NON-LOCALITY IN THEORIES OF GRAVITY*

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We consider curvature-based gravity depending on non-local terms as $\Box^{-1}R$, with $\Box^{-1}$ being the inverse of the d'Alembert operator and R the Ricci curvature scalar. Specifically, we select the functional form of the effective Lagrangians by the Noether symmetries and then find out exact solutions in cosmological and spherically symmetric backgrounds. A comparison with experimental data is provided.

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1. Introduction

Gravitational interaction is described by Einstein's General Relativity (GR), where dynamics is given by the space-time curvature. Throughout the years, it obtained several experimental confirmations, providing evidence of its validity at different scales of energy. For instance, GR satisfactory traces a self-consistent cosmic history, ranging from the early stages of the Universe up to the current epoch. Recently, the direct detection of gravitational waves and black holes provided further probes of its validity. Nevertheless, it is a matter of fact, both from the theoretical and experimental side, that GR manifests some shortcomings. Concerning the large-scale structures, some problems occur in fitting experimental observations, such as the galaxy rotation curves (which led to the introduction of dark matter) or the Universe accelerating expansion (which led to the introduction of dark energy). The latter can be taken into account by introducing the cosmological constant Λ. However, the experimental value of Λ, coming from the cosmological Friedmann equations, hugely differs from that evaluated by the Quantum Field Theory in curved space-time. These problems, in principle, can be ascribed to the lack of a unified theory comprehending both Quantum Mechanics and GR. However, these are not the only shortcomings. For instance, GR can

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be renormalized up to the second-loop level, meaning that ultraviolet divergences occur in the renormalization procedure [1]. On the other hand, any attempt to describe gravity under the same standard of other interactions has failed so far, meaning that a self-consistent theory of Quantum Gravity is still missing. The need of addressing the above-mentioned (and several other) issues, led to the introduction of different gravity theories, alternatives, or extensions of GR [2–4]. Most of them extend the Einstein–Hilbert action considering some functions of the curvature scalar or other geometric invariants, with the consequence that the resulting theory is often capable of addressing the accelerated expansion of the Universe or the dark matter issues. Within the theories of gravity which extend or modify the gravitational action, particular interest has been recently gained by the so-called Non-local Gravity. This is mainly due to the effective capability of matching Quantum Mechanics, which is intrinsically a non-local theory. In fact, several one-loop effective Lagrangians of non-gravitational fundamental interactions exhibit dynamical non-locality. This is the case, for instance, of the low-energy limit of the Yukawa theory with a massive scalar field, where the non-locality in the effective Lagrangian is due to the term $(\Box + m^2)^{-1}$, where m is the scalar field mass. Non-local theories of gravity can be divided into two main categories: Integral Kernel Theories of Gravity (IKGs) and Infinite Derivative Theories of Gravity (IDGs). Both introduce non-local terms in the gravitational action, such as the d'Alembert operator $\Box$ or its inverse $\Box^{-1}$. An example of the former is given by the Deser–Woodard model, proposed in Ref. [5], while the latter was considered for instance in Refs. [6–8]. It turns out that, under given limits, these models are both unitary and renormalizable. Here, we consider applications to cosmology of IKGs containing functions the non-local term $\Box^{-1}R$. Specifically, the paper is organized as follows: in Sec. 2 we provide exact cosmological solutions for curvature-based models, meaning that the corresponding action depends on the function $F(R, \Box^{-1}R)$. In Sec. 3, we find out spherically symmetric solutions in the weak-field limit and compare the results with data provided by S2 star orbit around Sagittarius A*. Finally, in Sec. 4, we discuss possible future perspectives.

2. Cosmology in $F(R, \Box^{-1}R)$ gravity

Let us begin by considering a class of extended non-Local models described by the action

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\, F\left(R, \Box^{-1}R\right)\,, \tag{1}$$

which extends the standard $f(R)$ gravity by introducing a non-local term $\Box^{-1}R$. In order to select theories with symmetries, we need to *localize* the model by introducing the auxiliary scalar field ϕ, defined as $\phi \equiv \Box^{-1}R$, so that $R \equiv \Box\phi$. In this way, it is possible to use the Lagrange Multipliers Method and find out a point-like Lagrangian suitable for cosmology. In a Friedmann–Lemaître–Robertson–Walker space-time and integrating out second derivatives, the cosmological Lagrangian turns out to be

$$\mathcal{L} = a^3F - a^3\dot{\phi}\dot{\epsilon} - a^3RF_R + 6a\dot{a}^2F_R - 6a\dot{a}^2\epsilon + 6a^2\dot{a}\dot{R}\,F_{RR} + 6a^2\dot{a}\dot{\phi}F_{R\phi} - 6a^2\dot{a}\dot{\epsilon}\,, \tag{2}$$

where $a(t)$ is the scale factor F_X the derivative of F with respect to the given field X, and $\epsilon(t)$ the Lagrange multiplier promoted to a time-dependent field. To select the functional form of the latter, we apply the Noether Symmetry Approach [9, 10]. By imposing the existence of Noether symmetry

$$X^{[1]}\mathcal{L} + \mathcal{L}\,\dot{\xi} = \dot{g}\,, \tag{3}$$

where ξ and g are functions of cosmic time, the first prolongation of the Noether vector reads

$$\begin{aligned} X^{[1]} &= \alpha\frac{\partial}{\partial a} + \beta\frac{\partial}{\partial R} + \gamma\frac{\partial}{\partial \phi} + \delta\frac{\partial}{\partial \epsilon} \\ &\quad + \left(\dot{\alpha} - \dot{\xi}\dot{a}\right)\frac{\partial}{\partial \dot{a}} + \left(\dot{\beta} - \dot{\xi}\dot{R}\right)\frac{\partial}{\partial \dot{R}} + \left(\dot{\gamma} - \dot{\xi}\dot{\phi}\right)\frac{\partial}{\partial \dot{\phi}} + \left(\dot{\delta} - \dot{\xi}\dot{\epsilon}\right)\frac{\partial}{\partial \dot{\epsilon}}\,. \end{aligned} \tag{4}$$

The computation results in a system of 28 partial differential equations, which can be reduced to six, after neglecting linear combinations (see [11, 12] for details). It turns out that a possible function containing symmetries is [11]

$$F(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)}R + [2\xi_0R]^n\,\mathcal{F}\left(\phi + \frac{(1-n)}{\ell}\log\left[2\xi_0R\right]\right). \tag{5}$$

Here, $\mathcal{F}$ is a general function of $\left(\phi + \frac{(1-n)}{\ell}\log[2\xi_0R]\right)$ and ξ_0, ℓ, δ_1, n integration constants. Assuming the function to be linearly dependent on its argument and replacing this model into the point-like Lagrangian (2), the Euler–Lagrange equations can be solved analytically, providing

$$\begin{aligned} &a(t) = a_0\mathrm{e}^{\Lambda t}\,, \qquad R(t) = -12\Lambda^2\,, \qquad \phi(t) = -\frac{1}{3}(40+3q) - 4\Lambda t\,, \\ &\epsilon(t) = 9\,(8\xi_0)^3\,\Lambda^5 t - \frac{C_3\mathrm{e}^{-3\Lambda t}}{3\Lambda} + \frac{\delta_1}{4\xi_0}\,, \qquad \Lambda = \sqrt{-\frac{1}{24\xi_0 e}} \quad (\xi_0 < 0)\,, \end{aligned} \tag{6}$$

with q, a_0, C_3 integration constants. This solution, describing a general de Sitter expansion, only holds when $n = 3$. Another solution of the Euler–Lagrange equations yields a power-law scale factor and holds only when

$R = 0$. This means that the corresponding model reduces to GR minimally coupled to a scalar field. It reads

$$a(t) = a_0 t^{\frac{1}{2}}\,, \qquad R(t) = 0\,, \quad \phi(t) = C_2\,, \qquad \epsilon(t) = \frac{\delta_1}{2\xi_0(n-1)} - \frac{2C_3}{\sqrt{t}}\,. \quad (7)$$

Finally, the last solution holds for general values of the constant n and reads

$$\begin{aligned} a(t) &= a_0 t^{-10}\,, \qquad \phi(t) \sim C_2 + \log(t)\,, \\ \epsilon(t) &= \frac{\delta_1}{2\xi_0(n-1)} + C_3 t^{31} + c_4 \left(2\xi_0\right)^3 t^{-4}\,. \end{aligned} \quad (8)$$

The above solutions show that the non-local models, selected by the Noether symmetry, can trigger both an inflationary phase and a late-time cosmic acceleration, depending on the energy regime considered.

3. $F(R, \Box^{-1}R)$ gravity in spherical symmetry and S2 star orbit

In the above section, we considered a curvature-based non-local gravity model in a cosmological background. Here, we start from the Deser–Woodard action and constrain its functional form in spherical symmetry, with a line element given by $\mathrm{d}s^2 = e^{\nu(r,t)}\mathrm{d}t^2 - \mathrm{e}^{\lambda(r,t)}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2$. Specifically, as before, we introduce an auxiliary scalar field $\phi \equiv \Box^{-1}R$ aimed at localizing the theory and use the Lagrange multipliers method. The Lagrange multiplier, as in the previous case, can be promoted to a radius-dependent field $\varepsilon(r,t)$, so that the action can be written as

$$S = \frac{1}{2\kappa}\int \sqrt{-g}\,\Big\{R[1+f(\phi)] + \varepsilon(r,t)(\Box\phi - R)\Big\}\mathrm{d}^4x\,, \quad (9)$$

where $\kappa = 1/(8\pi G_N)$, with G_N being the gravitational constant. After selecting symmetries and finding the corresponding point-like Lagrangian, we consider the weak-field limit and constrain the free parameters by observations on S2 star orbit around our galactic center SgrA*. In this framework, corrections to the Newtonian potential can be addressed to non-locality. Replacing the above interval into Eq. (9) and integrating by parts higher-order derivatives, it is possible to find the point-like Lagrangian and to apply the symmetry existence condition (see [13] for details). The Noether Symmetry Approach selects the two functions

$$f(\phi) = \delta_0\phi + f_1\,, \qquad f(\phi) = \delta_0 + f_1 \mathrm{e}^{\frac{\gamma_0}{\xi_1}\phi}\,, \quad (10)$$

where $\delta_0, f_1, \gamma_0, \xi_1$ are integration constants. To the purpose of finding analytic solutions, we consider a subclass of solutions in which the Birkhoff theorem holds. Let us, therefore, recast the line element as $\mathrm{d}s^2 = A(r)\mathrm{d}t^2 - B(r)\,\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2$. Considering thus the second function of (10) and setting $\delta_0 = f_1 = 1$ and $\gamma_0 = \xi_1$, in the post-Newtonian limit, the Euler–Lagrange equations can be solved analytically providing the expression of A and B, that is [13]

$$\begin{aligned} A(r) &= 1 - \frac{2G_N M\phi_c}{c^2 r} + \frac{G_N^2 M^2}{c^4 r^2}\left[\frac{14}{9}\phi_c^2 + \frac{18r_\varepsilon - 11r_\phi}{6r_\varepsilon r_\phi} r\right] \\ &\quad - \frac{G_N^3 M^3}{c^6 r^3}\left[\frac{50r_\varepsilon - 7r_\phi}{12r_\varepsilon r_\phi}\phi_c r + \frac{16\phi_c^3}{27} - \frac{r^2\left(2r_\varepsilon^2 - r_\phi^2\right)}{r_\varepsilon^2 r_\phi^2}\right], \\ B(r) &= 1 + \frac{2G_N M\phi_c}{3c^2 r} + \frac{G_N^2 M^2}{c^4 r^2}\left[\frac{2\phi_c^2}{9} + \left(\frac{3}{2r_\varepsilon} - \frac{1}{r_\phi}\right) r\right]. \end{aligned}$$

Here, r_ε and r_ϕ are free parameters with dimension of length, respectively related to the two scalar fields ε and ϕ. They occur due to the weak-field modifications provided by non-locality. The length scales r_ε and r_ϕ, thus, can be constrained by the two-body simulation of S2. In particular, considering the data in Ref. [14], the free parameters can be chosen to provide the lowest χ^2 [13]. As a consequence, it is possible to consider different regions of the parameter space r_ϕ–r_ε, providing a lower χ^2 than the Keplerian orbit, as shown in Fig. 1.

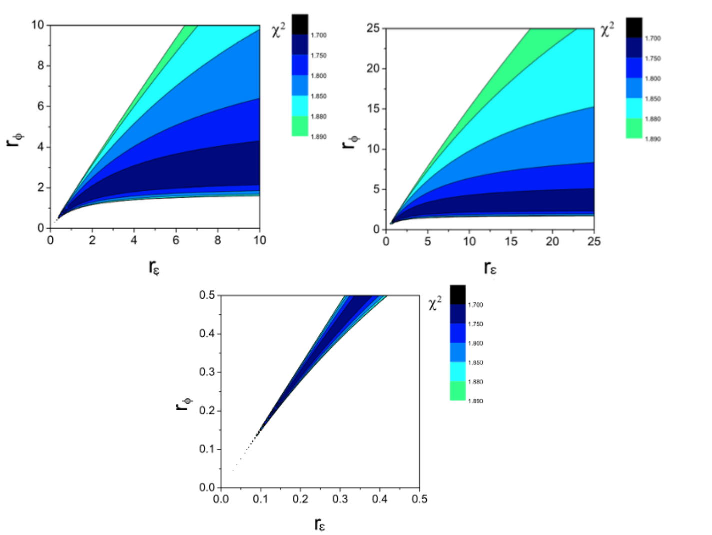

Fig. 1. Reduced χ^2 in different regions of the parameter space $r_\phi - r_\varepsilon$ (in AU). Regions with darker colors provide a better fit.

As reported in [13], r_ϕ is constrained such that the values providing the best fit are in the range of $r_\phi \sim 0.1$–2.5 AU, that is in the capabilities of modern observational tools. Moreover, considering $r_\phi \sim 1.2$ AU and $r_\varepsilon \sim 1.1$ AU, corresponding to $\chi^2 \sim 1.72$, it turns out that some regions are in agreement with observations better than the Keplerian case [13]. This is a preliminary indication that effects of gravitational non-locality could be detected also at galactic scales. See also [15].

4. Conclusions and perspectives

We considered a non-local curvature-based theory of gravity belonging to the class of IKGs. First, we focused on a cosmological spatially-flat background and applied the Noether Symmetry Approach to constrain the functional form of the starting action. We showed that selected models containing symmetries lead to exact cosmological solutions, capable of naturally addressing issues related to cosmic accelerated expansion. This means that gravitational non-locality can reproduce dark energy effects. In the second part of the paper, we considered a spherically symmetric background. Also here, the presence of Noether symmetries selects the model as a Schwarzschild-like solution. Moreover, we used observations of S2 star orbit around Sgr A* to constrain the free length parameters related to the non-local terms. It turns out that, for some values of the free parameters, non-local gravity can fit the S2 star orbit better than the Keplerian case. This result points out that non-local gravity effects could be detected at infrared scales and could represent a link with the ultraviolet behavior of gravity.

REFERENCES

[1] F. Bajardi, F. Bascone, S. Capozziello, «Renormalizability of Alternative Theories of Gravity: Differences between Power Counting and Entropy Argument», *Universe* **7**, 148 (2021).

[2] S. Capozziello, M. De Laurentis, «Extended Theories of Gravity», *Phys. Rep.* **509**, 167 (2011).

[3] F. Bajardi, D. Vernieri, S. Capozziello, «Exact solutions in higher-dimensional Lovelock and AdS$_5$ Chern–Simons gravity», *J. Cosmol. Astropart. Phys.* **11**, 057 (2021).

[4] S. Capozziello, F. Bajardi, «Gravitational waves in modified gravity», *Int. J. Mod. Phys. D* **28**, 1942002 (2019).

[5] S. Deser, R. Woodard, «Nonlocal Cosmology», *Phys. Rev. Lett.* **99**, 111301 (2007).

[6] L. Modesto, «Super-renormalizable quantum gravity», *Phys. Rev. D* **86**, 044005 (2012).

[7] T. Biswas, A. Mazumdar, W. Siegel, «Bouncing universes in string-inspired gravity», *J. Cosmol. Astropart. Phys.* **03**, 009 (2006).

[8] L. Buoninfante, G. Lambiase, A. Mazumdar, «Ghost-free infinite derivative quantum field theory», *Nucl. Phys. B* **944**, 114646 (2019).

[9] K.F. Dialektopoulos, S. Capozziello, «Noether Symmetries as a geometric criterion to select theories of gravity», *Int. J. Geom. Meth. Mod. Phys.* **15**, 1840007 (2018).

[10] Z. Urban, F. Bajardi, S. Capozziello, «The Noether–Bessel–Hagen symmetry approach for dynamical systems», *Int. J. Geom. Meth. Mod. Phys.* **17**, 2050215 (2020).

[11] A. Acunzo, F. Bajardi, S. Capozziello, «Non-local curvature gravity cosmology via Noether symmetries», *Phys. Lett. B* **826**, 136907 (2022), `arXiv:2111.07285 [gr-qc]`.

[12] S. Capozziello, F. Bajardi, «Non-local gravity cosmology: An overview», to appear in *Int. J. Mod. Phys. D*, DOI: 10.1142/S0218271822300099, `arXiv:2201.04512 [gr-qc]`.

[13] K.F. Dialektopoulos *et al.*, «Constraining nonlocal gravity by S2 star orbits», *Phys. Rev. D* **99**, 044053 (2019).

[14] S. Gillessen *et al.*, «Monitoring Stellar Orbits around the Massive Black Hole in the Galactic Center», *Astrophys. J.* **692**, 1075 (2009).

[15] S. Capozziello *et al.*, «Logarithmic corrections to Newtonian gravity and Large Scale Structure», *Eur. Phys. J. C* **81**, 352 (2021).