QCD IN THE CORES OF NEUTRON STARS*

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I discuss why the state-of-the art perturbative QCD calculations of the equation of state at large chemical potential that are reliable at asymptotically high densities constrain the same equation of state at neutron-star densities. I describe how these theoretical calculations affect the EOS at lower density. I argue that the *ab initio* calculations in QCD offer significant information about the equation of state of the neutron-star matter, which is complementary to the current astrophysical observations.

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1. Introduction

The equation of state (EOS) of the dense matter at zero temperature is a necessary input for the neutron-stars (NS) physics. Theoretical calculations of the EOS can be done only at the two opposite (low- and high-density) limits. At the low-density limit, the matter can be described within the chiral effective field theory (CET) [1, 2]. Those calculations are reliable up to around nuclear saturation density $n_\mathrm{s} = 0.16/\mathrm{fm}^3$. On the other hand, we can access the EOS using perturbative Quantum Chromodynamics (pQCD) at the asymptotically high densities, above $\sim 40\, n_\mathrm{s}$ [3, 4]. Central densities of maximally massive neutron stars are around 4–8 n_s, which is not reachable within CET or pQCD. Therefore, there are no tools in our possession to compute the EOS of the cores of NS from the first principles.

However, we can obtain empirical access to the cores of NSs using recent astrophysical observations. The most important probes of NS physics are the discovery of massive NSs [5–7], mass–radius measurements [8, 9], and the gravitational-wave and multi-messenger astronomy [10, 11]. Utilizing all constraints coming from astrophysical observation as well as first principle

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calculations narrows down dramatically the range of possible EOSs, which allows us to use the densest objects in the Universe to test independently various beyond Standard Model scenarios and/or general relativity.

The majority of the EOS studies extrapolate CET EOS up to NS densities of 5–10 $n_{\rm s}$ and conditioning it with observational inputs. The results differ from the works that include high-density limit and interpolate between two orders of magnitude. The qualitative difference is in the softening of the EOS happening around $\epsilon \sim 750$ MeV/fm^{-3}, which can be interpreted as quark-matter cores inside the most massive NS [12].

In this work, I answer why and how the pQCD input offers significant information about the EOS at NS densities. I find that the pQCD input propagates non-trivial constraints all the way down to 2.2 $n_{\rm s}$ just by using solely thermodynamic stability, consistency, and causality [13]. In addition, the complementariness of the pQCD input to the astrophysical observations was studied in [14]. I show that pQCD is responsible for the softening of the EOS at the NS densities. Therefore, it is essential to include pQCD input in any inference study of the EOS.

2. Setup

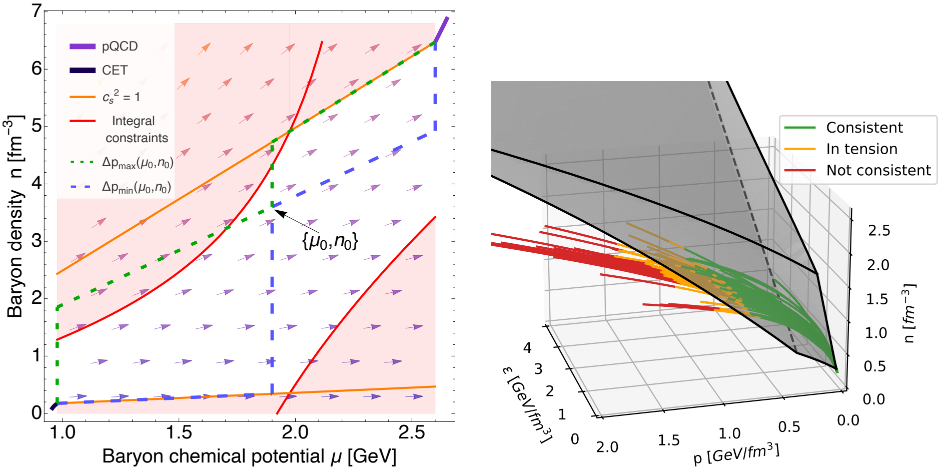
All technical details as well as analytical formulas are presented in [13]. In this section, I describe the conditions I use, in particular, stability, consistency, and causality, and the resulting propagation of the pQCD input down to lower densities. Let us start with the baryon density n as a function of the chemical potential μ as shown in Fig. 1 (a). The goal is to find all possible lines that connect the endpoint of CET results (dark blue line in the bottom left corner) with the first point of pQCD calculations (purple line in the upper right corner) using 3 conditions.

The first condition is thermodynamic stability, which implies concavity of the grand canonical potential $\partial_\mu^2 \Omega(\mu) \leq 0$. At zero temperature, $\Omega(\mu) = -p(\mu)$, which implies that the number density is a monotonically increasing function of the chemical potential $\partial_\mu n(\mu) \geq 0$.

The second condition is causality — the sound speed cannot exceed the speed of light $c_{\rm s}^2 \leq 1$. This provides constraints on the first derivative of the number density with respect to the chemical potential

$$c_{\rm s}^{-2} = \frac{\mu}{n}\frac{\partial n}{\partial \mu} \leq 1 \,. \tag{1}$$

For each point on the μ–n plane, we can calculate the least allowed slope coming from causality, which is represented by the arrows in Fig. 1 (a). This cuts the upper (lower) region of the plane, because any points from the area above (below) the solid gray/orange lines, $c_{\rm s}^2 = 1$, cannot be connected to pQCD (CET) in a casual way.



(a) (b)

Fig. 1. (Colour on-line) (a) Baryon density as a function of chemical potential. Simultaneous fulfilment of thermodynamic consistency, stability, and causality narrows down the allowed region (white area) of the EOS. (b) Zero temperature EOSs from CompOSE database plotted with the allowed area (gray shape) arising from the new constraints in ϵ–p–n space. Consistent/in tension/not consistent means that EOS is consistent with integral constraints for all/some/none X values in a range [1,4].

The third condition is thermodynamic consistency. In addition to n and μ, we need to match pressure p at the low- and high-density limits. The pressure is given by the integral of the number density

$$\int_{\mu_{\rm CET}}^{\mu_{\rm QCD}} n(\mu)\mathrm{d}\mu = p_{\rm QCD} - p_{\rm CET} = \Delta p\,. \tag{2}$$

This implies that the area under the curve for any EOS is fixed by our input parameters. For each arbitrary point μ_0, n_0, we can construct the EOS that maximize/minimize the area under the curve $\Delta p_{\rm max\,/\,min}(\mu_0, n_0)$ shown as a short-dashed green/long-dashed blue lines in Fig. 1 (a). If $\Delta p_{\rm max}(\mu_0, n_0) < \Delta p$, then any EOS that goes through the point μ_0, n_0 does not have enough area under the curve. This discards the region in the lower right corner in Fig. 1 (a) under the red line called “integral constraints”. If $\Delta p_{\rm min}(\mu_0, n_0) > \Delta p$ then any EOS that goes through the point μ_0, n_0 has too much area under the curve. This cuts area in the upper left corner above the thin solid black/red lines. The integral constraints can be obtained without any assumptions of the interpolation function in a completely general and analytical way.

We can map the allowed region from μ–n to ϵ–p plane. The results of such mapping are shown in Fig. 2. The green envelope corresponds to the white area in Fig. 1 (a) restricted by the causality and the integral constraints. The shapes of allowed regions with and without pQCD are shown for the fixed number density $n = 2$, 3, 5, and $10\, n_{\mathrm{s}}$. This explicitly shows how the pQCD input can propagate information down to lower density starting from $2.2\, n_{\mathrm{s}}$. Strikingly, at $5\, n_{\mathrm{s}}$, it excludes 75% of the otherwise allowed area.

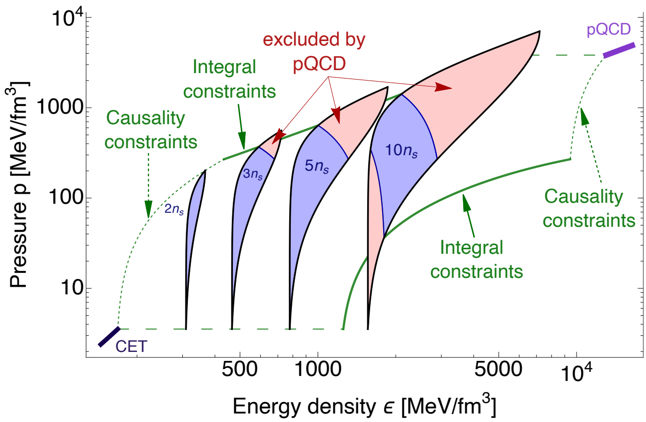


Fig. 2. (Colour on-line) Constraints on the ϵ–p plane coming from low- and high-density limits. Shapes outlined by solid black line are the allowed areas for fixed number density without pQCD. Blue/dark gray shapes are allowed regions after imposing the pQCD input.

Using the new constraints, we can check the consistency of publicly available EOSs. Results for all zero temperature EoSs in β-equilibrium from the public CompOSE database [15, 16] are shown in Fig. 1 (b). Almost all of the EOSs start to be inconsistent with pQCD input at some density within the provided range.

3. Bayesian inference of EOS

With the construction described above, we can propagate information from *ab initio* QCD calculations down to NS densities, where we already have constraints from astrophysical observations. To understand if the new constraints from pQCD go beyond the constraints coming from the NS measurements, we construct a Bayesian-inference framework. This was done in [4], where we generate a large ensemble of different EOSs using Gaussian-process regression. We anchor the ensemble to CET calculations and extrapolate it up to $10 n_{\mathrm{s}}$, where we impose pQCD input as a blue/dark gray shape from Fig. 2. We condition the ensemble sequentially with the astrophysical observations. With this setup, we can turn on and off the pQCD input in order to study its effect on our posterior distribution after imposing the astrophysical observation.

The results are present in Fig. 3. The reduction of the pressure (green/gray arrow on the right plot), which is caused by the QCD input, happens before the density reaches its maximal central value. In other words, the prediction of QCD input is the softening of the EOS that happens inside the most massive neutron stars.

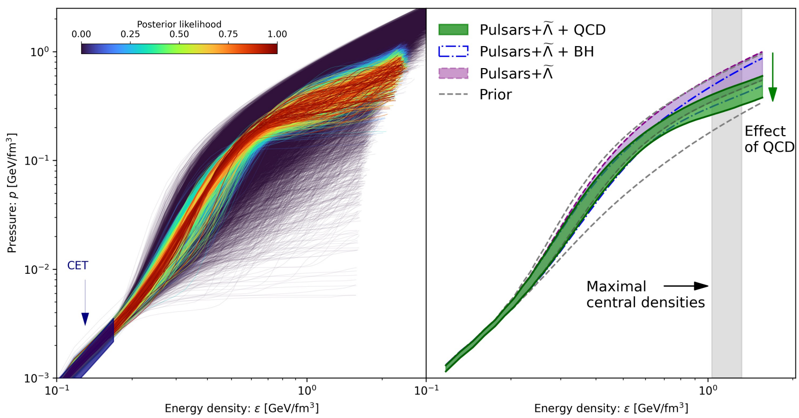


Fig. 3. (Colour on-line) Left plot shows the sample of 10 k EOSs. The colouring represents the likelihood after imposing all observations as well as pQCD input. Right plot shows 67%-credible intervals conditioned with the different astrophysical observations and high-density limit. The gray band shows a 67%-credible interval for the maximal central energies density reached in NSs.

4. Conclusion

In this work, I show how QCD calculations at the asymptotically high densities can propagate information down to lower densities using solely thermodynamic consistency, stability, and causality. This information offers significant constraints to the EOS at NS density, which is complementary to the current astrophysical observations. In addition, I show that the prediction of QCD input is the softening of the EOS that happens in the most massive NSs. An easy-to-use python script is provided to check consistency of the EOS with the pQCD input, available on Github [17].

In order to achieve an accurate determination of the EOS, it is crucial to utilize all available controlled measurements and theoretical calculations. This strategy either helps us to understand the matter of the densest objects in the Universe or find a discrepancy between different inputs, which allows us to use NS as a tool for fundamental discoveries.

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