MULTIWAVELENGTH AND MULTIMESSENGER OBSERVATIONS OF BLAZARS AND THEORETICAL MODELING: BLAZARS AS ASTROPHYSICAL NEUTRINO SOURCES*

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Received 15 April 2022, accepted 26 April 2022, published online 9 September 2022

This contribution reviews recent advances in the possible identification of blazars as potential sources of at least some of the very-high-energy neutrinos detected by the IceCube neutrino detector at the South Pole. The basic physical requirements for neutrino production and physics constraints that may be drawn from neutrino–blazar associations are reviewed. Several individual cases of possible associations will be discussed in more detail. It is emphasized that due to $\gamma\gamma$ opacity constraints in efficiently neutrino-producing blazars, an association between X-ray to soft γ-ray activity and very-high-energy neutrino production is more naturally expected than a connection between neutrino and high-energy/very-high-energy γ-ray activity.

DOI:10.5506/APhysPolBSupp.15.3-A8

1. Introduction

Since the first announcement of the detection of very-high-energy neutrinos of astrophysical origin by the IceCube neutrino detector at the South Pole [1, 2], the identification of the sources of these neutrinos has been one of the highest-priority quests in modern astrophysics. Various high-energy sources, including supernovae and their remnants, tidal disruption events, and active galactic nuclei (AGN) have been suggested as possible sources of very-high-energy neutrinos (see [3] and [4] for recent reviews). Among the most promising candidates are blazars, a class of jet-dominated

* Presented at the 28th Cracow Epiphany Conference on *Recent Advances in Astroparticle Physics*, Cracow, Poland, 10–14 January, 2022.

AGN with their relativistic jets pointing close to our line of sight, resulting in significant Doppler boosting of their emission. They are known to be variable throughout the electromagnetic spectrum, from radio to high-energy (HE: $E > 100$ MeV), and in some cases even very-high-energy (VHE: $E > 100$ GeV) γ-rays, on a variety of time scales, from months to years down to minutes. Blazars consist of flat-spectrum radio quasars (FSRQs) and BL Lac objects, depending on the presence or absence of bright optical/UV emission lines, respectively. Their broad-band spectral energy distributions are dominated by two broad, non-thermal emission components: The low-frequency component (radio through optical/UV/X-rays) is likely due to synchrotron emission by relativistic electrons in the jet, while the high-frequency (X-rays to γ-rays) component may be either due to inverse Compton scattering of low-energy target photons by the same relativistic electrons, or due to hadronic processes (for a recent review see, *e.g.*, [5]). Depending on the location of the peak of the synchrotron component, blazars are further sub-divided into low-synchrotron-peaked (LSP; $\nu_{\rm sy} < 10^{14}$ Hz), intermediate-synchrotron-peaked (ISP; 10^{14} Hz $< \nu_{\rm sy} < 10^{15}$ Hz), and high-synchrotron-peaked (HSP; $\nu_{\rm sy} > 10^{15}$ Hz) blazars. All known FSRQs belong to the LSP class, while BL Lacs display synchrotron peaks in all ranges.

While it has been demonstrated that blazars cannot be responsible for the entire IceCube astrophysical neutrino excess (*e.g.*, [6]), the number of plausible spatial and temporal associations between blazars in multi-wavelength outbursts and IceCube neutrino alerts is steadily increasing (see, *e.g.*, [7, 8] and [9] for comprehensive multi-wavelength studies of plausible associations).

This review will provide an introduction to the physical conditions and energetics required for VHE neutrino production in blazar jets (Section 2), and then discuss the results of detailed studies of individual cases of potential blazar-flare–neutrino-alert associations (Section 3). Section 4 will finally provide a summary of lessons learned from these studies.

2. General considerations: Physical conditions and energetics requirements

The production of > 100 TeV neutrinos (which have a high probability of being of astrophysical origin) requires the acceleration of protons or nuclei to at least $\sim$ PeV energies. Neutrinos are a byproduct of the decay of charged pions and muons produced in hadronic interactions dominated by the excitation and subsequent decay of the Δ^+ (1232 MeV) resonance (see, *e.g.*, Chapter 3 of [10]). In about 1/3 of the interactions, the Δ^+ decay results in the production of a charged pion

$$p + p \to p + \Delta^+ \to p + n + \pi^+ \quad \text{or} \quad p + \gamma \to \Delta^+ \to n + \pi^+ \qquad (1)$$

followed by

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \,, \\ \mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \,. \end{aligned} \tag{2}$$

In the jets of AGN, the number of potential target protons for pp interactions is expected to be many orders of magnitude smaller than the number of available target photons for $p\gamma$ interactions. Almost all works studying neutrino production in blazars therefore consider only the photo-pion production channel (see, however, *e.g.*, [11] for an alternative, hadronuclear, beam-dump-like scenario). The excitation of the Δ^+ resonance requires the corresponding center-of-momentum energy of the reaction, thus constraining the product of the proton ($E'_{\rm p}$) and target photon ($E'_{\rm t}$) energies in the co-moving frame of the emission and neutrino production region, which moves with relativistic speed along the jet:

$$s \sim E'_{\rm p}\, E'_{\rm t} = E^2_{\Delta^+} = (1232\,{\rm MeV})^2 \,. \tag{3}$$

The relativistic motion of the emission region along the jet closely aligned with our line of sight leads to relativistic Doppler boosting of both electromagnetic and neutrino emissions by a Doppler factor $\delta \equiv 10\,\delta_1$. In the photopion reactions and subsequent pion and muon decay, each neutrino takes up on average about 5% of the proton's energy. Consequently, the production of an IceCube neutrino with observed energy $E_\nu = 100 E_{14}$ TeV (which is produced with $E'_\nu = 10\,\delta_1^{-1}\,E_{14}$ TeV in the emission-region rest frame) requires the acceleration of protons to $E'_{\rm p} \sim 200\,E_{14}\,\delta_1^{-1}$ TeV. The Delta resonance condition (3) then implies that target photons of energies $E'_{\rm t} \sim 1.6\,E_{14}^{-1}\,\delta_1$ keV (*i.e.*, X-rays) are needed.

Different constraints on the power in relativistic protons in the jet can be derived, depending on whether the target photon field originates within the jet, co-moving with the emission region (*e.g.*, the primary electron–synchrotron radiation), or outside the jet, stationary in the AGN rest frame. In the former case, the co-moving target photons are Doppler boosted into the observer's frame to an energy of $E_{\rm t}^{\rm obs} \sim 16\,E_{14}^{-1}\,\delta_1^2/(1+z)$ keV. In addition to the frequency shift, the hard X-ray flux corresponding to this target photon field will be directly observed, with the νF_ν flux enhanced by a factor δ^4. The co-moving target photon density is therefore tightly limited by the observed hard X-ray flux (or upper limits thereof). In turn, a low target photon density necessitates a high co-moving density of ultrarelativistic protons in order to produce a given neutrino flux. This often leads to the requirement of far super-Eddington jet powers. The power requirements can be greatly reduced if the target photons originate outside the jet, *e.g.*, in the Broad Line Region (*e.g.*, [12]), in a non-relativistic sheath surrounding the relativistic inner jet (*e.g.*, [13]), in a radiatively inefficient accretion

flow (*e.g.*, [14]), or in a scenario of a possible binary or strongly bent jet structure [15]. In this case, if the radiation field is approximately isotropic (*i.e.*, with anisotropies on much larger angular scales than the $1/\Gamma$ angular pattern of relativistic beaming, where Γ is the bulk Lorentz factor of the motion along the jet) in the AGN rest frame, then target photon energies will be boosted by a factor $\sim\delta$ into the emission-region rest frame, so the observer will see them unboosted at energies of $E_{\rm t}^{\rm obs}\sim 160\,E_{14}^{-1}/(1+z)$ eV, *i.e.*, in the UV to soft X-rays. Equally, the target photon energy density will be enhanced by a factor $\sim\Gamma^2$ into the emission-region rest frame, while the observer will see the corresponding UV-soft X-ray flux without Doppler enhancement. This allows for a much denser target photon field in the emission-region rest frame than in the case of a co-moving field and therefore greatly relaxes the hadronic jet power constraints. See, *e.g.*, [16] for the numerical evaluation of these jet power constraints in the specific case of modeling the 2014–2015 neutrino flare from the direction of TXS 0506+056.

2.1. $\gamma\gamma$ opacity constraints

The efficient production of neutrinos through photo-pion interactions in a blazar jet inevitably leads to the production of ultra-high-energy and VHE γ-rays through the decay of neutral pions and synchrotron emission of secondary decay products. This has motivated systematic searches for γ-ray flaring blazars coincident with neutrino alerts (*e.g.*, [8]). However, the same target photon field that leads to photo-pion production also acts as a source of $\gamma\gamma$ opacity for co-spatially produced γ-rays (*e.g.*, [6, 16]). Most notably, the peak cross section (at the Δ^+ resonance) for protons to interact through photo-pion production is about 300 times smaller than the peak $\gamma\gamma$ absorption cross section at a peak energy corresponding to $E_\gamma\sim(m_ec^2)^2/E_{\rm t}\sim 3.3\times10^{-5}\,E_\nu$. For $\gtrsim$ 100 TeV neutrinos, this corresponds to $\sim$ GeV–TeV γ-rays, in the energy ranges observed by *Fermi*-LAT and ground-based atmospheric Cherenkov telescopes. The energy in these $\gamma\gamma$ absorbed photons will emerge from the emission region, through electromagnetic cascading processes, in the X-rays and soft γ-rays. Consequently, a correlation between the HE and VHE γ-ray emission and neutrino activity is not necessarily expected, especially if γ-rays and neutrinos are produced in the same emission region. On the other hand, X-ray and $\sim$ MeV γ-ray activity is expected as a natural by-product of photo-pion processes (*e.g.*, [6, 16, 17]). This is also in line with the finding by [18, 19] that AGN with bright (strongly Doppler boosted) radio cores are likely sources of IceCube neutrinos, with radio brightness often increasing in correlation with neutrino alerts, while no systematic correlation with γ-ray activity is found.

3. Discussion of individual cases

This section discusses those tentative associations between IceCube neutrino alerts and multi-wavelength flaring blazars which have been studied in some detail in the literature. They are discussed in chronological order of the neutrino alerts.

3.1. IC-35 ("Big Bird") and PKS 1424-418

IC-35 was the third PeV ($E \sim 2$ PeV) neutrino detected by IceCube, on 4 December 2012. Its arrival direction, although with large uncertainty, coincided with the FSRQ PKS 1424-418 at a redshift of $z = 1.522$. Its arrival time coincided with the rising phase of a multi-wavelength flare, including HE γ-rays, X-rays and radio, although *Swift*-XRT observations only commenced several days after the alert (see Fig. 1). In particular, the authors of [20] found that the radio outburst contemporaneous with the neutrino alert was dominated by an increase in the flux of the radio core, possibly marking the ejection of a new jet component.

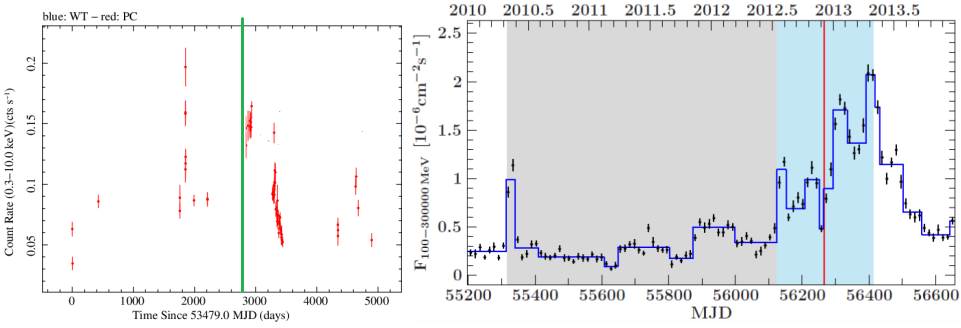


Fig. 1. (Color online) *Swift*-XRT (left — from `https://www.swift.psu.edu/monitoring/`) and *Fermi*-LAT (right) light curves of PKS 1424-418. The vertical green (left) and red (right) lines, respectively, indicate the arrival time of IC-35. *Fermi*-LAT light curve adapted from [20] — see `arXiv:1602.02012 [astro-ph.HE]`.

A phenomenological, calorimetric estimate of the expected neutrino flux by [21] found a very small probability of an IceCube neutrino detection during multi-wavelength flares of this source. More detailed physical modeling of the SED and neutrino production through photo-hadronic processes was performed by [22]. They demonstrate that neither a photo-pion-dominated nor a proton-synchrotron-dominated lepto-hadronic model can simultaneously explain the SED and the PeV neutrino event. Instead, a model with primary-electron-SSC-dominated γ-ray emission and only a subdominant hadronic contribution to the SED can successfully fit the SED and predict up to 0.3 neutrino events detected by IceCube during the outburst phase.

3.2. IceCube-170922A and TXS 0506+056 (and PKS 0502+049?)

The Ice-Cube detection of an $E \sim 290$ TeV neutrino on 22 September 2017, coincident with the flaring BL Lac type LSP blazar TXS 0506+056 [23], is by far the most well-studied and highest-confidence association between a neutrino event and a flaring blazar. What makes this association particularly compelling is the fact that an archival search for neutrino emission from the direction of this blazar led to the identification of an excess of $\sim 13 \pm 5$ lower-energy neutrinos during a ~ 4 months period around December 2014 from a direction also spatially consistent with TXS 0506+056, which, for the first time, allowed the IceCube Collaboration to derive a meaningful neutrino flux and spectrum (based on more than just a single neutrino) [24]. These associations have been discussed in a large volume of literature which has been summarized in several review papers (*e.g.*, [3]). A recent, detailed multi-epoch study of the multi-wavelength and multi-messenger emission of TXS 0506+056 by [25] also provides a comprehensive review of the previous modeling works with relevant references. Most works on the theoretical modeling of the multi-wavelength and neutrino emission agree that it is very difficult, if not impossible, to model the high-energy emission as dominated by hadronic processes, but instead, leptonic-dominated multi-wavelength emission is favored, with only a sub-dominant hadronic component responsible for the detected neutrino flux, as in the case of IC-35/PKS 1424-418 described above. Neutrino production is likely due to ultra-relativistic protons interacting with a UV/soft X-ray target photon field originating external to the jet (*e.g.*, [16, 25–28]).

A very recent finding and a suggestion that has so far received little attention in the literature is based on a detailed re-analysis of the pc-scale jet structures using archival 8 and 15 GHz very-long-baseline interferometry data, not only of TXS 0506+056, but also of another nearby γ-ray blazar, namely PKS 0502+049 [29]. Estimating the ejection times of 12 and 7 jet components in TXS 0506+056 and PKS 0502+049, respectively, they find that the arrival time of IceCube-170922A coincides with the estimated ejection times of two components in the jet of TXS 0506+056, while no such coincidence was found for the 2014–2015 neutrino flare. On the contrary, the 2014–2015 neutrino flare was coincident with the ejection time of a jet component in PKS 0502+049 (see Fig. 2). The authors of [29] thus argue that IceCube-170922A and the 2014–2015 neutrino flare might actually originate from two different blazars. However, it seems that the decomposition of the VLBI images into individual components as well as the identification of components across epochs appears very difficult in the case of these blazars, and superluminal-motion speeds up to $\gtrsim 60\ c$ have been found for both sources, which should be taken with great caution. Nevertheless, the suggestion of the two neutrino signals originating from two different sources should be seriously considered.

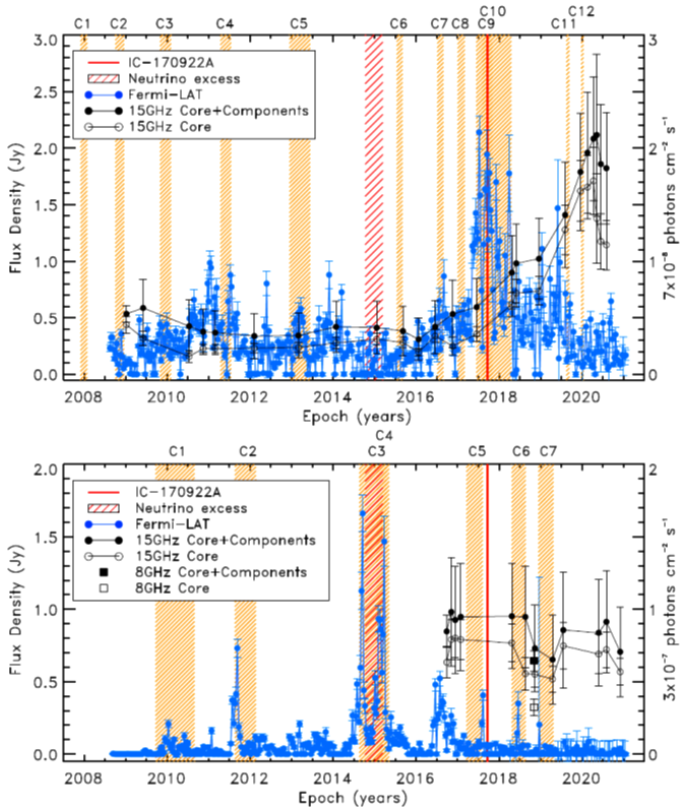


Fig. 2. (Color online) 15 GHz radio and *Fermi*-LAT γ-ray light curves of TXS 0506+056 (top) and PKS 0502+049 (bottom). Vertical strips (orange) indicate the error regions of ejection times of new jet components. The vertical solid line (red) indicates the arrival time of IceCube-170922A, while the dashed strip (red) indicates the time of the 2014–2015 neutrino flare (from [29]).

3.3. IceCube-190730A and PKS 1502+106

Another well-studied potential coincidence was the $\sim$ 300 TeV (67% probability of being astrophysical) neutrino alert IceCube-190730A which was spatially coincident with the FSRQ PKS 1502+106 ($z = 1.84$). The coincidence attracted particular attention in the literature as this FSRQ is the 15$^{\text{th}}$ brightest HE γ-ray source in the 4LAC [30]. This coincidence is further remarkable since the high-energy (X-ray and γ-ray) emission shows no sign of flaring during the arrival time of IceCube-190730A, but instead, it occurs near the peak of a several-year-long radio outburst (see Fig. 3, from [8]). This is in contrast to almost all other known neutrino–blazar coincidences for which contemporaneous X-ray coverage exists, which consistently shows X-ray elevated states during the time of the neutrino alert [31].

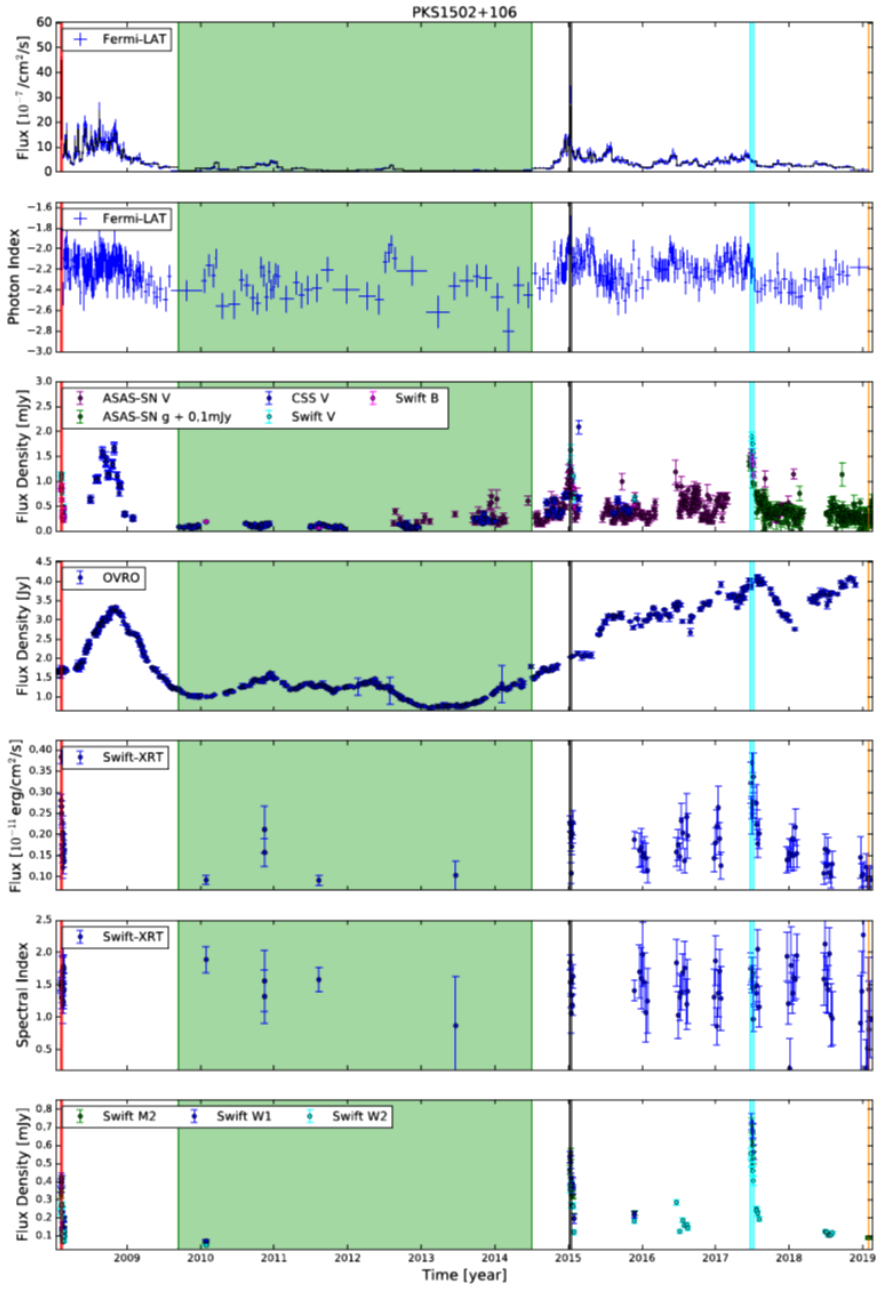


Fig. 3. (Color online) Multi-wavelength light curves of PKS 1502+106. The solid lines (orange) towards the end of the light curves indicate the arrival time of IceCube-190730A (from [8]).

The multi-wavelength and neutrino emission of this source was studied in detail by [32] and [33]. The authors of [32] modeled the SED and neutrino emission studying various lepto-hadronic and proton-synchrotron-dominated models and found viable model solutions — with X-ray and γ-ray emission dominated by external Compton and proton-synchrotron processes, respectively (see Fig. 4) — both for the quiescent and flaring states. They provide arguments in favor of a lepto-hadronic scenario, in particular, a lower power demand, but such a scenario also predicts detectable neutrino emission in flaring states (in particular, prior to 2010), which should be found in archival searches.

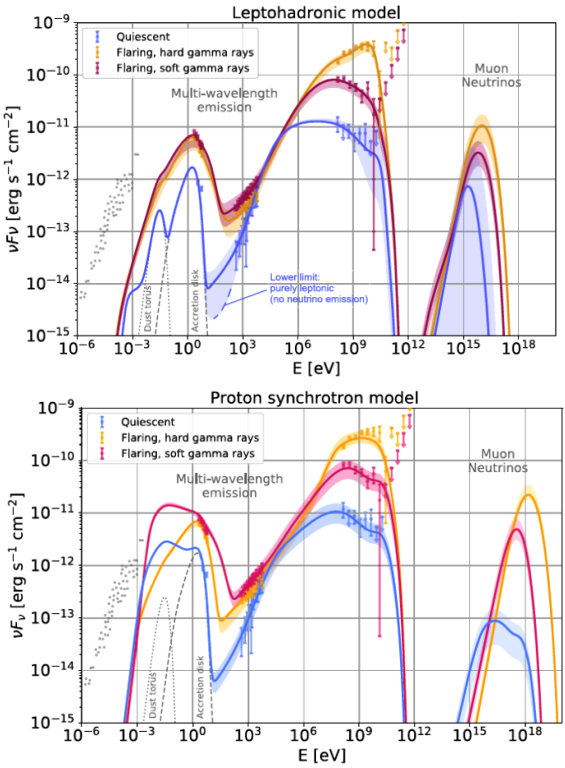


Fig. 4. Model fits to the SEDs and neutrino emission from PKS 1502+106 during quiescent and flaring states, using a lepto-hadronic (left) and a proton-synchrotron-dominated (right) model (from [32]).

In contrast, the authors of [33] investigated various scenarios with leptonic (SSC and external-Compton) scenarios for the high-energy emission from PKS 1502+106 with only a sub-dominant hadronic component responsible for neutrino production (more in line with the majority of findings for TXS 0506+056). They found that both an SSC and an EC scenario are

viable and may predict ~ 1 muon neutrino during the lifetime of IceCube, with power requirements significantly below the source's Eddington limit, while at the same time being consistent with blazars powering the observed ultra-high-energy cosmic-ray flux.

3.4. IceCube-200107 and 3HSP J095507.9+35510

IceCube-200107 was an IceCube High-Energy Starting Event (HESE) with highly uncertain energy ($E_\nu \sim 330^{+2230}_{-270}$ TeV). It was found to be spatially coincident with the high-synchrotron-peaked BL Lac object 3HSP J095507.9+35510 (= BZB J0955+3551) [34] at a redshift of $z = 0.5573$ [35, 36]. *Swift*-XRT ToO observations one day and NuSTAR ToO observations 4 days after the neutrino alert found the blazar in a bright X-ray flare, but exhibiting only moderate γ-ray activity [34, 36]. Incidentally, there was one other γ-ray blazar within the error region of the neutrino event, namely 4FGL J0957.8+3423, which, however, did not exhibit any significant flux enhancement in any waveband [37]. 3HSP J095507.9+35510 is, therefore, considered the more likely counterpart of IceCube-200107.

Estimates of requirements on jet power in relativistic protons and on the target photon field in BZB J0955+3551 to produce a neutrino flux corresponding to an association with IceCube-200107, along with numerical simulations, were presented in [36]. They found that the resulting $p\gamma$-induced cascade emission would significantly overshoot the observed optical flux from the source if the neutrino flux estimate is based on the occurrence of 1 HESE

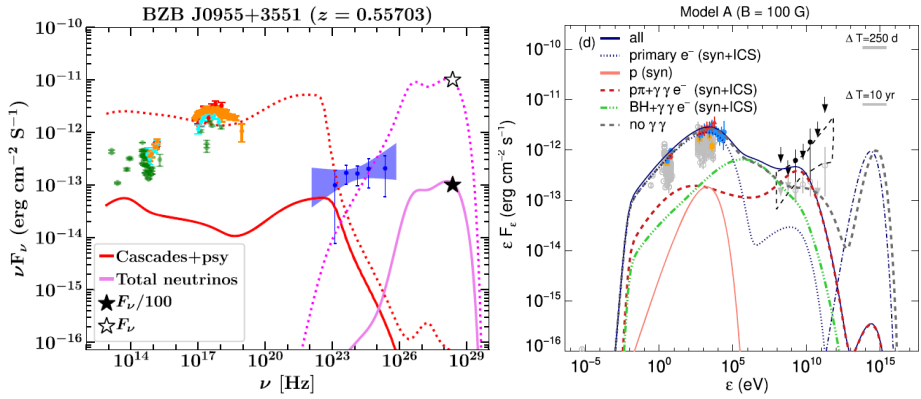


Fig. 5. Left: Model calculations of the neutrino flux and resulting $p\gamma$-induced cascade emission assuming a neutrino flux corresponding to 1 neutrino in the lifetime of the current neutrino alert system (dashed) and accounting for the Eddington bias by reducing the flux by a factor of 100 (solid), respectively (from [36]). Right: An example of a lepto-hadronic one-zone model SED fit + neutrino expectations from [39].

during the lifetime of the alert system (see Fig. 5, left panel). However, the actual neutrino flux may be significantly lower than this estimate since there are likely many blazars with similar properties, also exhibiting a similar neutrino flux, from which no neutrinos have been detected. This is known as the Eddington bias [38] and motivates a lowering of the neutrino flux estimate by a factor of ~ 100 (*i.e.*, assuming a probability of $\sim 1\%$ of detecting a HESE muon neutrino from this particular source). This Eddington-bias-corrected neutrino flux can be produced without violating any observational constraints and with moderate proton power requirements of $\lesssim 10^{47}$ erg s^{-1}. While a target photon field originating external to the jet is preferred, a co-moving target photon field cannot be ruled out, contrary to the case of TXS 0506+056. Also, due to the required dense target photon field, $\gamma\gamma$ absorption suppresses any co-spatially produced HE γ-ray emission, implying that no neutrino–γ-ray correlation is necessarily expected, as discussed above in Section 2.1 [36].

Various scenarios for the production of the multi-wavelength emission of 3HSP J095507.9+35510 and the IceCube-200107 event were explored in detail by [39]. They investigate one-zone lepto-hadronic models and also consider alternative scenarios, such as an intergalactic cascade scenario based on a high-energy cosmic-ray beam originating from the blazar jet, interacting with the intergalactic medium (see Fig. 5, right panel, for an example of a lepto-hadronic model fit). In agreement with the Eddington-bias argument mentioned above, they find a probability of ~ 1 — a few % of detecting 1 muon neutrino in 10 years from 3HSP J095507.9+35510 for the most optimistic lepto-hadronic models, in which they also predict strong internal $\gamma\gamma$ absorption and, hence, a suppression of HE and VHE γ-ray emission from the neutrino production region. The same issue was encountered by [40], who investigated both a hadronic (proton-synchrotron dominated) and a lepto-hadronic (SSC dominated γ-ray emission) scenario for this source. While moderate power requirements (of the order of the Eddington luminosity) were found for their hadronic model, their lepto-hadronic model required proton kinetic jet powers of $\sim 10^{51}$ erg s^{-1} which seem unrealistic.

3.5. IceCube-211125 and AT2021afpi/4FGL J0258.1+2030

Two potential counterparts were identified for the IceCube-211125 neutrino, with an estimated energy of $E \sim 117$ TeV (39% probability of being astrophysical). One is the young, narrow-line, He-rich nova AT2021afpi [41, 42], which was detected as an optical transient by the MASTER network of robotic telescopes [43]. It was found to be in an X-ray outburst around the time of the IceCube-211125 alert [44]. This would be the first such association of a classical nova outburst with a neutrino event.

The second potential counterpart is the *Fermi*-LAT source 4FGL J0258.1 +2030, a blazar of an unknown type with uncertain redshift ($z \sim 2.2$). It is consistent with a compact radio source in a high state, detected by the TELAMON program, which is also spatially coincident with another IceCube Bronze neutrino event, namely IceCube-191231A [45]. If this association is true, it would be the first-ever source of more than one single > 100 TeV neutrino, in which case the Eddington-bias argument would no longer apply. Unfortunately, there is very little information about this source at other wavelengths, and, to our knowledge, no theoretical modeling of this source and the potentially associated neutrino emission has been published.

3.6. IceCube-211208 and PKS 0735+17

IceCube-211208 was a track-like event with an estimated energy of $E \sim 172$ TeV with a relatively large statistical 90% localization error region of ~ 13 square degrees. The BL Lac type ISP blazar PKS 0735+17 ($z = 0.45$) is actually located slightly outside this error region, but was in a historical outburst in HE γ-rays (*Fermi*-LAT), X-rays (*Swift*-XRT), and optical — UV at the time of the neutrino alert (see Fig. 6, from [46]). In addition, the source is positionally consistent with the (rather large) localization error regions of a Baikal-GVD detected neutrino event ($E \sim 43$ TeV) 3.95 hours

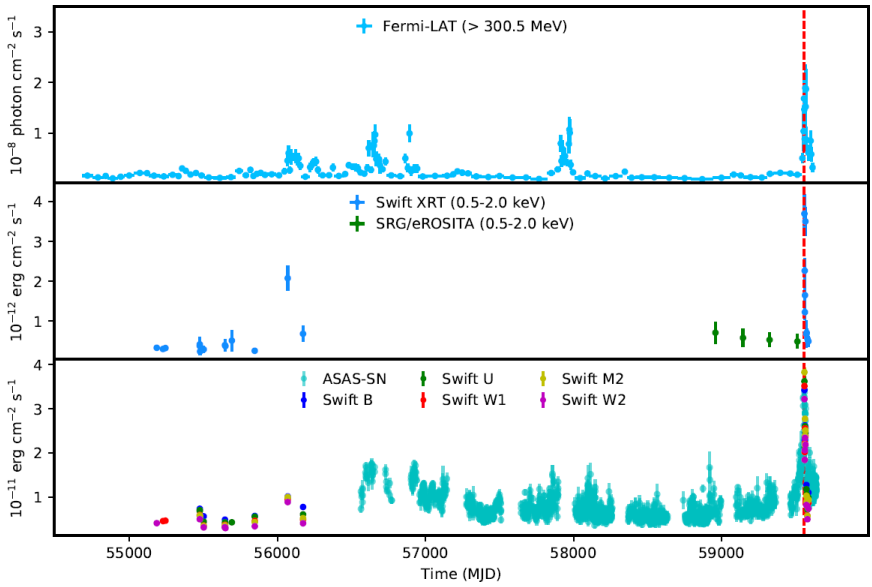


Fig. 6. (Color online) *Fermi*-LAT (top), X-ray (middle), and optical-UV (bottom) light curves of PKS 0735+17 (from [46]). The dashed vertical line (red) indicates the arrival time of IceCube-211208.

after the IceCube-211208 event [47], of a GeV neutrino detected 4 days earlier by the Baksan Underground Scintillation Telescope [48], as well as an $\sim$ 18 TeV neutrino a week after IceCube-211208, found in an archival search of data from the Km3NeT undersea neutrino detectors, currently under construction in the Mediterranean Sea [49]. It is this multitude of potential neutrino associations as well as the exceptional flaring state of PKS 0736+17 which drew the attention of the community to this source as a possible, prolific VHE neutrino source.

The authors of [46] investigated several lepto-hadronic single-zone emission scenarios for the multi-wavelength and neutrino emission from PKS 0735 +178. Their most optimistic scenario requires a jet with a kinetic luminosity close to the Eddington limit of the source, where relativistic protons interact with an external radiation field from the BLR (see Fig. 7), resulting in a 6.7% probability of detecting one muon/antimuon neutrino during a 3-week flaring period.

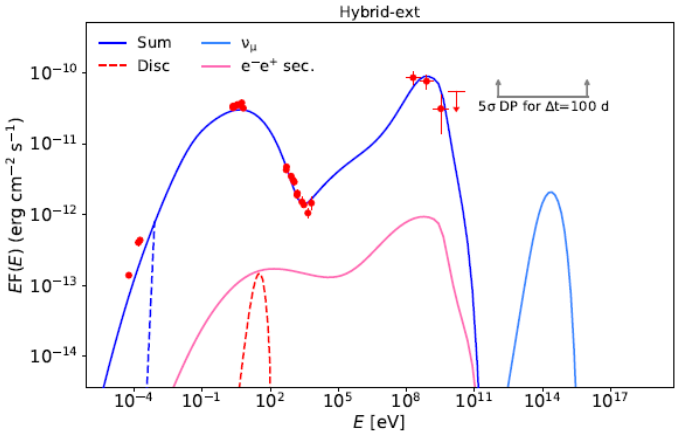


Fig. 7. SED of PKS 0735+17, along with the most favorable lepto-hadronic single-zone model fit, including a BLR radiation field as the target photon field for $p\gamma$ processes, from [46].

4. Summary and discussion

We have provided an overview of the physical conditions required in blazar jets to produce astrophysical VHE neutrinos detected by IceCube. We have demonstrated that protons need to be accelerated to $\gtrsim$ PeV energies, and a target photon field of UV/X-ray energies in the co-moving frame of the neutrino production regions is required. On energetics grounds, photon fields originating external to the jet are generally favored. Due to the dense target photon field required for efficient neutrino production, the emission

region is likely to be opaque to internal $\gamma\gamma$ absorption. Therefore, a correlation between neutrino production and γ-ray flaring activity is not necessarily expected. However, due to cascading of the internally absorbed HE and VHE γ-ray emission, enhanced X-ray to soft γ-ray is a natural byproduct of $p\gamma$-induced neutrino production. Another pattern that seems to emerge is the coincidence of radio brightening and the ejection of new jet components with neutrino emission, indicating that neutrino production is likely occurring within the central $\sim$ pc of the jet.

After the first relatively-high-confidence association between IceCube-170922A and TXS 0506+056, the number of potential associations between flaring blazars and IceCube neutrino alerts has been steadily increasing. We have provided a brief overview of the best-known associations and, where available, results of model calculations to reproduce coincident multi-wavelength and neutrino emission. In most cases, the detailed modeling results confirm the general expectations outlined in Section 2. In particular, it is often found that internal $\gamma\gamma$ absorption suppresses the co-spatially produced HE and VHE γ-ray emission and that neutrino production is accompanied by high states in X-rays and soft γ-rays. Also, in many cases, $p\gamma$ interactions with external radiation fields are indeed favored over co-moving (primary electron synchrotron) target photon fields.

A particularly exciting development is the possibility that at least one blazar (4FGL J0258.1+2030) could be associated with two IceCube neutrino events and PKS 0735+17 could be the counterpart not only to one IceCube event, but also to neutrino events detected by Baikal-GVD, Baksan, and KM3NeT. If such multiple associations are confirmed by future observations, it would allow for a more meaningful determination of VHE neutrino fluxes from such sources, and it would invalidate the Eddington-bias argument that models can be considered successful if they predict only a probability of a few % to detect a neutrino from a single source, accounting for the fact that there are many other sources with similar properties which may exhibit the same neutrino flux.

The work of M.B. is supported by the South African Research Chairs Initiative (SARChI) of the Department of Science and Innovation and the National Research Foundation[1] of South Africa through SARChI grant UID 64789. We thank Matthias Kadler, Viktor Sumida, Anna Franckowiak, Xavier Rodrigues, Maria Petropoulou, and Narek Sahakyan for granting us permission to use figures from their publications in this review, and for helpful comments on the manuscript.

[1] Any opinion, finding, and conclusion or recommendation expressed in this material is that of the authors, and the NRF does not accept any liability in this regard.

REFERENCES

[1] IceCube Collaboration (M.G. Aartsen *et al.*), «Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector», *Science* **342**, 1242856 (2013).

[2] M.G. Aartsen *et al.*, «First Observation of PeV-Energy Neutrinos with IceCube», *Phys. Rev. Lett.* **111**, 021103 (2013).

[3] F. Halzen, A. Kheirandish, in: F.W. Stecker (Ed.) «Neutrino Physics and Astrophysics», *World Scientific Publishing Company*, Singapore , in: G.G. Fazio (Ed.) «The Encyclopedia of Cosmology II», 2022.

[4] K. Murase, F.W. Stecker, in: F.W. Stecker (Ed.) «Neutrino Physics and Astrophysics», *World Scientific Publishing Company*, Singapore , in: G.G. Fazio (Ed.) «The Encyclopedia of Cosmology II», 2022.

[5] M. Böttcher, «Progress in Multi-Wavelength and Multi-Messenger Observations of Blazars and Theoretical Challenges», *Galaxies* **7**, 20 (2019).

[6] K. Murase, F. Oikonomou, M. Petropoulou, «Blazar Flares as an Origin of High-energy Cosmic Neutrinos?», *Astrophys. J.* **865**, 124 (2018).

[7] S. Garrappa *et al.*, «Investigation of Two *Fermi*-LAT Gamma-Ray Blazars Coincident with High-energy Neutrinos Detected by IceCube», *Astrophys. J.* **880**, 103 (2019).

[8] A. Franckowiak *et al.*, «Detectability of Life Using Oxygen on Pelagic Planets and Water Worlds», *Astrophys. J.* **893**, 163 (2020).

[9] P. Giommi *et al.*, «Dissecting the regions around IceCube high-energy neutrinos: growing evidence for the blazar connection», *Mon. Not. R. Astron. Soc.* **497**, 865 (2020).

[10] M. Böttcher, D. Harris, H. Krawczynski, «Relativistic Jets from Active Galactic Nuclei», *Wiley-VCH*, Berlin 2012.

[11] R.-Y. Liu, «Hadronuclear interpretation of a high-energy neutrino event coincident with a blazar flare», *Phys. Rev. D* **99**, 063008 (2019).

[12] P. Padovani *et al.*, «TXS 0506+056, the first cosmic neutrino source, is not a BL Lac», *Mon. Not. R. Astron. Soc.* **484**, L104 (2019).

[13] F. Tavecchio, G. Ghisellini, «Spine–sheath layer radiative interplay in subparsec-scale jets and the TeV emission from M87», *Mon. Not. R. Astron. Soc.* **385**, L98 (2008).

[14] C. Righi, F. Tavecchio, S. Inoue, «Neutrino emission from BL Lac objects: the role of radiatively inefficient accretion flows», *Mon. Not. R. Astron. Soc.* **483**, L127 (2019).

[15] S. Britzen *et al.*, «A cosmic collider: Was the IceCube neutrino generated in a precessing jet-jet interaction in TXS 0506+056?», *Astron. Astrophys.* **630**, A103 (2019), *Corrigendum ibid.* **632**, C3 (2019).

[16] A. Reimer, M. Böttcher, S. Buson, «Cascading Constraints from Neutrino-emitting Blazars: The Case of TXS 0506+056», *Astrophys. J.* **881**, 46 (2019).

[17] S.I. Stathopoulos *et al.*, «High-energy neutrinos from X-rays flares of blazars frequently observed by the *Swift* X-ray Telescope», *Mon. Not. R. Astron. Soc.* **510**, 4063 (2021).

[18] A. Plavin, Yu.Y. Kovalev, Yu.A. Kovalev, S.V. Troitsky, «Observational Evidence for the Origin of High-energy Neutrinos in Parsec-scale Nuclei of Radio-bright Active Galaxies», *Astrophys. J.* **894**, 101 (2020).

[19] A. Plavin, Yu.Y. Kovalev, Yu.A. Kovalev, S.V. Troitsky, «Directional Association of TeV to PeV Astrophysical Neutrinos with Radio Blazars», *Astrophys. J.* **908**, 157 (2021).

[20] M. Kadler *et al.*, «Coincidence of a high-fluence blazar outburst with a PeV-energy neutrino event», *Nature Phys.* **12**, 807 (2016).

[21] M. Kreter *et al.*, «On the Detection Potential of Blazar Flares for Current Neutrino Telescopes», *Astrophys. J.* **902**, 133 (2020).

[22] S. Gao, M. Pohl, W. Winter, «On the Direct Correlation between Gamma-Rays and PeV Neutrinos from Blazars», *Astrophys. J.* **843**, 109 (2017).

[23] M.G. Aartsen *et al.*, «Multimessenger observations of a flaring blazar coincident with high-energy neutrino IceCube-170922A», *Science* **361**, 1378 (2018).

[24] IceCube Collaboration (M.G. Aartsen *et al.*), «Neutrino emission from the direction of the blazar TXS 0506+056 prior to the IceCube-170922A alert», *Science* **361**, 147 (2018).

[25] M. Petropoulou *et al.*, «Multi-epoch Modeling of TXS 0506+056 and Implications for Long-term High-energy Neutrino Emission», *Astrophys. J.* **891**, 115 (2020).

[26] M. Cerruti *et al.*, «Leptohadronic single-zone models for the electromagnetic and neutrino emission of TXS 0506+056», *Mon. Not. R. Astron. Soc.* **483**, L12 (2019).

[27] S. Gao, A. Fedynitch, W. Winter, M. Pohl, «Modelling the coincident observation of a high-energy neutrino and a bright blazar flare», *Nature Astron.* **3**, 88 (2019).

[28] A. Keivani *et al.*, «A Multimessenger Picture of the Flaring Blazar TXS 0506+056: Implications for High-energy Neutrino Emission and Cosmic-Ray Acceleration», *Astrophys. J.* **864**, 84 (2018).

[29] V.Y.D. Sumida, A. Schutzer, A. Caproni, Z. Abraham, «The relativistic parsec-scale jets of the blazars TXS 0506+056 and PKS 0502+049 and their possible association with gamma-ray flares and neutrino production», *Mon. Not. R. Astron. Soc.* **509**, 1646 (2022).

[30] Fermi-LAT Collaboration (S. Abdollahi *et al.*), «Large Area Telescope Fourth Source Catalog», *Astrophys. J. Suppl. Ser.* **247**, 33 (2020).

[31] M. Fu *et al.*, talk at First Pan-African Astroparticle and Collider Physics Workshop (PACP2022), virtual, 21–23 March, 2022, https://indico.tlabs.ac.za/event/109/contributions/2246/

[32] X. Rodrigues *et al.*, «Multiwavelength and Neutrino Emission from Blazar PKS 1502+106», *Astrophys. J.* **912**, 54 (2021).

[33] F. Oikonomou *et al.*, «Multi-messenger emission from the parsec-scale jet of the flat-spectrum radio quasar PKS 1502+106 coincident with high-energy neutrino IceCube-190730A», *J. Cosmol. Astropart. Phys.* **2021**, 082 (2021).

[34] P. Giommi *et al.*, «3HSP J095507.9+355101: A flaring extreme blazar coincident in space and time with IceCube-200107A», *Astron. Astrophys.* **640**, L4 (2020).

[35] S. Paiano *et al.*, «The redshift and the host galaxy of the neutrino candidate 4FGL J0955.1+3551 (3HSP J095507.9+355101)», *Mon. Not. R. Astron. Soc.* **495**, L108 (2020).

[36] V.S. Paliya *et al.*, «Multifrequency Observations of the Candidate Neutrino-emitting Blazar BZB J0955+3551», *Astrophys. J.* **902**, 29 (2020).

[37] F. Krauß *et al.*, «Multimessenger observations of counterparts to IceCube-190331A», *Mon. Not. R. Astron. Soc.* **497**, 2553 (2020).

[38] N.L. Strotjohann, M. Kowalski, A. Franckowiak, «Eddington bias for cosmic neutrino sources», *Astron. Astrophys.* **622**, L9 (2019).

[39] M. Petropoulou *et al.*, «Comprehensive Multimessenger Modeling of the Extreme Blazar 3HSP J095507.9+355101 and Predictions for IceCube», *Astrophys. J.* **899**, 113 (2020).

[40] S. Gasparyan, D. Bégué, N. Sahakyan, «Time-dependent lepto-hadronic modelling of the emission from blazar jets with: the case of TXS 0506+056, 3HSP J095507.9+355101, and 3C 279», *Mon. Not. R. Astron. Soc.* **509**, 2102 (2022).

[41] R. Stein *et al.*, «Classification of AT2021afpi, a possible counterpart to IC211125A, as a classical nova», *The Astronomer's Telegram* No. **15069** (2021).

[42] K. Taguchi *et al.*, «Spectroscopic and Photometric follow-up of MASTER OT J030227.28+191754.5: a Possible "Narrow-lined He Nova" or Large-amplitude Dwarf Nova», *The Astronomer's Telegram* No. **15072** (2021).

[43] K. Zhirkov *et al.*, «MASTER OT J030227.28+191754.5 — 10 mag outburst detection during an inspect of IceCube-211125A», *The Astronomer's Telegram* No. **15067** (2021).

[44] V.S. Paliya, «Classical nova AT2021afpi, a possible EM counterpart to IC211125A, is in X-ray outburst», *The Astronomer's Telegram* No. **15073** (2021).

[45] TELAMON Team (M. Kadler *et al.*), «TELAMON Detection of a High-State Compact Radio Source and Gamma-Bright AGN Positionally Coincident with Two IceCube Bronze Neutrino Events (IceCube-191231A and IceCube-211125A)», *The Astronomer's Telegram* No. **15076** (2021).

[46] N. Sahakyan *et al.*, arXiv:2204.05060 [astro-ph.HE], submitted to *Mon. Not. R. Astron. Soc.*.

[47] Baikal-GVD Collaboration (Zh.-A. Dhilkibaev, O. Suvarova), «Baikal-GVD observation of a high-energy neutrino candidate event from the blazar PKS 0735+17 at the day of the IceCube-211208A neutrino alert from the same direction», *The Astronomer's Telegram* No. **15112** (2021).

[48] Baksan Underground Scintillation Telescope Group (V.B. Petkov, Yu.F. Novoseltsev, R.V. Novoseltseva), «Baksan Underground Scintillation Telescope observation of a GeV neutrino candidate event at the time of a gamma-ray flare of the blazar PKS 0735+17, a possible source of coinciding IceCube and Baikal high-energy neutrinos», *The Astronomer's Telegram* No. **15143** (2021).

[49] F. Filippini *et al.*, «Search for neutrino counterpart to the blazar PKS0735+178 potentially associated with IceCube-211208A and Baikal-GVD-211208A with the KM3NeT neutrino detectors», *The Astronomer's Telegram* No. **15290** (2022).