THEORETICAL ASPECTS OF RELATIVISTIC PERFECT-FLUID SPIN HYDRODYNAMIC FRAMEWORK*

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We review the recently proposed perfect-fluid spin hydrodynamic formalism, which provides a new tool for the description of the spin polarization of $\Lambda(\bar{\Lambda})$ particles. This formalism is based on the de Groot–van Leeuwen–van Weert definitions of the energy-momentum and spin tensors.

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1. Introduction

Relativistic hydrodynamics has been quite successful in describing phenomena in various areas of physics [1–4], consequently making the path in explaining the interesting properties of quark–gluon plasma [5]. Spin polarization measurements in relativistic heavy-ion collisions [6–10] provided a new area for study [11]. These measurements initiated vast theoretical developments concerning spin-vorticity coupling [12–40]. However, after explaining global spin polarization, spin-thermal-based frameworks have not been able to explain longitudinal spin polarization [7, 20, 41]. This discrepancy instigated a new formalism of spin hydrodynamics, first proposed in Ref. [42], see also Refs. [30, 43–70] for further developments and new approaches. The evolution of spin polarization in spin hydrodynamics [42–44] is controlled by an antisymmetric spin polarization tensor $\omega^{\alpha\beta}$ (independent of thermal vorticity), introducing six spin polarization components which need to be determined along with the background parameters from the conservation laws. The framework of spin hydrodynamics [42, 43] uses the de Groot–van Leeuwen–van Weert (GLW) [71] definitions of energy-momentum and spin tensors. Keeping in mind the smallness of spin polarization magnitude in the experiments, the spin polarization tensor [42, 43] is assumed to be of leading order, hence having contributions only in the spin tensor,

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making the spin dynamics decouple from the perfect-fluid background dynamics [44, 45, 63]. After obtaining the dynamics of spin, we then move on to calculate the mean spin polarization per particle in the particle rest frame (PRF) to be compared with the experimental data [63].

2. Spin hydrodynamic equations

The formalism of the perfect-fluid spin hydrodynamics for particles with spin-1/2 is based on the conservation of charge, energy-momentum, and angular momentum with the GLW [71] definitions of the energy-momentum tensor, $T^{\alpha\beta}_{\rm GLW}$, and spin tensor, $S^{\alpha\beta\gamma}_{\rm GLW}$ [1], such as [42, 43, 45]

$$\partial_\mu N^\mu = 0\,, \qquad \partial_\mu T^{\mu\nu}_{\rm GLW} = 0\,, \qquad \partial_\lambda S^{\lambda,\alpha\beta}_{\rm GLW} = T^{\beta\alpha}_{\rm GLW} - T^{\alpha\beta}_{\rm GLW}\,, \tag{1}$$

where

$$N^\alpha = nU^\alpha\,, \qquad T^{\alpha\beta}_{\rm GLW} = (\varepsilon + P)U^\alpha U^\beta - Pg^{\alpha\beta}\,, \tag{2}$$

with N^α being the net baryon charge current while ε, P, and n are the energy density, pressure, and baryon density, respectively. U^β represents the four-vector fluid flow which is time-like, hence $U\cdot U = 1$.

The symmetric nature of GLW energy-momentum tensor (1) implies separate spin conservation [44, 45] with the spin tensor expressed by $S^{\alpha,\beta\gamma}_{\rm GLW} = \cosh(\xi)\left(n_{(0)}(T)U^\alpha\omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta{\rm GLW}}\right)$, where [43, 45]

$$S^{\alpha,\beta\gamma}_{\Delta{\rm GLW}} = \mathcal{A}_{(0)}U^\alpha U^\delta U^{[\beta}\omega^{\gamma]}{}_\delta + \mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_\delta + U^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}{}_\delta + U^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}{}_\delta\right),$$

and the thermodynamic coefficients are $\mathcal{B}_{(0)} = -\frac{2}{\hat{m}^2}s_{(0)}(T)$ and $\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T)$, with $n_{(0)}(T)$ and $s_{(0)}(T)$ being the number density and entropy density for the case of neutral and massive spin-less Boltzmann particles, $\Delta^{\alpha\beta}$ is the spatial projector operator transverse to U, ξ denotes the ratio between the baryon chemical potential (μ) and temperature (T) whereas $\hat{m}$ denotes the ratio between the mass of the particle and temperature [45, 63]. The quantity $\omega_{\mu\nu}$ is called the spin polarization tensor $\omega_{\mu\nu}$, antisymmetric and rank-two, that can be written in terms of κ^μ and ω^μ four-vectors [42, 63]

$$\omega_{\mu\nu} \;=\; \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta}U^\alpha\omega^\beta\,. \tag{3}$$

κ^μ and ω^μ being parallel to fluid flow have no contribution to the R.H.S. of Eq. (3), thus they fulfill[2]

$$\kappa\cdot U = 0\,, \qquad \omega\cdot U = 0 \tag{4}$$

[1] Spin polarization is assumed to be small ($|\omega_{\mu\nu}| < 1$) in this formalism.

[2] κ^μ and ω^μ together form six independent components of $\omega_{\mu\nu}$ [63].

allowing us to write these four-vectors as [44, 45, 63] $\kappa_\mu = \omega_{\mu\alpha}U^\alpha$, $\omega_\mu = \frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}\omega^{\alpha\beta}U^\gamma$ κ^μ, and ω^μ can be decomposed again in terms of the basis vectors X, Y, Z [3] and scalar spin coefficients $(C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z}, C_{\omega X}, C_{\omega Y}, C_{\omega Z})$ after using orthogonality conditions (4) as [63]

$$\kappa^\alpha = C_{\kappa X}X^\alpha + C_{\kappa Y}Y^\alpha + C_{\kappa Z}Z^\alpha\,, \qquad \omega^\alpha = C_{\omega X}X^\alpha + C_{\omega Y}Y^\alpha + C_{\omega Z}Z^\alpha\,, \tag{5}$$

which then help us to obtain the general form of $\omega_{\alpha\beta}$ [63]

$$\begin{aligned}\omega_{\alpha\beta} &= 2\left(C_{\kappa X}X_{[\alpha}U_{\beta]} + C_{\kappa Y}Y_{[\alpha}U_{\beta]} + C_{\kappa Z}Z_{[\alpha}U_{\beta]}\right)\\ &\quad +\epsilon_{\alpha\beta\gamma\delta}U^\gamma\left(C_{\omega X}X^\delta + C_{\omega Y}Y^\delta + C_{\omega Z}Z^\delta\right).\end{aligned} \tag{6}$$

3. Spin polarization of particles at the freeze-out

With the information about the evolution of the spin polarization components and background parameters, we can now calculate the mean spin polarization per particle defined as $\langle\pi_\mu\rangle_p = E_p\frac{\mathrm{d}\Pi^*_\mu(p)}{\mathrm{d}^3p}/E_p\frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3p}$, where [44, 63]

$$E_p\frac{\mathrm{d}\Pi^*_\mu(p)}{\mathrm{d}^3p} = -\frac{1}{(2\pi)^3 m}\int \cosh(\xi)\Delta\Sigma_\lambda p^\lambda\, \mathrm{e}^{-\beta\cdot p}\left(\widetilde{\omega}_{\mu\beta}p^\beta\right)^*\,, \tag{7}$$

$$E_p\frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3p} = \frac{4}{(2\pi)^3}\int \Delta\Sigma_\lambda p^\lambda \cosh(\xi)\,\mathrm{e}^{-\beta\cdot p}\,, \tag{8}$$

and $E_p\frac{\mathrm{d}\Pi^*_\mu(p)}{\mathrm{d}^3p}$ is the total Pauli–Lubański (PL) vector in the PRF (after integrating over the freeze-out hyper-surface element $\Delta\Sigma_\lambda$) for particles having momentum p, while $E_p\frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3p}$ represents the momentum density of all particles [45, 63]. The quantity $\langle\pi_\mu\rangle_p$, the average spin polarization per particle, is a function of momentum coordinates p_x, p_y, p_z [63] which will allow us to obtain the local spin polarization (polarization along the beam direction), whereas to calculate the global spin polarization (along the $-y$ axis) we must integrate over the momentum variable such as [63]

$$\langle\pi_\mu\rangle = \frac{\int \mathrm{d}P\langle\pi_\mu\rangle_p E_p\frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3p}}{\int \mathrm{d}P E_p\frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3p}} \equiv \frac{\int \mathrm{d}^3p\, \frac{\mathrm{d}\Pi^*_\mu(p)}{\mathrm{d}^3p}}{\int \mathrm{d}^3p\, \frac{\mathrm{d}\mathcal{N}(p)}{\mathrm{d}^3p}}\,. \tag{9}$$

[3] X, Y, Z are the space-like four-vectors which span the space transverse to U [63].

4. Summary

In this article, we have reviewed the theoretical aspects of the spin hydrodynamic formalism [42–44]. Considering the spin polarization tensor of the leading order, we find the decoupling nature between the background and the spin dynamics. We then put forward the expressions required to calculate the mean spin polarization of the particles for the comparison with the experimental data [63].

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