# STRONG EVIDENCE OF THE $\rho(1250)$ FROM A UNITARY MULTICHANNEL REANALYSIS OF ELASTIC SCATTERING DATA WITH CROSSING-SYMMETRY CONSTRAINTS*

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We present an analysis of elastic $P$-wave $\pi\pi$ phase shifts and inelasticities up to 2 GeV in order to identify the corresponding $J^{PC} = 1^{--}$ excited $\rho$ resonances focusing on the $\rho(1250)$ *vs.* $\rho(1450)$ controversy. In our approach, we employed an improved parametrization in terms of a manifestly unitary and analytic three-channel S-matrix with its complex-energy pole positions. The included channels were $\pi\pi$, $\rho 2\pi$, and $\rho\rho$. The improvement with respect to prior work amounts to the enforcement of maximum crossing symmetry through once-subtracted dispersion relations called GKPY equations. A clear picture emerges from this analysis, identifying five vector $\rho$ states below 2 GeV which are $\rho(770)$, $\rho(1250)$, $\rho(1450)$, $\rho(1600)$, and $\rho(1800)$, with $\rho(1250)$ being indisputably the most important excited $\rho$ resonance.

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## 1. Introduction

The experimental status of meson resonances with masses ranging from 1 to 2 GeV is very poor. Many states expected from the quark model have not been observed so far, whereas several apparently normal resonances listed in the PDG tables [1] do not fit in with mainstream quark models like, for instance, the relativized meson model by Godfrey and Isgur (GI) [2]. One of these disagreements is the first radial excitation of $\rho(770)$, $\rho'$, that is listed by the PDG as $\rho(1450)$ [1], which is difficult to reconcile with a lighter $K^\star(1410)$, as the latter state contains one strange quark and one light quark instead of two light quarks. However, one can find many indications of a lighter $\rho'$, roughly in the range of 1.25–1.3 GeV listed in the

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PDG under the $\rho(1450)$ entry. In our work, we aimed at clarifying the status of $\rho'$ and also the higher vector $\rho$ excitations, by reanalyzing old data on $\pi\pi$ scattering, *viz.* elastic phase shifts and inelasticities up to about 2 GeV.

## 2. Methodology

We used a manifestly unitary three-channel S-matrix parametrization, in which the complex pole positions of the different $\rho$ resonances are explicitly included through generalized BW-type expressions. The three included channels were $\pi\pi$, as well as the effective channels $\rho 2\pi$ and $\rho\rho$, with the latter ones mimicking $4\pi$ final states.

Our approach included a careful analysis of the S-matrix poles of certain resonances on different Riemann sheets of the complex energy plane. In order to get model-independent results *i.e.* the positions of the complex poles, we fitted the free parameters of the amplitude to the experimental data and to dispersion relations called GKPY equations [3]. The amplitudes are fully unitary and analytical, of the form of

$$A_{kl}(s) = \frac{1}{2i}\frac{S_{kl}-\delta_{kl}}{1-\frac{4m^2}{s}}\,, \tag{2.1}$$

where $s$ is the effective two-pion mass squared, $\delta_{kl}$ is the phase shift, $m$ is the pion mass and the $S_{kl}$ are S-matrix elements. For example, in the case of the $\pi\pi$ channel, such an element reads

$$S_{11} = S_{11}^{\mathrm{res}} S_1^{\mathrm{bgr}} = \frac{d_{\mathrm{res}}^*(-w^*)}{d_{\mathrm{res}}(w)}\frac{d_{\mathrm{bgr}}(-k_1)}{d_{\mathrm{bgr}}(k_1)}\,, \tag{2.2}$$

and expressions for other matrix elements are given in Eq. (6) of Ref. [4]. The S-matrix factors $S^{\mathrm{res}}$ and $S^{\mathrm{bgr}}$ stand for resonant and background parts, respectively, while $d_{\mathrm{res}}$ and $d_{\mathrm{bgr}}$ are the corresponding Jost functions, which contain all the dynamics of the interacting particles, both in individual channels and between them. The momenta in a given channel are denoted by $k_i$ and the uniformizing variable $w$ is defined as

$$w = \frac{\sqrt{s-s_2}+\sqrt{s-s_3}}{\sqrt{s_3-s_2}}\,, \tag{2.3}$$

where $s_2$ and $s_3$ are the thresholds of the second and third channel, respectively. The variable $w$ transforms the eight-sheeted Riemann surface into a simpler complex plane. A resonance pole is given by $\sqrt{s_{\mathrm{r}}} = E_{\mathrm{r}} - i\Gamma_{\mathrm{r}}/2$, with $E_{\mathrm{r}}$ the resonance mass and $\Gamma_{\mathrm{r}}$ its full width. So for $s = s_{\mathrm{r}}$, we have

$$w_{\mathrm{r}} = \frac{\sqrt{s_{\mathrm{r}}-s_2}+\sqrt{s_{\mathrm{r}}-s_3}}{\sqrt{s_3-s_2}}\,, \tag{2.4}$$

and the resonance contributions $S^{\mathrm{res}}$ are defined as

$$d_{\mathrm{res}}(w) = w^{\frac{-M}{2}} \prod_{r=1}^{M} (w + w_{\mathrm{r}}^{*}) \,, \tag{2.5}$$

where $M$ is the number of resonances. The background Jost function has the form of

$$d_{\mathrm{bgr}}(k_1) = \exp\left[2ia - 2b\left(\frac{k_1}{m_1}\right)^3 \Theta(s, s_2)\right] , \tag{2.6}$$

with $\Theta(s, s_2)$ the Heaviside function ($= 1$ for $s > s_2$), and where $a$ and $b$ are real numbers.

This simplest possible background in Eq. (2.6) was introduced and fitted to the data as well as the GKPY equations in order to efficiently take into account the influence of all higher $\rho$ decay channels not included in the $d_{\mathrm{res}}(w)$ Jost function in Eq. (2.5). As a result, a constant and small phase of almost $-20^\circ$ and a smoothly increasing small inelasticity are obtained.

In order to improve the near-threshold behavior of the amplitudes, we replaced the original amplitude in Eqs. (2.1)–(2.6) by a polynomial below about 640 MeV (this value resulted from fits to the data and the GKPY equations). The polynomial is merely a generalized near-threshold expansion in powers of the pion momentum $k_1$,

$$\mathrm{Re}A(s) = \frac{\sqrt{s}}{4k} \sin 2\delta = m_\pi k^2 \left[a + bk^2 + ck^4 + dk^6 + O\left(k^8\right)\right] , \tag{2.7}$$

where $\delta$ denotes the phase shift and $a$, $b$ are just scattering length and effective range, respectively, which were fixed. However, the parameters $c$ and $d$ were free in the fits to the data and to the GKPY equations. We used these two parameters in order to match the phase shifts from the polynomial (*i.e.*, their values and first derivatives) to the multichannel ones determined by Eqs. (2.1)–(2.6) at the matching energy of about 640 MeV.

## 3. Results

The results presented in Figs. 1–4 are based on the fits carried out in Refs. [5, 6]. Figure 1 shows the $P$-wave $\pi\pi$ phase shifts due to the individual resonances, corresponding to poles (all members of a cluster for a given resonance) on different Riemann sheets. Of course, only the full phase shift has the correct threshold behavior, given by a polynomial with fixed scattering length and effective range. As one would expect, $\rho(770)$ has by far the largest influence on the overall phase, dominating the contributions of the $\rho$ excitations. Moreover, the second most important resonance is clearly $\rho(1250)$, whereas the smallest effect is due to $\rho(1450)$.

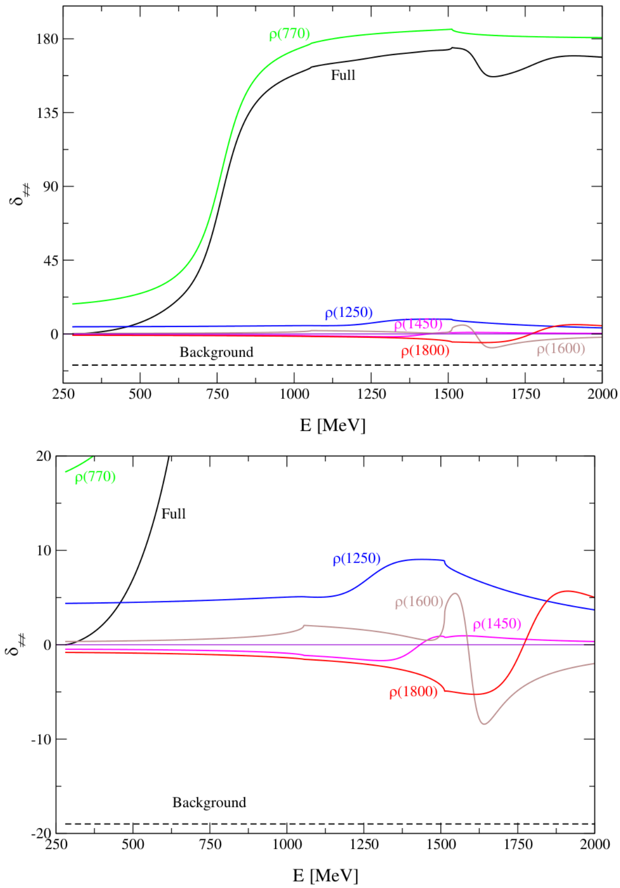


Fig. 1. Upper figure: phase shift due to individual resonances, as well as full phase and background; lower figure: enlarged fragment from the upper figure.

To properly assess the contribution of individual resonances to the full amplitude, it is very clarifying to compute it before and after removing those resonances. Figure 2 displays phase shifts for the full amplitude and that without terms from individual resonances as the amplitude of $\rho(770)$ is strongly dominant for the whole energy range. One can notice that below 1 GeV, all other resonances besides $\rho(770)$ have an almost negligible effect on the full phase shift. Changes by removing this resonance are not shown, because they would be too large. Once again, one can clearly see how important the role of $\rho(1250)$ is, in contrast with most notably $\rho(1450)$. Its influence dominates between 1.0 GeV and 1.5 GeV, being comparable to that of $\rho(1600)$ and $\rho(1800)$ there-above. The $\rho(1450)$ contribution is quite small over the entire tested energy range.

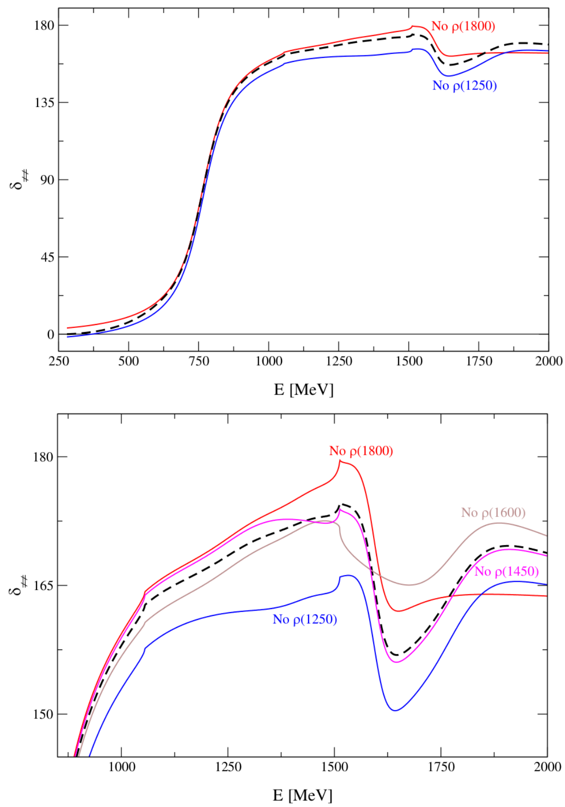


Fig. 2. Upper figure: phase shift from full amplitude (dashed line) and the same but without $\rho(1250)$ and $\rho(1800)$; lower figure: as the upper figure but enlarged over a reduced energy interval and also without $\rho(1450)$ and $\rho(1600)$.

Figure 3 shows the $\pi\pi$ inelasticity $\eta$ for the full amplitude and also the individual resonances. One can see very well that even below 1.5 GeV (near the $\rho\rho$ threshold) inelasticity due to the $\rho(1250)$ amplitude significantly differs from 1 and together with part from that of $\rho(770)$ almost completely determines the inelasticity of the full amplitude. Contributions from $\rho(1450)$ and $\rho(1600)$ largely cancel each other between the $\rho 2\pi$ and $\rho\rho$ thresholds *i.e.* between 1.05 GeV and 1.55 GeV respectively. Above roughly 1.5 GeV, $\rho(1800)$ determines the energy dependence of $\eta$ almost entirely, interfering with the still large but already rather unstructured $\rho(1250)$ part comparable in size to that of $\rho(770)$. The contribution of $\rho(1450)$ to $\eta$ is very small above 1.5 GeV. As expected, it only has a minor maximum at about 1.4 GeV. The significant drop in the full inelasticity at about 1.6 GeV is mostly determined by $\rho(1600)$, after the opening of the $\rho\rho$ channel. The role of the background in building $\eta$ is small, showing a slow and smooth rise.

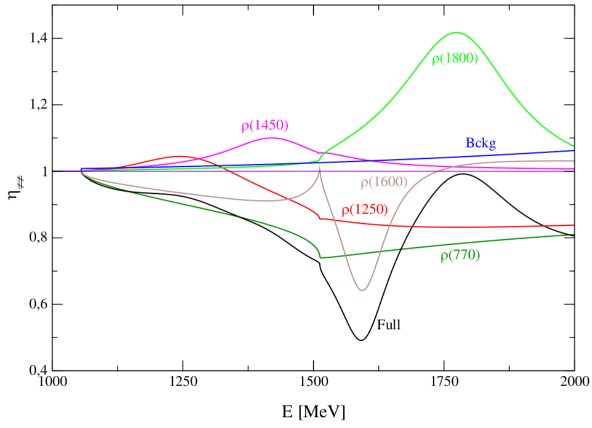


Fig. 3. Inelasticity due to individual resonances, full amplitude, and background.

Just as in the case of the phase shift, the energy dependence of the inelasticity for the amplitude without a given resonance, *i.e.*, by omitting all poles associated with it on the different Riemann sheets, is very informative. In Fig. 4, we see that removing $\rho(1250)$ would cause the largest change after that caused by $\rho(770)$ to the inelasticity curve as compared to the one due to the full amplitude. Similarly, a significant modification would be caused by leaving out $\rho(1600)$ or $\rho(1800)$, but only around 1600 MeV or there-above, respectively. Finally, also here we observe that without $\rho(1450)$, there would only be a modest change to $\eta$, over a relatively small energy region below 1.5 GeV, having little effect on the shape of the inelasticity curve.

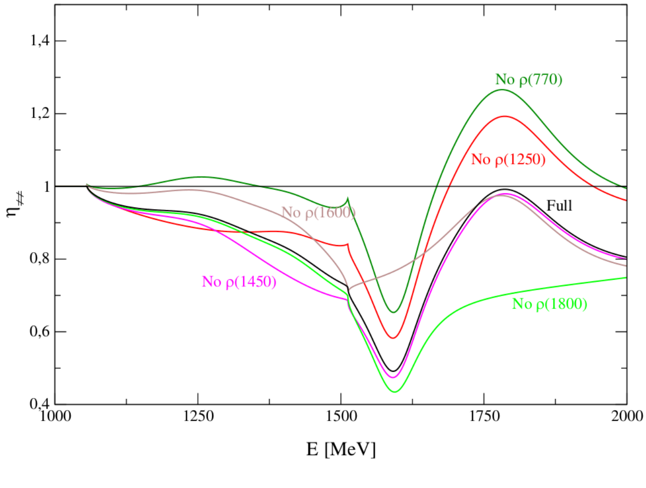


Fig. 4. Inelasticity due to full amplitude and the same without individual resonances.

## 4. Conclusion

The results of our combined analyses unmistakable demonstrate the necessity to include a $\rho'$ resonance at about 1.26 GeV. The stability of the fitted pole positions as well as the manifest fulfillment of multichannel unitarity and optimized crossing symmetry in our approach lend strong support to the reliability of our excited $\rho$ states, including the ones at about 1.42 GeV, 1.60 GeV, and 1.78 GeV. Straightforward spectroscopic arguments then impose the following quark-model assignments: $\rho(1250)/2\,^3S_1$, $\rho(1450)/1\,^3D_1$, $\rho(1600)/3\,^3S_1$, and $\rho(1800)/2\,^3D_1$. Confirmation of these four states, which were already found in a previous analysis [4], poses serious problems to mainstream quark models, unless at least $\rho(1250)$ is interpreted as a crypto-exotic tetraquark state, for which there is no experimental or theoretical support.

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