A MARGINALLY FAST-COOLING PROTON-SYNCHROTRON MODEL FOR PROMPT GRB EMISSION*

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A small fraction of Gamma Ray Bursts (GRBs) with available data down to soft X-rays (~ 0.5 keV) have been detected to feature a spectral break in the low-energy part of their prompt emission spectrum. The overall spectral shape is consistent with optically thin synchrotron emission from a population of marginally fast cooling particles. In this work, we consider that such radiating particles are hadrons and investigate the idea of the marginally fast-cooling proton-synchrotron emission. We compute the source parameters required for such a scenario to work and investigate numerically how additional processes, namely photohadronic interactions and $\gamma\gamma$ pair production, contribute to the overall spectrum. We also construct analytically and numerically the expected neutrino flux from this model.

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1. Introduction

Gamma-Ray Bursts (GRBs), which are brief flashes of γ-rays observed all over the Universe, are considered to be one of the most energetic transient explosive phenomena. The prompt phase of the GRB emission is associated with a high photon luminosity and a usually highly variable light curve, consisting of many pulses, each of them lasting from 1 msec to 1 sec. The total prompt emission duration can last from seconds to minutes reaching luminosities up to $L_\gamma = 10^{54}$ erg s^{-1} (for more information, see [1]).

On a large scale, GRBs are puzzling phenomena since there are a lot of missing links in our understanding of them, such as the radiative mechanism responsible for their production. The fact that the overall spectrum of the GRB prompt emission might be non-thermal had led to the suggestion that

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the radiation is dominated by synchrotron emission from a power-law distribution of relativistic electrons. However, a major criticism of this model is the predicted low-energy spectral slope, which is usually flatter than the respective synchrotron one. This is known as the synchrotron *line of death* problem [2], making synchrotron radiation still a debated process.

Some recent observations with available data down to optical and soft X-rays (see [3]) might be able to determine in a more robust way the low-energy photon index of the GRB spectrum. Instead of the classic Band function [4], *i.e.* a smoothly broken power law, which is used to fit the GRB spectra, [3] have proposed another phenomenological function that consists of three power laws joined at two energies, $E_{\rm cool,obs}$ and $E_{\rm pk,obs}$. This distribution function could be also physically interpreted as the synchrotron radiation of electrons that do not cool completely, but their cooling stops at an electron Lorentz factor $\gamma_{\rm cool}$, which corresponds to the synchrotron energy $E_{\rm cool,obs}$ and is comparable to the minimum electron Lorentz factor, $\gamma_{\rm min}$. Generally, $\gamma_{\rm min}/\gamma_{\rm cool} = \mathcal{O}(10)$, where $\gamma_{\rm min} \gg 1$, and this is the marginally fast-cooling regime (see [5]). [6] examined the idea that synchrotron radiation from marginally fast-cooling particles is able to reproduce GRB prompt emission. They found out that the synchrotron radiation from marginally fast-cooling electrons is able to reproduce the GRB prompt emission, if the radius of the emitting region is $R_\gamma \geq 10^{16}$ cm and the comoving magnetic field strength is $B \leq 1$ G, which, however, leads to the conclusion that the minimum variability timescale should be much larger than the one observed.

In what follows, we work on the marginally fast-cooling proton-synchrotron scenario for the interpretation of GRB prompt emission and compute analytically and numerically the photon and neutrino spectra expected from such a model.

2. Model parameters

We assume that relativistic protons are injected inside a spherical region that moves with a bulk Lorentz factor $\mathit{\Gamma}$ at a distance R_γ from the central engine of the GRB. This spherical region can be thought of as the shell of the shocked ejecta in the internal shock GRB model. It has got a comoving width $r_{\rm b} = R_\gamma/\mathit{\Gamma}$ and contains a tangled magnetic field of comoving strength B. The injected protons have a power-law distribution of spectral index p, starting from a minimum Lorentz factor $\gamma_{\rm min}$ up to a maximum value $\gamma_{\rm max}$. Upon entering the source, they interact with the magnetic field and any soft photons present, creating radiation and secondary particles through proton synchrotron radiation and photohadronic interactions.

In order to compute the source parameters, we use the proton-synchrotron relations. We express the three observable parameters given in [3] ($E_{\rm pk,obs}$, $E_{\rm cool,obs}$, and $F_{\rm cool}$[1]) as a function of the bulk Lorentz factor as well as B, $\gamma_{\rm min}$, $r_{\rm b}$, and F_γ

$$E_{\rm pk,obs} = \frac{m_e}{m_p}\frac{3qhB\gamma_{\rm min}^2}{4\pi m_e c}\frac{\Gamma}{1+z}\,, \tag{1}$$

$$E_{\rm cool,obs} = \left(\frac{m_p}{m_e}\right)^5 \frac{27\pi q h m_e c(1+z)}{\sigma_{\rm T}^2 B^3 t_{\gamma,\rm obs}^2 \Gamma}\,, \tag{2}$$

$$F_{\rm cool} = F_\gamma \left(\frac{E_{\rm cool,obs}}{h}\right)^{-1}\left(\frac{3}{4}+2\sqrt{\frac{E_{\rm pk,obs}}{E_{\rm cool,obs}}}-2+\frac{2}{p-2}\sqrt{\frac{E_{\rm pk,obs}}{E_{\rm cool,obs}}}\right)^{-1}\,, \tag{3}$$

$$t_{\gamma,\rm obs} = \frac{r_{\rm b}(1+z)}{c\Gamma}\,, \tag{4}$$

where z is the redshift of the source and F_γ the bolometric photon luminosity. We assume that the variability timescale is fixed at $t_{\gamma,\rm obs} = 1$ sec. In order to compute the comoving proton luminosity, we assume that the observed luminosity is approximately equal to the injection luminosity of particles. Using the observational parameters for all the GRBs of the sample given in [3], we compute the source parameters for each of them. In Fig. 1, we show the resulting values of B and $\gamma_{\rm min}$, *i.e.* $\langle B\rangle \approx 10^7$ G and $\langle\gamma_{\rm min}\rangle \approx 10^4$ respectively. These values correspond to all the GRBs of our sample and $\Gamma = 300$.

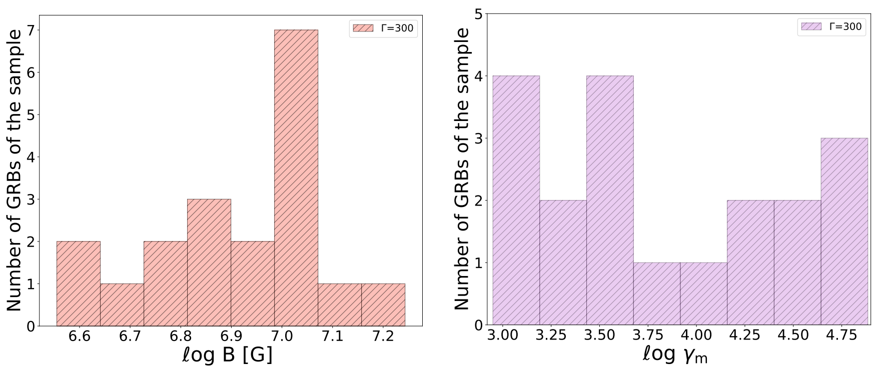

Fig. 1. Logarithmic histograms of the inferred B and $\gamma_{\rm min}$ values for the GRB sample and $\Gamma = 300$ [3]. Results for another choice of Γ can be obtained using the scaling relations $B \propto \Gamma^{-1/3}$, $\gamma_{\rm min} \propto \Gamma^{-1/3}$.

[1] $F_{\rm cool}$ is the flux at the cooling break frequency $\nu_{\rm cool,obs} = h^{-1}E_{\rm cool,obs}$.

3. Photohadronic interactions and neutrino spectra

Relativistic protons, with energy ε_p can become targets to their own radiation, interacting with the synchrotron photons, via photohadronic interactions. In what follows, we show some analytical calculations based on the above assumption and derive an expression for the all-flavour neutrino flux. We assume that the differential proton number density at steady-state follows the expression given by [7] and can be approximated by the following expression:

$$n_p(\gamma) \approx \frac{Q_0\ t_{p,\mathrm{esc}}}{p-1}\ \gamma_{\mathrm{cool}}\, \mathrm{e}^{-\frac{\gamma_{\mathrm{cool}}}{\gamma}} \begin{cases} \left(\frac{\gamma}{\gamma_{\mathrm{min}}}\right)^{-2} \ \gamma_{\mathrm{min}}^{-p+1}, & \gamma \leq \gamma_{\mathrm{min}}\,, \\ \gamma^{-p-1}, & \gamma > \gamma_{\mathrm{min}}\,, \end{cases} \tag{5}$$

where $t_{p,\mathrm{esc}}$ is the proton escape timescale. From now on, we will assume that $p = 2.6$. The relativistic protons cool via synchrotron until $\gamma = \gamma_{\mathrm{cool}} \approx \gamma_{\mathrm{min}}$, and the photon flux per unit energy produced via proton-synchrotron radiation in the observer's frame is

$$F_{\varepsilon_{\mathrm{obs}}} = \begin{cases} h^{-1} F_{\mathrm{cool}} \left(\frac{\varepsilon_{\mathrm{obs}}}{E_{\mathrm{cool,obs}}}\right)^{1/3}, & \varepsilon_{\mathrm{obs}} < E_{\mathrm{cool,obs}}\,, \\ h^{-1} F_{\mathrm{cool}} \left(\frac{\varepsilon_{\mathrm{obs}}}{E_{\mathrm{cool,obs}}}\right)^{-1/2}, & E_{\mathrm{cool,obs}} < \varepsilon_{\mathrm{obs}} < E_{\mathrm{pk,obs}}\,, \\ h^{-1} F_{\mathrm{cool}} \left(\frac{\varepsilon_{\mathrm{obs}}}{E_{\mathrm{pk,obs}}}\right)^{-p/2} \left(\frac{E_{\mathrm{pk,obs}}}{E_{\mathrm{cool,obs}}}\right)^{-1/2}, & \varepsilon_{\mathrm{obs}} > E_{\mathrm{pk,obs}}\,. \end{cases} \tag{6}$$

The same relativistic protons interact with the isotropic synchrotron photon field via the photopion interaction resulting in the production of γ-rays, secondaries, and high-energy neutrinos. The all-flavour neutrino flux can be approximately computed as

$$\varepsilon_{\nu,\mathrm{obs}} F_{\varepsilon_{\nu,\mathrm{obs}}} \approx \frac{3}{8} \mathit{\Gamma}^4 f_{p\pi}(\varepsilon_p) \frac{r_{\mathrm{b}}^2 c \varepsilon_p U_p(\varepsilon_p)}{d_{\mathrm{L}}^2}\,. \tag{7}$$

Here, d_{L} is the luminosity distance, $U_p(\varepsilon_p) = m_p c^2 n_p(\varepsilon_p)$ is the differential proton energy and $\varepsilon_{\nu,\mathrm{obs}} \approx \eta_{p\pi} \mathit{\Gamma} \varepsilon_p (1+z)$ is the neutrino energy produced, with $\eta_{p\pi} = 1/20$ the fraction of proton energy that is transferred to each neutrino. Moreover, $f_{p\pi}(\gamma)$ is the fractional energy loss rate of a proton due to photopion interactions, which is computed analytically by the following relation:

$$f_{p\pi}(\gamma) = \begin{cases} \frac{8}{p(p+4)}\, \alpha_1 \left(\frac{\gamma}{\epsilon_{\mathrm{th}}}\right)^{p/2}, & \gamma < \frac{\epsilon_{\mathrm{th}}}{x_{\mathrm{pk}}}\,, \\ \frac{8}{5} c_1\, \alpha_2 \left(\frac{\gamma}{\epsilon_{\mathrm{th}}}\right)^{1/2}, & \frac{\epsilon_{\mathrm{th}}}{x_{\mathrm{pk}}} \leq \gamma < \frac{\epsilon_{\mathrm{th}}}{x_{\mathrm{cool}}}\,, \\ \frac{9}{10} c_2\, \alpha_3 \left(\frac{\gamma}{\epsilon_{\mathrm{th}}}\right)^{-1/3} \left((\gamma_{\mathrm{max}}\ x_{\mathrm{cool}})^{1/3} \epsilon_{\mathrm{th}}^{-1/3} - 1 \right), & \gamma \geq \frac{\epsilon_{\mathrm{th}}}{x_{\mathrm{cool}}}\,, \end{cases} \tag{8}$$

where $\hat{\sigma}_{p\pi} \equiv \sigma_{p\pi}\xi_{p\pi}$, $\sigma_{p\pi} \simeq 0.34$ mb is the cross section for pion production for a photon with energy ε in the proton rest frame (in $m_e c^2$ units), $\xi_{p\pi} \simeq 0.2$ is the average fraction of energy lost by the proton per interaction, $x_{\rm pk} = E_{\rm pk,obs}(1+z)/\Gamma m_e c^2$, $x_{\rm cool} = E_{\rm cool,obs}(1+z)/\Gamma m_e c^2$. Moreover, α_1, α_2, α_3 are normalisation parameters related to the photon number density namely:

$$\alpha_1 = \frac{3d_{\rm L}^2 F_{\rm cool}\hat{\sigma}_{p\pi}}{h r_{\rm b} c}\left(\frac{1+z}{m_e c}\right)^{p/2}\left(\frac{\gamma_{\rm min}}{\gamma_{\rm cool}}\right)^{-1} E_{\rm pk,obs}^{p/2}\,\Gamma^{-3-p/2}\,, \tag{9}$$

$$\alpha_2 = \frac{3d_{\rm L}^2 F_{\rm cool}\hat{\sigma}_{p\pi}}{h r_{\rm b} c}\left(\frac{1+z}{m_e c}\right)^{p/2} E_{\rm cool,obs}^{1/2}\,\Gamma^{-7/2}\,, \tag{10}$$

and

$$\alpha_3 = \frac{3d_{\rm L}^2 F_{\rm cool}\hat{\sigma}_{p\pi}}{h r_{\rm b} c}\left(\frac{1+z}{m_e c}\right)^{-1/3} E_{\rm cool,obs}^{-1/3}\,\Gamma^{-8/3}\,, \tag{11}$$

while c_1 and c_2 are defined as

$$c_1 = \frac{\alpha_1}{\alpha_2}\,\frac{5}{p(p+4)}\,x_{\rm pk}^{-p/2+1/2}$$

and

$$c_2 = \frac{\alpha_2}{\alpha_3}\,\frac{16}{9}\,c_1\,x_{\rm cool}^{-5/6}\left((\gamma_{\rm max}\,x_{\rm cool})^{1/3}\epsilon_{\rm th}^{-1/3} - 1\right)^{-1}.$$

After taking all the above into account, the relation of the neutrino flux (7) reads

$$\varepsilon_{\nu,\rm obs}F_{\varepsilon_{\nu,\rm obs}} \approx F_0\,{\rm e}^{-\frac{\varepsilon_{\nu,\rm c,obs}}{\varepsilon_{\nu,\rm obs}}}\begin{cases} G_1\varepsilon_{\nu,\rm obs}^{p/2}\,, & \varepsilon_{\nu,\rm obs} < \varepsilon_{\rm obs,1}\,,\\ G_2\varepsilon_{\nu,\rm obs}^{-p/2+1}\,, & \varepsilon_{\rm obs,1} \le \varepsilon_{\nu,\rm obs} < \varepsilon_{\rm obs,2}\,,\\ G_3\varepsilon_{\nu,\rm obs}^{-p+3/2}\,, & \varepsilon_{\rm obs,2} \le \varepsilon_{\nu,\rm obs} < \varepsilon_{\rm obs,3}\,,\\ G_4\varepsilon_{\nu,\rm obs}^{-p+2/3}\,, & \varepsilon_{\nu,\rm obs} \ge \varepsilon_{\rm obs,3}\,,\end{cases} \tag{12}$$

where $F_0 = (3/8)\Gamma^4\,r_{\rm b}^2 c\,m_p c^2 d_{\rm L}^{-2}$. The break neutrino energies are

$$\varepsilon_{\nu,\rm c,obs} = \eta_{p\pi}\Gamma m_p c^2\gamma_{\rm cool}\,, \tag{13}$$

$$\varepsilon_{\rm obs,1} = \eta_{p\pi}\Gamma m_p c^2\gamma_{\rm min}\,, \tag{14}$$

$$\varepsilon_{\rm obs,2} = \eta_{p\pi}\Gamma m_p c^2(\epsilon_{\rm th}/x_{\rm pk})\,, \tag{15}$$

$$\varepsilon_{\rm obs,3} = \eta_{p\pi}\Gamma m_p c^2(\epsilon_{\rm th}/x_{\rm cool})\,, \tag{16}$$

while G_1, G_2, G_3, G_4 are normalised relations

$$G_1 = \frac{8A\,\alpha_1}{p(p+4)(p-1)}\gamma_{\rm min}^{-p+3}\,\epsilon_{\rm th}^{-p/2}\left(\eta_{p\pi}\,\Gamma m_p c^2\right)^{-p/2}\,, \tag{17}$$

$$G_2 = \frac{8A\ \alpha_1}{p(p+4)(p-1)}\ \epsilon_{\rm th}^{-p/2}\left(\eta_{p\pi}\ \Gamma m_p c^2\right)^{p/2-1}, \tag{18}$$

$$G_3 = \frac{8A\ c_1\ \alpha_2}{5}\ \epsilon_{\rm th}^{-1/2}\left(\eta_{p\pi}\ \Gamma m_p c^2\right)^{p-3/2}, \tag{19}$$

$$G_4 = \frac{9A\ c_2\ \alpha_3}{10}\ \epsilon_{\rm th}^{1/3}\left(\eta_{p\pi}\ \Gamma m_p c^2\right)^{p-2/3}. \tag{20}$$

In the above relations, $A = Q_0 \gamma_{\rm cool} t_{p,\rm esc} r_{\rm b}\ \hat{\sigma}_{p\pi}$.

We apply Eq. (7) to each GRB of the sample using the parameters that correspond to $\Gamma = 300$. The resulting all-flavour neutrino fluxes are shown in the left panel of Fig. 2. The charged byproducts of photopion interaction do not always decay instantaneously, but if their synchrotron cooling timescale is shorter than their decay timescale, they cool via synchrotron radiation. They can, therefore, contribute to the overall GRB photon spectrum while suppressing the neutrino flux above a certain energy. In this case, the neutrino spectrum steepens by a factor of 2 (for more information, see [8]). This correction is not shown in Eq. (12) but it is taken into account in Fig. 2 (see coloured diamonds).

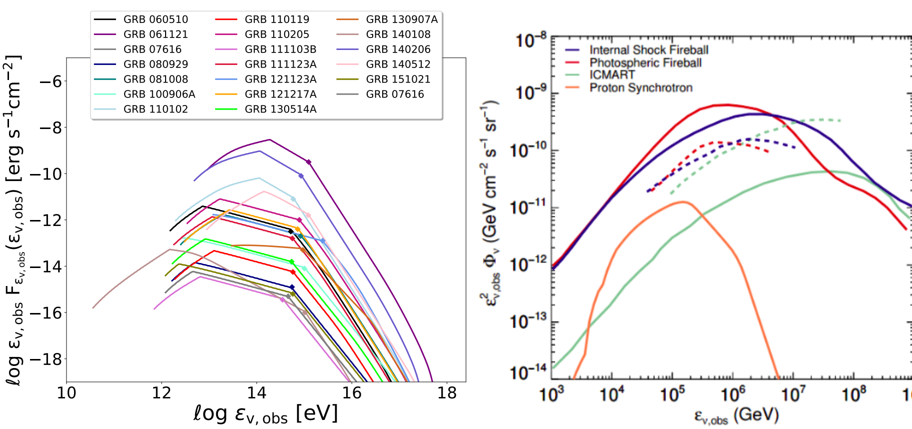

Fig. 2. (Color online) Left panel: Analytical results for the all-flavour neutrino energy flux spectra in the proton-synchrotron model for the GRB sample of [3], assuming $\Gamma = 300$. Right panel: The all-sky quasi-diffuse neutrino flux (orange solid line), computed from the neutrino spectra of the marginally fast-cooling proton-synchrotron model (left panel), assuming that the GRB sample used is representative of the entire GRB population. The rest coloured lines are neutrino spectra computed by other models.

In the right panel of Fig. 2, we compute the all-sky quasi-diffuse neutrino flux by using the neutrino spectra computed by our model and assuming that the GRB sample used is representative of the entire GRB population. Given a rate of long GRBs $\dot{N} = 667\ \mathrm{yr}^{-1}$ [9], the stacking flux for muon neutrinos over the whole sky can be written as

$$\Phi_{\nu_\mu+\bar{\nu}_\mu} \simeq \left(\dot{N}/4\pi\right)\left(\mathcal{F}_{\nu+\bar{\nu}}/3\right) \simeq 10^{-11}\ \mathrm{GeV\ cm^{-2}s^{-1}sr^{-1}}\,, \tag{21}$$

where we assumed vacuum flavour mixing. For comparison reasons, we overplot the quasi-diffuse neutrino fluxes computed by some other standard neutrino models *i.e.* Internal Shock Fireball model (blue line) [10], Photospheric-Fireball (red line) [11], ICMART (green line) [12]. It is evident that the proton-synchrotron model provides neutrino spectra that are one order of magnitude lower than the respective ones produced by the other neutrino models and peak at a neutrino energy approximately two orders of magnitude below the corresponding peak energy of the rest models (~ 1 PeV). Our results are compatible with [13] who also examined the neutrino production from the proton-synchrotron marginally fast-cooling scenario.

4. Numerical investigation

We next verify our analytical results by utilising the numerical code ATHEνA [14] using the parameter values obtained in Section 2 [2]. This code follows the evolution of spatially averaged particle populations (photons, protons, electrons, neutrons, neutrinos) inside a homogeneous spherical emitting region. The numerical approach to the problem gives us also the opportunity to extend the work of [6] by investigating the contribution of photohadronic interactions and $\gamma\gamma$ pair production to the overall photon spectrum and by computing the associated neutrino flux. In the radiative processes, we take into account also the synchrotron cooling of kaons, pions, and muons, and check how they affect the photon spectrum. The numerical results are shown in Fig. 3.

It seems that the low-energy part of the photon spectrum is modified in the case the photohadronic interactions and the $\gamma\gamma$ absorption are taken into account. While the proton-synchrotron spectrum (see blue dotted lines in Fig. 3) fits perfectly the observations, the other physical processes that cannot be neglected reshape the photon spectrum in a way that it becomes incompatible with the soft X-ray and optical observations (see blue solid lines in Fig. 3). These results strongly relate to the bulk Lorentz factor and they alleviate when Γ becomes very large ($\Gamma \geq 1000$). In both cases, the neutrino energy spectrum (dashed lines) peaks at PeV energies, as we showed also in the analytical approach in the previous section. Moreover, the neutrino peak flux is many orders of magnitude lower than the peak γ-ray flux.

[2] We use the observational parameters of GRB 061121 from [3].

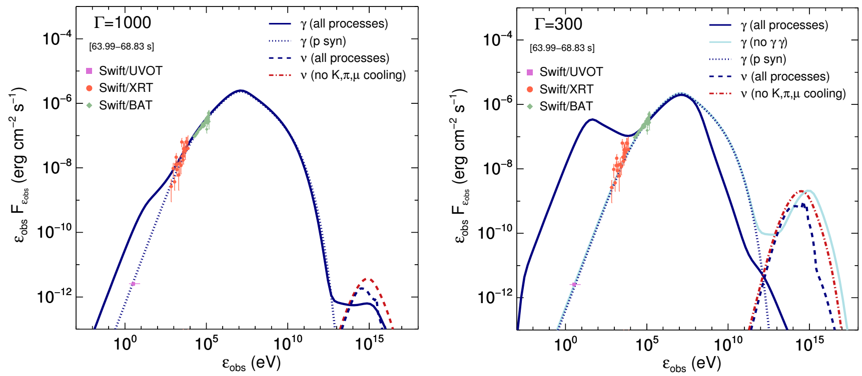

Fig. 3. (Color online) Predicted photon (solid blue line) and all-flavour neutrino (dashed blue line) energy spectra of GRB 061121 in the proton-synchrotron prompt emission model for $\Gamma = 1000$ (left panel) and $\Gamma = 300$ (right panel).

5. Conclusion

The main conclusions of the marginally fast-cooling proton-synchrotron model are the following:

— The low-energy part of the spectrum is not in good agreement with the observations when photomeson production and photon–photon pair production processes are taken into account for $\Gamma < 1000$.

— The proton-synchrotron scenario for GRB prompt emission demands very strong magnetic fields $B \sim 10^7$ G, resulting in very high magnetic jet powers $P_{B,\mathrm{jet}} \sim 10^{54}\,\mathrm{erg\,s^{-1}}$.

— The predicted diffuse neutrino flux is much lower that the respective γ-ray flux. This is due to the fact that protons are cooling mainly via synchrotron radiation in the presence of very strong magnetic fields.

— The predicted neutrino flux peaks at energies ~ 1 PeV, two orders of magnitude lower than the one of standard neutrino production models. This is attributed to the values of $\gamma_{\mathrm{min}} \approx 10^4$, used by the proton-synchrotron model.

In conclusion, the first two results strongly disfavour the marginally fast-cooling protons as an explanation of the low-energy spectral break in the prompt GRB spectra, which still remains a puzzle.

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