ELECTROWEAK EFFECTS IN ASYMMETRIES OF ELECTRON–POSITRON ANNIHILATION PROCESSES*

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Asymmetries in processes of e^+e^- annihilation into a pair of fermions are considered. Left–right and forward–backward asymmetries are calculated for the polarized initial and/or final particles. Effects due to 1-loop electroweak radiative corrections are scrutinized. Numerical results are presented for the Z-boson peak. Higher energies relevant for future colliders are also covered. Electroweak scheme dependence and interplay of QED and weak-interaction corrections are discussed.

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1. Introduction

This study is motivated by the current development of several electron–positron collider projects. The physics of fundamental interactions is now in a deep crisis. We see that the Standard Model (SM) of elementary particles works extremely well almost for all accessible for us observables. Nevertheless, we cannot believe the SM is the true fundamental theory, first of all, because of its unnatural complexity. Other reasons for that lie beyond the SM, they include, *e.g.*, the problem of gravity quantization and the nature of dark matter and dark energy phenomena. Thus, we assume that the SM is an effective theory, *i.e.*, a low-energy approximation of a more fundamental theory. All that motivates both theoretical and experimental searches for

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physics beyond the Standard Model. In spite of many efforts, new physics phenomena are not yet directly observed. The situation is unfair since we even do not have indications for the energy range of the SM applicability. In other words, we do not have any clear hint for the energy range of new physics (except the Planck mass scale which is too high for a direct verification in our experiments). All that makes the future of high-energy physics unclear. In particular, we do not know what should be the next high-energy collider to find new physics and make substantial progress in studying the microworld.

However, searches for new physics are certainly not the ultimate task for us. In any case, the SM will remain relevant as a tool to describe the bulk of physical phenomena around us. Besides the principal problem to define the energy domain of the SM applicability, we still have a lot of questions concerning its structure and features including the origin of three fermion generations, the hierarchy of fermion masses, confinement in QCD, quantization of the electric charge *etc.* The central point of the Standard Model construction is the electroweak symmetry breaking which is presently described by the Brout–Englert–Higgs mechanism at the semi-classical level. Exploring the phenomenon of electroweak symmetry breaking is crucial for understanding of the most fundamental properties of Nature. To our mind, this is the principal motivation to build a new high-energy collider where the Higgs and electroweak sectors of the SM will be studied with high precision. An electron–positron (or $\mu^+\mu^-$) collider seems to be the best option from this point of view.

There are several projects of future electron–positron colliders, including CLIC, ILC, FCC-ee, CEPC *etc.* All of them suppose to devote a special run to re-visit the Z-boson peak and then go to higher energies, where the properties of the Higgs boson and top quark can be studied. Hopefully, new physical phenomena will be discovered there as well. Nevertheless, we should be prepared for the scenario in which only the SM physics would be accessible at the new collider.

One of the most advanced tools in exploring the electroweak (EW) sector of the SM is the analysis of asymmetries in different processes at energies of the EW scale order. Asymmetries provide direct access to symmetry-breaking phenomena. They are typically defined as ratios of certain cross section, in which the bulk of experimental and theoretical uncertainties are cancelled out. That is why asymmetries are really suited to perform high-precision studies of the SM features. On the other hand, looking for deviations from the SM predictions in asymmetries can help to find the SM applicability limit and to find new physics. In general, asymmetries are sensitive to:

— parameters of the EW sector including the EW mixing angle ϑ_W;
— C and CP parity violation effects;
— lepton universality;
— many kinds of new physics.

In the past, asymmetries in processes of electron–positron annihilation were extensively studied at LEP and SLC devices. The latter had an advantage because of having polarized beams. Obviously, collisions of polarized particles are convenient to verify the V–A structure of weak interactions in the SM. Most projects of future e^+e^- colliders plan to have polarized beams (at least the electron one). Here, we report on the results of revisiting a set of asymmetries in processes of electron–positron annihilation into a pair of leptons. A few remarks on the quark pair case are also done. The report is based on the results presented in our article [1]. Our aim is to analyse the current status of theoretical uncertainties and to look for further steps in increasing the precision of theoretical predictions in order to match the requirements of future experiments. In particular, we keep in mind the plans of the FCC-ee project [2] to collect the statistics at the Z-boson peak about two orders of magnitude more than at LEP. THus, the experimental precision will be below 1 per mille level and the corresponding theoretical uncertainty should be even less in order not to spoil the resulting data analysis accuracy.

2. Preliminaries and notation

Let us introduce the basic notation. For longitudinally polarized beams with polarization degrees P_{e^-} and P_{e^+}, an annihilation cross section can be decomposed as follows:

$$\begin{aligned}\sigma(P_{e^-},P_{e^+}) \;=\; &\sigma_{\mathrm{RR}}(1+P_{e^-})(1+P_{e^+})+\sigma_{\mathrm{LR}}(1-P_{e^-})(1+P_{e^+})\\ &+\sigma_{\mathrm{RL}}(1+P_{e^-})(1-P_{e^+})+\sigma_{\mathrm{LL}}(1-P_{e^-})(1-P_{e^+})\,. \qquad (1)\end{aligned}$$

It is convenient to introduce the so-called effective polarization

$$P_{\mathrm{eff}} \equiv \frac{P_{e^-}-P_{e^+}}{1-P_{e^-}P_{e^+}}\,, \qquad (2)$$

which will systematically appear in expressions for asymmetries below.

There will appear also the typical combination of EW couplings

$$A_f \equiv 2\frac{g_{\mathrm{V}_f}g_{\mathrm{A}_f}}{g^2_{\mathrm{V}_f}+g^2_{\mathrm{A}_f}}\,, \qquad (3)$$

where g_{V_f} and g_{A_f} are the vector and axial-vector couplings of the given fermion $f=e,\mu,\tau$.

Since we perform calculations within the perturbation theory in a fixed order, our results depend on the EW scheme choice, *i.e.*, on the way to define the set of input parameters. For calculations at EW-scale energies, the following three schemes are often used:

1. The $\alpha(0)$ scheme takes the fine-structure constant $\alpha(0)$ as input. In this scheme, running of α from zero to high energies gives a large effect in radiative corrections.
2. The $\alpha(M_Z^2)$ scheme takes the effective electromagnetic constant $\alpha(M_Z^2)$ as input at the Born level, while virtual 1-loop and real photon bremsstrahlung contributions are proportional to $\alpha^2(M_Z^2)\alpha(0)$.
3. The G_μ scheme uses the Fermi coupling constant G_μ to define the EW coupling constants at the Born level, while the virtual 1-loop and real photon bremsstrahlung contributions are proportional to $G_\mu^2\alpha(0)$.

Variation of EW scheme choices simulates higher-loop effects which are not taken into account by the fixed order calculation and thus can be used for the estimates of theoretical uncertainties.

3. Examples of asymmetries

3.1. Forward–backward asymmetry A_{FB}

The forward–backward asymmetry is defined with respect to the angle ϑ_f between the produced fermion and the incoming electron beam

$$\begin{aligned} A_{\mathrm{FB}} &= \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}\,, \\ \sigma_{\mathrm{F}} &= \int\limits_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\vartheta_f}\mathrm{d}\cos\vartheta_f\,, \qquad \sigma_{\mathrm{B}} = \int\limits_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\vartheta_f}\mathrm{d}\cos\vartheta_f\,, \end{aligned} \tag{4}$$

For high-precision tests, the most convenient channels are $f = e, \mu$. The cases of $f = \tau,\ b,\ c$ are also very interesting. Remind that the forward–backward asymmetry of bottom quarks A_{FB}^b at LEP provided a lot of physical information relevant both for EW and QCD studies.

At the Born level, the asymmetry is proportional to the product of the initial and final fermion coupling combinations:

$$A_{\mathrm{FB}} \approx \frac{3}{4} A_e A_f\,. \tag{5}$$

The relation is approximate since it does not take into account radiative corrections and the dependence on fermion masses. Moreover, it holds only at the Z-boson peak as can be seen from Fig. 1. From Fig. 2, one can see that there is sensitivity with respect to the EW scheme choice.

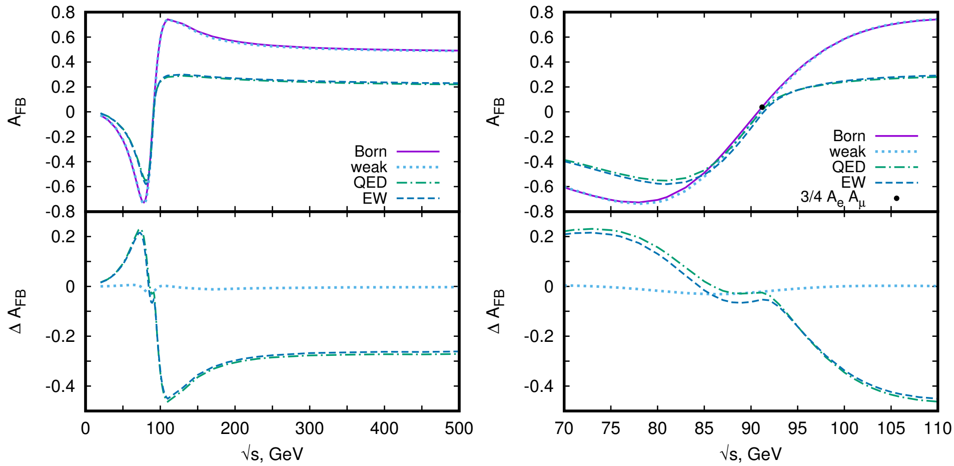

Fig. 1. The A_{FB} asymmetry at the Born and 1-loop (weak, QED, EW) levels and the corresponding shifts ΔA_{FB} for a wide c.m.s. energy range (left) and for the Z-peak region (right).

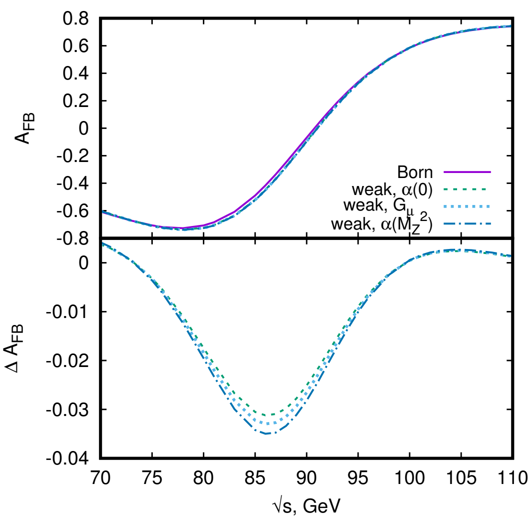

Fig. 2. The A_{FB} asymmetry and ΔA_{FB} in the Born and 1-loop EW approximations within the $\alpha(0)$, G_μ, and $\alpha(M_z^2)$ EW schemes *versus* the c.m.s. energy.

Our analysis shows the following features of the forward–backward asymmetry:

— Pure weak contributions are rather small at all energies.

— They are, however, numerically relevant at the Z-boson peak region due to the high statistics to be collected there. EW scheme dependence is also small but visible.

— $A_{\rm FB}$ is strongly dependent on beam polarization degrees. Thus, it is sensitive to the uncertainties in the beam polarimetry.

— Pure QED corrections to $A_{\rm FB}$ in higher orders are known with high precision, see *e.g.* [3] and [4].

— There is an interesting idea [5] to use the $A_{\rm FB}$ asymmetry for extraction of $\alpha(M_Z)$.

— One-loop corrections to $A_{\rm FB}$ contain contributions proportional to the first power of m_f, which are relevant, *e.g.*, for the channel with b-quark production.

3.2. Left–right asymmetry $A_{\rm LR}$

An annihilation cross section with polarized initial particles (for $m_e \to 0$) can be represented as

$$\sigma(P_{e^-}, P_{e^+}) = (1 - P_{e^-}P_{e^+})[1 - P_{\rm eff}A_{\rm LR}]\sigma_0\,. \tag{6}$$

In the general case with partial polarizations,

$$A_{\rm LR} = \frac{1}{P_{\rm eff}}\frac{\sigma(-P_{\rm eff}) - \sigma(P_{\rm eff})}{\sigma(-P_{\rm eff}) + \sigma(P_{\rm eff})}\,. \tag{7}$$

For fully polarized initial particles ($|P_{e^\pm}| = 1$),

$$A_{\rm LR} = \frac{\sigma_{{\rm L}_e} - \sigma_{{\rm R}_e}}{\sigma_{{\rm L}_e} + \sigma_{{\rm R}_e}}\,. \tag{8}$$

At the Born level, $A_{\rm LR} \approx A_e$. Thus, this asymmetry pretends to provide the direct access to the electron EW couplings and the combination of $A_{\rm LR}$ and $A_{\rm FB}$ will allow to get the couplings for the initial and final particles separately.

Figure 3, as well as all other numerical results in this work, were produced with the help of the e^+e^- branch [6] of the MCSANC Monte Carlo integrator [7]. For the $A_{\rm LR}$ asymmetry, we see the following features:

— A_{LR} is almost insensitive to the details of particle detection since they tend to cancel out in the ratio.

— A_{LR} (almost) does not depend on the final-state fermion couplings in the vicinity of the Z-boson peak and can be measured for any final state with large statistics.

— Therefore, A_{LR} is suited for extraction of $\sin^2 \vartheta_W^{\mathrm{eff}}$.

— At large energies, both QED and weak corrections ΔA_{LR} are large.

— At the Z peak, pure QED corrections are small but the weak ones are large.

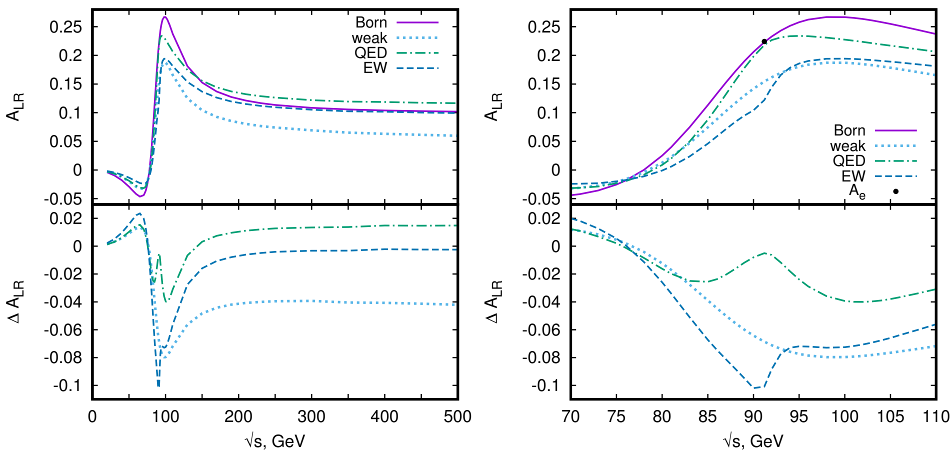

Fig. 3. The A_{LR} asymmetry in the Born and 1-loop (weak, pure quantum electrodynamics (QED), and electroweak (EW)) approximations and ΔA_{LR} *versus* c.m.s. energy in a wide range (left) and for the Z-peak region (right).

3.3. Left–right FB asymmetry A_{LRFB}

To measure the weak couplings of the final-state fermions, it was suggested to analyse the so-called left–right forward–backward asymmetry

$$A_{\mathrm{LRFB}} = \frac{(\sigma_{\mathrm{L}_e} - \sigma_{\mathrm{R}_e})_{\mathrm{F}} - (\sigma_{\mathrm{L}_e} - \sigma_{\mathrm{R}_e})_{\mathrm{B}}}{(\sigma_{\mathrm{L}_e} + \sigma_{\mathrm{R}_e})_{\mathrm{F}} + (\sigma_{\mathrm{L}_e} + \sigma_{\mathrm{R}_e})_{\mathrm{B}}}\,, \tag{9}$$

where σ_{L} and σ_{R} are the cross sections with left- and right-handed helicities of the initial electrons.

At the Z-resonance peak, the Born-level asymmetry is

$$A_{\mathrm{LRFB}} \approx \frac{3}{4} A_f\,. \tag{10}$$

Figure 4 illustrates the effects of radiative corrections in this asymmetry, and we have following notes here:

— The A_{LRFB} asymmetry is more affected by weak-interaction corrections than A_{LR}.

— Formula $A_{\mathrm{LRFB}} \approx \frac{3}{4} A_f$ is very rough and should not be used directly in the data analysis.

— Shifts ΔA_{LRFB} only slightly depend on the EW scheme choice.

— A_{LRFB} at the Z-boson peak can be used to measure weak couplings of μ and τ and to compare them with the electron one, *i.e.*, to check the lepton universality.

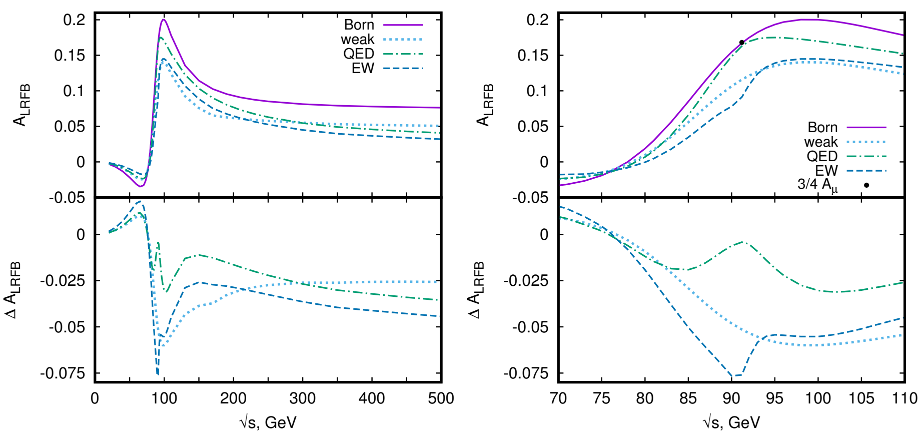

Fig. 4. The A_{LRFB} asymmetry in the Born and 1-loop (weak, QED, EW) approximations and ΔA_{LRFB} for c.m.s. energy range (left) and for the Z-peak region (right).

3.4. Final-state fermion polarization P_f

The polarization of a final-state fermion $P_{f=\mu,\tau}$ can be found as

$$P_f = \frac{\sigma_{\mathrm{R}_f} - \sigma_{\mathrm{L}_f}}{\sigma_{\mathrm{R}_f} + \sigma_{\mathrm{L}_f}} \,. \tag{11}$$

Experimentally, it can be measured for the $\tau^+\tau^-$ channel by reconstructing τ polarization from the pion spectrum in the $\tau \to \pi\nu$ decay. At LEP, TAUOLA [8] and KORALZ [9] programs were used for such an analysis.

For unpolarized beams near the Z peak

$$P_\tau(\cos\vartheta_\tau) \approx -\frac{A_\tau + \frac{2\cos\vartheta_\tau}{1+\cos^2\vartheta_\tau} A_e}{1 + \frac{2\cos\vartheta_\tau}{1+\cos^2\vartheta_\tau} A_e A_\tau} \,. \tag{12}$$

Thus, both A_τ and A_e can be extracted simultaneously.

Let us again look at the effects of EW radiative corrections. They can be seen in Fig. 5. We would like to underline the following features:

— The P_τ asymmetry is very sensitive to weak-interaction corrections. Therefore, they should be accurately computed and taken into account.

— The P_τ asymmetry is very sensitive also to the initial beam polarizations and thus to uncertainties in the beam polarimetry.

— Near the Z resonance, the resulting theoretical uncertainty of P_τ is determined by an interplay of uncertainties of large QED and weak-radiative correction contributions.

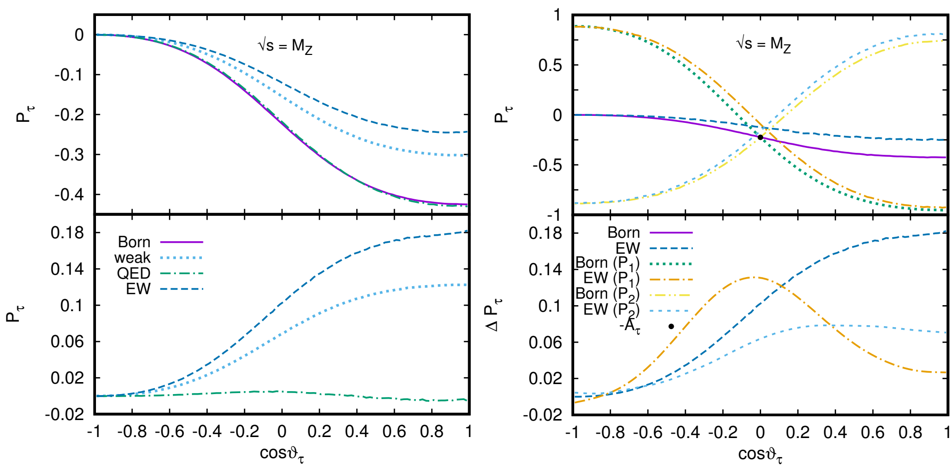

Fig. 5. P_τ polarization in the Born and 1-loop (weak, pure QED, and EW) approximations as a function of $\cos\vartheta_\tau$ at $\sqrt{s} = M_Z$ (left), P_τ polarization for unpolarized and polarized cases with $P_1 = (-0.8, 0.3)$ and $P_2 = (0.8, -0.3)$ degrees of initial beam polarizations in the Born and EW 1-loop approximations *versus* cosine of the final τ lepton angle (right).

3.5. Conclusions and outlook

The future e^+e^- colliders pose a challenge for high-precision theoretical calculations. We need to increase substantially the precision of annihilation processes description especially at the Z-boson peak. The new TeV energy scale also requires an update of calculations. Here, we presented results for A_{LR}, A_{FB}, and A_{LRFB} for the $e^+e^- \to \mu^+\mu^-$ channel and polarization P_τ for the final state in $e^+e^- \to \tau^+\tau^-$ channel around the Z-boson peak and in the high-energy region up to 500 GeV. We evaluated the resulting shifts of asymmetries due to the complete one-loop corrections in different EW schemes. In our results, we clearly see a non-trivial interplay between pure QED and weak-interaction effects. This fact means that those contributions should be treated always in a combined way.

Studies of asymmetries in different channels of e^+e^- annihilation processes will allow to verify the lepton universality hypothesis at a new precision level. The asymmetries are also very sensitive to the possible presence of hypothetical extra neutral Z'-vector bosons [10]. Since the new bosons can have couplings to left and right fermions different from the ones in the SM, the asymmetries can contribute to the search for new Z' bosons.

The experimental precision tag of FCC-ee in the measurement of $\sin^2\vartheta_W^{\rm eff}$ is about 5×10^{-6}, which is about thirty times better than the present precision 1.6×10^{-4}. Then, it is required to provide theoretical calculations for the considered asymmetries with a precision that would not spoil the accuracy of the future experiments. Our study clearly shows that the one-loop precision (within the complete SM) is not enough. To meet the requirements of the future experiments we need in addition:

- higher-order QED corrections with resummation of large logarithmic contributions;
- complete two-loop and enhanced higher-order electroweak corrections;
- perturbative and nonperturbative QCD effects in loop corrections;
- new efficient Monte Carlo event generators and integrators.

Challenges in calculations of the higher-order QED corrections for FCC-ee were analyzed in [3, 11]. Higher order next-to-leading large logarithmic QED corrections should be taken into account. The complete two-loop electroweak corrections in the Z-boson peak energy region were calculated in [12] for a set of (pseudo)observables in annihilation channels.

Our suggestion is to perform calculations of higher-order loop corrections not for an individual process but for form factors, *i.e.*, coefficients (functions of kinematical invariants) in front of different Lorentz structure in amplitudes. A library of such two-loop (or even multi-loop) form factors can be created and used further in construction of cross sections and other observables.

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