energies of the states of the rotational bands and the ratios of the probabilities of electromagnetic transitions from the states of the bands built on vibrational bases.

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In Refs. [7, 8], within the framework of the phenomenological model [6], which takes into account the Coriolis mixing of the states of rotational bands, the low-lying states of the ^{156}Gd nucleus were investigated. The energies, wave functions, and electrical characteristics of the states of the rotational bands were calculated. It has been established that the reasons leading to nonadiabatic effects are manifested in the energies, wave functions of rotational states, and ratios of electrical transitions from vibrational states to levels of the main band.

In this work, we continue to study the properties of the rotational states of the ^{156}Gd nucleus. Using the wave functions obtained in [7], we calculated the reduced probabilities of electromagnetic transitions and investigated nonadiabaticities that appear in the coefficients of the multipole mixture of δ(E2/M1) states of the rotational bands. The influence of the rotation on the electromagnetic characteristics of excited states is studied.

The phenomenological model used is described in detail in review [6]. Earlier, we have successfully applied this model to study the Coriolis mixing of bands of states in isotopes ^{158,160}Gd [9–11].

2. Electromagnetic transitions and coefficients mixtures of multipoles δ(E2/M1)

Along with the given probabilities of M1-transitions, the coefficients of the mixture of δ(E2/M1) multipoles are investigated, which are calculated using the following formula:

$$\delta\left(I_{\mathrm{i}}K_{\mathrm{i}} \to I_{\mathrm{f}}K_{\mathrm{f}}\right) = 0.834 E_\gamma\ (\mathrm{MeV}) \frac{\langle I_{\mathrm{f}}K_{\mathrm{f}} \| \hat{m}(\mathrm{E2}) \| I_{\mathrm{i}}K_{\mathrm{i}}\rangle}{\langle I_{\mathrm{f}}K_{\mathrm{f}} \| \hat{m}(\mathrm{M1}) \| I_{\mathrm{i}}K_{\mathrm{i}}\rangle} \left(\frac{e\mathrm{b}}{\mu_{\mathrm{N}}}\right), \tag{1}$$

where $\hat{m}$(E2) and $\hat{m}$(M1) are electric quadrupole and magnetic dipole operators, respectively, E_γ is the γ-transition energy, b is barn, μ_{N} is a nuclear magneton.

The matrix element of the operator of the quadrupole electric transition $\hat{m}$(E2) between these states is determined by formula [6]

$$\begin{aligned}
\langle I_{\mathrm{f}} 0_1 \| \hat{m}(E2) \| I_{\mathrm{i}}K_{\mathrm{i}}\rangle &= (2I_{\mathrm{i}}+1)^{1/2} \left\{ \sqrt{\frac{5}{16\pi}} Q_0 \left[\Psi^{I_{\mathrm{f}}}_{0_1,0_1} \Psi^{I_{\mathrm{i}}}_{0_1,K_{\mathrm{i}}} C^{I_{\mathrm{f}}0}_{I_{\mathrm{i}}0;20} \right.\right. \\
&+ \left. \sum_{K_n} \Psi^{I_{\mathrm{f}}}_{K_n,0_1} \Psi^{I_{\mathrm{i}}}_{K_n,K_{\mathrm{i}}} C^{I_{\mathrm{f}}K_n}_{I_{\mathrm{i}}K_n;20} \right] + \sqrt{2} \left[\Psi^{I_{\mathrm{f}}}_{0_1,0_1} \sum_n \frac{(-1)^{K_n} m_{K_n} \Psi^{I_{\mathrm{i}}}_{K_n,K_{\mathrm{i}}}}{\sqrt{1+\delta_{K_n,0}}} C^{I_{\mathrm{f}}0}_{I_{\mathrm{i}}K_n;2-K_n} \right. \\
&+ \left.\left. \Psi^{I_{\mathrm{f}}}_{0_1,K_{\mathrm{i}}} \sum_{K_n} \frac{m_{K_n} \Psi^{I_{\mathrm{i}}}_{K_n,0_1}}{\sqrt{1+\delta_{K_n,0}}} C^{I_{\mathrm{f}}K_n}_{I_{\mathrm{i}}0;2K_n} \right] \right\}.
\end{aligned} \tag{2}$$

Here the quantum number K_n takes the values of $K_n = 0_2^+, 0_3^+, 0_4^+, 0_5^+, 1_\nu^+$, 2_1^+, and 2_2^+. The quantities $m_{K_n} = \langle 0_1^+|\hat{m}(\mathrm{E2})|K_n\rangle$ in (2) are the matrix elements of the operator between the internal wave functions of the main band ($K_\nu^\pi = 0_1^+$) and other bands included in the basis of the Hamiltonian of the model [7]; Q_0 — internal quadrupole moment of the nucleus; $\Psi_{KK'}^I$ — mixing amplitudes of states of different bands with the same angular momentum I due to the Coriolis interaction; $C_{I_\mathrm{i}K_\mathrm{i};2K_\mathrm{i}+K_\mathrm{f}}^{I_\mathrm{f}K_\mathrm{f}}$ — the Clebsch–Gordan coefficients.

In the adiabatic approximation, the following expression is valid for the reduced probability of an E2-transition from the vibrational band with $K^\pi = 0^+$ and $K^\pi = 2^+$:

$$B^{\mathrm{rot}}(\mathrm{E2}; I_\mathrm{i}K_\mathrm{i} \to I_\mathrm{f}0_1) = (2 - \delta_{K_\mathrm{i},0}) \left| m_{K_\mathrm{i}} C_{I_\mathrm{i}K_\mathrm{i};2-K_\mathrm{i}}^{I_\mathrm{f}0} \right|^2 . \tag{3}$$

The reduced matrix element of the M1-transition may be presented as follows:

$$\begin{aligned}\langle I'0_1\|\hat{m}(\mathrm{M1})\|IK\rangle \;=\; &\sqrt{\frac{3(2I+1)}{4\pi}} \left(\sum_{K_1=1}^{2} (g_{K_1} - g_\mathrm{R}) K_1 \Psi_{K_1K}^{I} \Psi_{K_10_1}^{I'} C_{IK_1;10}^{I'K_1} \right. \\ &\left. + \frac{\sqrt{6}}{10} \sum_\nu m'_{1_\nu} \Psi_{0_10_1}^{I'} \Psi_{1_\nu K}^{I} C_{I1;1-1}^{I'0} \right) , \end{aligned} \tag{4}$$

where $m'_{1_\nu} = \langle 0_1^+\|\hat{m}(\mathrm{M1})\|1_\nu^+\rangle$ are matrix elements between the internal wave functions of the main (0_1^+) and 1_ν^+-bands; g_K is the internal g-factor of the band with $K \neq 0, g_\mathrm{R} = Z/A$ is the gyromagnetic factor associated with the rotation. As to g_R, it may be fixed as $g_\mathrm{R} \approx 0.4 + 0.1$ from the systematics of gyromagnetic ratios for deformed nuclei of the rare-earth and transuranium regions.

In the adiabatic approximation, formula (1) for M1-transitions from states to ($I1_\nu^+$) states to $(I \pm 1)0_1^+$ states of the main band is given by

$$\delta(I1_\nu^+ \to (I \pm 1)0_1) = -9.855 E_\gamma \left(\frac{m_{1\nu}}{m'_{1\nu}} \right) \frac{C_{I1;2-1}^{(I\pm1)0}}{C_{I1;1-1}^{(I\pm1)0}} . \tag{5}$$

3. Results and discussions

The absolute values of parameters $m_{0_2}, m_{0_3}, m_{0_4}, m_{0_5}, m_{2_1}$, and m_{2_2} in (2) were calculated using relation (3) and experimental values of $B(\mathrm{E2}; 2K_\mathrm{i} \to 00_1)$ [1]. Since there are no experimental data on the probabilities of E2-transitions from states of bands with $K^\pi = 1_\nu^+$, the numerical values of

m_{1_ν} and the signs of the parameters m_{1_ν}, m_{2_1}, and m_{2_2} were determined from the condition of the best description of the data on the ratios R_{I2_1} for transitions from states with odd spins of the $K^\pi = 2_1^+$ band [1, 12]. Therefore, all matrix elements m_{1_ν} were assumed to be the same ($m_{1_\nu} = m_1$). Signs of $m_{0_2}, m_{0_3}, m_{0_4}, m_{0_5}$ were determined by the best agreement of the experimental values of the reduced probabilities of E2-transitions from the rotational levels of the bands with $K^\pi = 0_2^+, 0_3^+$ and from states with even spins of the band with $K^\pi = 2_1^+$ [1]. The value of the quadrupole moment Q_0 was taken from the experiment of [4]. The numerical values of parameters m_{K_n} and Q_0 are given in Table I. Signs of all parameters m_{K_n}, given in Table I, except for m_{0_2} coincide with the signs of the same parameters for the ^{158,160}Gd nuclei [9–11]. Apparently, the reason for this is that in contrast to the spectra of the ^{158,160}Gd isotopes, in the spectrum of the ^{156}Gd nucleus, the band with $K^\pi = 0_2^+$ is located lower than the band with $K^\pi = 2_1^+$.

TABLE I

The values of parameters m_K and intrinsic quadrupole moment Q_0 used in calculations of transition probabilities ^{156}Gd (in units of eFm^2).

Q_0[6]	m_{0_2}	m_{0_3}	m_{0_4}	m_{0_5}	m_{1_ν}	m_{2_1}	m_{2_2}
687	−14.0	14.4	10.0	−2.0	−13.0	25.0	8.0

Parameters g_K and m'_{1_ν} in (4) describing the magnetic characteristics were determined as follows. In fact, in the ^{156}Gd nucleus, there are no experimental data for M1-transitions within $K^\pi = 2_{1,2}^+$ bands. Therefore, the numerical value of the g_{R}-parameter could be determined by the well-known formula $g_{\mathrm{R}} = Z/A$ for the deformed nuclei. For the g_K-parameter, the values determined for the ^{158}Gd kernel were taken as in Ref. [9]. The role of these parameters in calculating the magnetic moments of the states of $K^\pi = 0_2^+, K^\pi = 2_1^+, K^\pi = 1_\nu^+$ bands and in-band transitions is important. However, in the transitions between the states of the rotation bands, their role is not significant [6]. For the interband transitions, the part of the formula containing the parameter m'_1 dominates. Numerically, the latter can be evaluated as follows:

$$m'_{1_\nu} = \sqrt{\frac{B\left(\mathrm{M1}; 00_1^+ \to 11_\nu^+\right)}{0.014325}}\,, \tag{6}$$

where one can exploit the experimental data on $B(\mathrm{M1}; 00_1^+ \to 11_\nu^+)$ [1].

Using the parameters defined above, the reduced probabilities of electromagnetic transitions and the coefficients of the $\delta(\mathrm{E2/M1})$ multipole mixture were calculated and are presented below.

Table II shows a comparison of the coefficients of the mixture of multipoles δ(E2/M1) calculated by formula (1) and experimental values [1, 4] for transitions from the $K^\pi = 0_2^+$, $K^\pi = 0_3^+$, $K^\pi = 2_1^+$ and $K^\pi = 1_\nu^+$ states to the baseband states. As it can be seen from Table II, our model satisfactorily reproduces both the numerical values and the signs of the coefficients of the δ(E2/M1) multipole mixture.

In the adiabatic approximation, there is no M1-transitions from the $K^\pi = 0^+$ and 2^+ band states. Within the framework of the present model, M1-transitions from the states of $K^\pi = 0^+$ and 2^+ bands arise due to the presence of the $K^\pi = 1_\nu^+$ components in the wave functions.

Table II presents the given matrix elements of E2- and M1-transitions. Note that the electrical properties of excited levels were discussed in detail in [8] and compared with available experimental data.

TABLE II

Multipole mixture coefficients δ(E2/M1) for ^{156}Gd. $\langle \mathrm{E2}\rangle_{\mathrm{if}}$ and $\langle \mathrm{M1}\rangle_{\mathrm{if}}$ are the reduced matrix elements of the E2- and M1-transitions, respectively, and E_{γ}- is the energy of the transition.

$I_\mathrm{i}K_\mathrm{i}$	$I_\mathrm{f}K_\mathrm{f}$	E_γ [MeV]	$\langle \mathrm{E2}\rangle_{\mathrm{if}}$ [eFm^2]	$\langle \mathrm{M1}\rangle_{\mathrm{if}}$ [μ_N]	δ_{exp} [1, 4]	δ_{theor}
22_1	20_1	1.0652	18.81	0.0412	16(5)	4.1
32_1	20_1	1.159	19.46	0.0313	11.8(+6, 7)	6.0
32_1	40_1	0.9598	16.23	0.0260	12(+13, 5)	5.0
42_1	40_1	1.0672	18.60	0.0639	+4.0(+9, 16)	2.6
52_1	40_1	1.2187	16.83	0.0488	$\delta > 7$	3.5
52_1	60_1	0.922	19.21	−0.0417	—	−3.5
62_1	60_1	1.060	17.00	−0.063	$\delta < 0.8$ or $\delta > 2.5$	−2.4
72_1	60_1	1.2648	15.06	−0.0634	—	−2.5
82_1	80_1	1.0457	15.84	−0.0584	$\delta < 0.6$ or $\delta > 1.6$	
92_1	80_1	1.2843	−13.73	0.0758	$\delta < -0.8, 0.39(6)$	−1.9
20_2	20_1	1.0405	10.31	−0.1011	+5.9(+14, 28)	0.9
40_2	40_1	1.0106	−12.75	0.2176	—	0.49
11_1	20_1	1.876	14.66	0.5503	+0.41(+25, 14) + 0.35(4)	0.41
11_2	20_1	1.938	14.61	0.3812	0.55(3)	0.63
11_3	20_1	2.0977	14.49	0.1888	1.2(2) or 1.08(+0.03, 0.22)	1.34
11_4	20_1	2.1807	14.44	0.3579	0.66(+0.06, 0.08)	0.73
20_3	20_1	1.1691	7.53	0.0539	0.38(6)	1.4
40_3	40_1	1.1741	8.65	0.0934	—	0.91

4. Conclusions

The effects of nonadiabaticity observed in the electromagnetic characteristics, in particular in the coefficients of the mixture of multipoles of δ(E2/M1) excited states, are explained by the Coriolis mixing of the states of the rotational bands. The reduced probabilities of E2- and M1-transitions are calculated. The calculated theoretical values of the reduced probabilities of E2- and M1-transitions from $K^\pi = 0_3^+$, $K^\pi = 2_1^+$, and $K^\pi = 1_\nu^+$ are in good agreement with the existing experimental measurements. The coefficients of the multipole mixture for the transitions from the states of the bands with $K^\pi = 0_2^+$, $K^\pi = 0_3^+$, $K^\pi = 2_1^+$ and $K^\pi = 1_\nu^+$ are calculated and compared with the experimental data. The capabilities of K forbidden transitions are described. The dependence of the coefficient δ(E2/M1) on the total angular momentum is discussed.

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