TRANSPORT PROPERTIES OF THE QCD MEDIUM*

Jacopo Ghiglieri

SUBATECH, Université de Nantes, IMT Atlantique, IN2P3/CNRS
4 rue Alfred Kastler, La Chantrerie BP 20722, 44307 Nantes, France

Received 31 July 2022, accepted 21 August 2022,
published online 14 December 2022

I present an overview of recent developments in the microscopic description of the quark–gluon plasma. I will concentrate on the *medium-induced emission* and *transverse momentum broadening*. These are two key ingredients of the theory of jet modifications in the QCD medium and of the kinetic theory used for transport and thermalisation. The main focus is on the progress towards a better understanding of theory and its uncertainties.

DOI:10.5506/APhysPolBSupp.16.1-A11

1. Introduction

As per the abstract, my main driver in discussing recent developments will be connecting the microscopic description to a better understanding and control of theory uncertainties. For a connection of these developments to jet quenching data — summarised at this conference in [1, 2] — I refer to [3]. Thermalisation and hydrodinamisation have been discussed in [4].

2. Medium-induced emission (MIE)

Hard jet partons of energy E much larger than the temperature T propagating through the QCD medium experience frequent soft interactions with the medium, which can eventually source collinear, medium-induced radiation. Its long formation time makes it sensitive to the Landau–Pomeranchuk–Migdal (LPM) effect, *i.e.* the quantum-mechanical interference of many soft scatterings. These effective $1 \leftrightarrow 2$ processes are not only the key in jet modification; through their number-changing nature, they ensure chemical equilibration and energy transport in *bottom-up thermalisation* [5].

The celebrated BDMPS-Z [6, 7] MIE probability, *i.e.*

$$\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{\alpha_\mathrm{s} P_{1\to 2}(x)}{[x(1-x)E]^2} \mathrm{Re} \int\limits_{t_1<t_2} \mathrm{d}t_1 \mathrm{d}t_2 \boldsymbol{\nabla}_{\boldsymbol{b}_2} \cdot \boldsymbol{\nabla}_{\boldsymbol{b}_1} \left[\left\langle \boldsymbol{b}_2, t_2 | \boldsymbol{b}_1, t_1 \right\rangle_{\boldsymbol{b}_2=0}^{\boldsymbol{b}_1=0} - \mathrm{vac} \right] \quad (1)$$

* Presented at the 29th International Conference on Ultrarelativistic Nucleus–Nucleus Collisions: Quark Matter 2022, Kraków, Poland, 4–10 April, 2022.

is factorised in a DGLAP splitting kernel $P_{1\to 2}(x)$ multiplying a propagator $\langle \boldsymbol{b}_2, t_2|\boldsymbol{b}_1, t_1\rangle$ that describes diffusion in transverse position space from the emission in the amplitude at time t_1 and vanishing $\boldsymbol{b}$ to the emission in the conjugate amplitude at time t_2. This propagator is a Green's function of

$$\mathcal{H} = -\frac{\nabla_{\boldsymbol{b}}^2}{2x(1-x)E} + \sum_i \frac{m_i^2}{2E_i} - i\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b})\,. \tag{2}$$

This 2D Hamiltonian has a real kinetic term with in-medium masses m_i. The imaginary part is the *scattering kernel.* It encodes jet–medium interactions and it is tightly related to the transverse momentum broadening (TMB).

Determining the propagator from Eq. (2) is difficult. Historically, approaches concentrated on a few limiting cases. For thin media, one can truncate the LPM resummation series at first order in the *opacity expansion* [8], while for thick media, one can perform the *harmonic oscillator approximation*, introducing the *momentum broadening coefficient* $\hat{q}$, *i.e.* $\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b}) \approx \mathcal{C}_{\mathrm{HO}} \equiv \frac{\hat{q}}{4}[b^2 + (xb)^2 + ((1-x)b)^2]$ [6]. The determination of the propagator simplifies also in an infinite, static medium [9].

The *improved opacity expansion*, introduced in [10, 11], is an economical, analytical prescription for overcoming these limiting cases. It corresponds to treating the non-harmonic parts of the scattering kernel as perturbations, *i.e.* $\mathcal{C} = \mathcal{C}_{\mathrm{HO}} + [\mathcal{C} - \mathcal{C}_{\mathrm{HO}}]$. In this way, the Coulomb logarithm, $\mathcal{C}(\boldsymbol{b}) \propto b^2 \ln(b)$, is captured, thus incorporating the rarer harder *Molière scatterings.* This new prescription was recently tested against new numerical determinations of the full propagator, obtained in [12–14], finding good, 10% or better agreement over its validity range, as shown in Fig. 1.

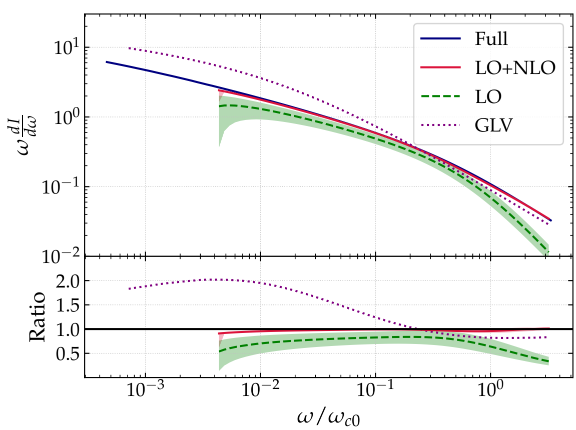

Fig. 1. The solid line is the numerical solution from [12, 13]. The LO+NLO is the improved opacity expansion, LO is the harmonic oscillator, and GLV the first order in opacity. Figure taken from [11].

3. Transverse momentum broadening

The discussion so far was agnostic to the specifics of the TMB kernel $\mathcal{C}$ and associated probability $\mathcal{P}(k_\perp)$, other than featuring a $1/k_\perp^4$ Coulomb tail in the UV and a diffusive Gaussian in the IR. If we were to determine the TMB kernel perturbatively, we would run into the known issues associated with the Linde problem: soft gluons ($\omega \ll T$) are *classical* high-occupancy modes, distributed on the T/ω IR tail of the Bose distribution. As the expansion parameter becomes g^2T/ω, convergence can be seriously hampered.

A breakthrough came from the realisation in [15] that these soft classical modes at space-like separations become Euclidean. As $\mathcal{C}$ is determined from Wilson lines at space-like separations, the large-distance contribution $b \gtrsim 1/gT$ can be determined non-perturbatively using the dimensionally-reduced theory on the lattice [16, 17]. At shorter distance, one can use pQCD and merge the two [18–20][1].

The resulting kernel is shown in Fig. 2 from [19]: a transition from $1/q_\perp^4$ Coulomb in the UV into a non-perturbative $1/q_\perp^3$ behaviour in the IR takes place, providing the bare minimum of "screening" to make $\hat{q}$, the second moment of the kernel, IR finite. In the $q_\perp \sim gT$ range, the non-perturbative curve differs appreciably from either the LO or NLO curves. References [18–20] further analysed the impact of the non-perturbative kernel on MIEs, both for infinite and finite media. They find that an $\mathcal{O}(1)$ difference between the MIE rates with NLO and non-perturbative kernels, a much larger effect than the improved opacity expansion compared to the full numerical propagator. See [24–29] for other non-perturbative approaches to $\hat{q}$ and transport.

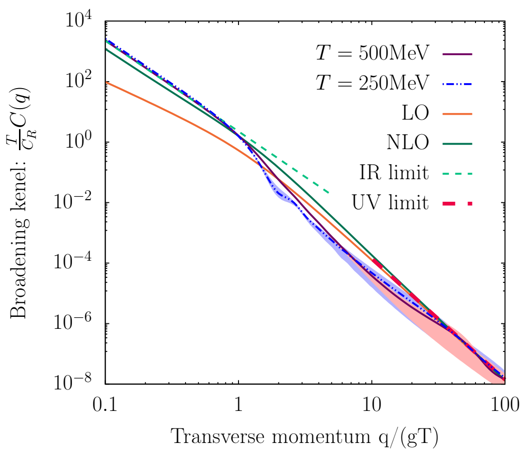

Fig. 2. The LO perturbative line comes from [30, 31], the NLO includes the $\mathcal{O}(g)$ corrections from [15] following [32]. Figure taken from [19].

[1] The same method is also being applied to in-medium masses [21–23].

Let us now shift to *quantum corrections*. In [33, 34], it was pointed out that radiative corrections to TMB from the recoil off the radiated gluon are responsible for a double-log enhanced, $\mathcal{O}(\alpha_\mathrm{s})$ correction to $\hat{q}$, arising from soft and collinear logarithms in the single-scattering regime, *i.e.*

$$\delta\hat{q} = \frac{\alpha_\mathrm{s} N_\mathrm{c}}{\pi}\hat{q}_0 \int_{\tau_0}^{\tau} \frac{\mathrm{d}\tau}{\tau} \int_{\hat{q}_0\tau}^{k_\perp^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,, \tag{3}$$

where the $\hat{q}_0\tau$ boundary keeps the phase space away from the $k_\perp^2 \sim \hat{q}\tau$ multiple scattering regime. This double-log correction can be understood as renormalizing $\hat{q}_0$, the LO value of $\hat{q}$. By promoting $\hat{q}_0$ to a variable and moving it (and the coupling) within the double integral, one obtains the resummation equation for these logs [35]. It was solved numerically and semi-analytically in [36–38], finding how an initial $\hat{q}(\tau_0, k_{\perp\,0}^2)$ evolves to a $\hat{q}(\tau, k_\perp^2)$ resumming arbitrary numbers of long duration, quantum fluctuations. One of the main results is that the non-local nature of these fluctuations affects the UV tail of the TMB probability, shifting it from its tree-level $\mathcal{P} \propto k_\perp^{-4}$ Coulomb form to a less steep $\mathcal{P} \propto k_\perp^{-4+2\alpha_\mathrm{s}N_\mathrm{c}/\pi}$. This larger probability for wider-angle scatterings corresponds to more efficient diffusion.

These quantum, radiative corrections are universal: as shown in [35, 39, 40], they also arise in the case of a double MIE with overlapping formation times in the soft limit. Over the past years, there has been an ongoing effort [41–46] to determine all these double-splitting real and virtual corrections, beyond the soft limit. The underlying goal is understanding whether the assumed Markovian nature of the MIE kernel holds when used to construct in-medium cascades. In the most recent developments [45, 47, 48], it was shown that, with some caveats, also the single-logarithmic radiative corrections determined in [33] are universal, in that they apply also to double splitting, opening a promising pathway for their resummation.

4. Kinetic theory, transport and thermalisation

As mentioned, MIE and TMB are the key ingredients of the kinetic description of QCD media. They are complemented by drag, longitudinal momentum broadening, and identity-change processes in the leading-order effective kinetic theory of QCD [49]. We refer to [50–54] for recent applications of the kinetic framework to jet modifications. Here, we instead connect the previous sections with transport coefficients and thermalisation.

Let us consider the shear viscosity η, which damps flow gradients. It is then clear that microscopic processes that isotropise momentum are dominant contributions. Hence, the direct isotropizing effect of TMB is more important for η than its indirect effect as the driver of MIEs, as seen also in the LO determination of η [55]. These were recently extended to (almost) NLO in [56]. These corrections are large — they reduce η/s by a factor of a few in the phenomenological $T \sim$ few T_c range — and are dominated by the $\mathcal{O}(g)$ classical corrections to $\hat{q}$ of [15] shown in Fig. 2. What should we make of this? Recently, it was pointed out in [57] that the problem might lie in the LO determination of TMB, which then affects η at LO. If that determination has excessive screening — see Fig. 2 — it underestimates broadening, potentially explaining qualitatively this LO–NLO discrepancy.

Several applications of kinetic theory to thermalisation have been discussed at this conference [58–65]. Can similarly problematic perturbative expansions arise in these kinetic frameworks? What are the systematics of extrapolations to $\alpha_s = 0.3$ ($g \approx 2$)? In [66, 67], NLO corrections to thermalisation for isotropic systems have been presented. Figure 3 shows the LO and NLO thermalisation times for an overoccupied initial condition. Two different NLO collision operators have been constructed, which resum differently higher-order effects. Their spread, indicated by the shaded band, is a proxy for the size of even higher-order corrections. This band is smaller than its spread from the LO result, as expected for a convergent expansion. Moreover, the extrapolation to intermediate coupling seems controlled, with a 40% correction for $g^2N_c = 10$. However, these isotropic systems are by their nature insensitive to the isotropizing effect of TMB, which we argued to play a determining role in corrections to transport. It remains then to be understood how reliable the extrapolation could be in situations typical of heavy-ion collisions with anisotropic initial conditions and expansions.

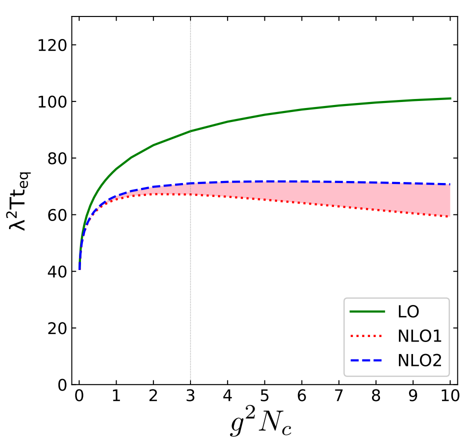

Fig. 3. Thermalisation time as a function of the coupling $\lambda = g^2N_c$ for an overoccupied initial condition. Figure adapted from [66].

In these systems, plasma instabilities [68–70] — another classical phenomenon — prevent at the moment consistent LO kinetic treatments. Recently, the instability-subtracted TMB kernel, together with a recipe for dealing with the unstable modes, was provided in [71], finding that anisotropy reduces the scattering kernel in the QGP phase. In the earlier glasma phase, large anisotropic TMB effects have been reported in [72–76].

5. Summary

The reviewed advances in the microscopic description of QCD media are instrumental in a better quantification of theory uncertainties and in narrowing the gap between the QCD Lagrangian and phenomenology.

REFERENCES

[1] L. Cunqueiro, talk presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[2] M. Rybar, talk presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[3] J.T. Brewer, talk presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[4] M. Spaliński, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A9 (2023), this issue.
[5] R. Baier, A.H. Mueller, D. Schiff, D. Son, *Phys. Lett. B* **502**, 51 (2001).
[6] R. Baier *et al.*, *Nucl. Phys. B* **483**, 291 (1997).
[7] B. Zakharov, *JETP Lett.* **63**, 952 (1996).
[8] M. Gyulassy, P. Levai, I. Vitev, *Nucl. Phys. B* **571**, 197 (2000).
[9] P.B. Arnold, G.D. Moore, L.G. Yaffe, *J. High Energy Phys.* **0206**, 030 (2002).
[10] Y. Mehtar-Tani, K. Tywoniuk, *J. High Energy Phys.* **2006**, 187 (2020).
[11] J.a. Barata, Y. Mehtar-Tani, A. Soto-Ontoso, K. Tywoniuk, *J. High Energy Phys.* **2109**, 153 (2021).
[12] C. Andres, L. Apolinário, F. Dominguez, *J. High Energy Phys.* **2007**, 114 (2020).
[13] C. Andres, F. Dominguez, M. Gonzalez Martinez, *J. High Energy Phys.* **2103**, 102 (2021).
[14] C. Andres, talk presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[15] S. Caron-Huot, *Phys. Rev. D* **79**, 065039 (2009).
[16] M. Panero, K. Rummukainen, A. Schäfer, *Phys. Rev. Lett.* **112**, 162001 (2014).
[17] G.D. Moore, N. Schlusser, *Phys. Rev. D* **101**, 014505 (2020); *Erratum ibid.* **101**, 059903 (2020).
[18] G.D. Moore, S. Schlichting, N. Schlusser, I. Soudi, *J. High Energy Phys.* **1010**, 059 (2021).
[19] S. Schlichting, I. Soudi, *Phys. Rev. D* **105**, 076002 (2022).

[20] S. Schlichting, I. Soudi, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A121 (2023), this issue.
[21] G.D. Moore, N. Schlusser, *Phys. Rev. D* **102**, 094512 (2020).
[22] J. Ghiglieri, G.D. Moore, P. Schicho, N. Schlusser, *J. High Energy Phys.* **2202**, 058 (2022).
[23] P. Schicho, talk presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[24] W. Florkowski *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A38 (2023), this issue.
[25] I. Grishmanovskii, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[26] A. Kumar, A. Majumder, J.H. Weber, arXiv:2010.14463 [hep-lat].
[27] I. Grishmanovskii *et al.*, *Phys. Rev. C* **106**, 014903 (2022), arXiv:2204.01561 [nucl-th].
[28] O. Soloveva, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[29] O. Soloveva, J. Aichelin, E. Bratkovskaya, *Phys. Rev. D* **105**, 054011 (2022).
[30] P. Aurenche, F. Gelis, H. Zaraket, *J. High Energy Phys.* **0205**, 043 (2002).
[31] P.B. Arnold, W. Xiao, *Phys. Rev. D* **78**, 125008 (2008).
[32] J. Ghiglieri, H. Kim, *J. High Energy Phys.* **1812**, 049 (2018).
[33] T. Liou, A. Mueller, B. Wu, *Nucl. Phys. A* **916**, 102 (2013).
[34] J.-P. Blaizot, F. Dominguez, E. Iancu, Y. Mehtar-Tani, *J. High Energy Phys.* **1406**, 075 (2014).
[35] E. Iancu, *J. High Energy Phys.* **1410**, 095 (2014).
[36] P. Caucal, Y. Mehtar-Tani, arXiv:2109.12041 [hep-ph].
[37] P. Caucal, Y. Mehtar-Tani, arXiv:2203.09407 [hep-ph].
[38] Y. Mehtar-Tani, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[39] J.-P. Blaizot, Y. Mehtar-Tani, *Nucl. Phys. A* **929**, 202 (2014).
[40] B. Wu, *J. High Energy Phys.* **1412**, 081 (2014).
[41] P. Arnold, S. Iqbal, *J. High Energy Phys.* **1504**, 070 (2015).
[42] P. Arnold, H.-C. Chang, S. Iqbal, *J. High Energy Phys.* **1609**, 078 (2016).
[43] P. Arnold, H.-C. Chang, S. Iqbal, *J. High Energy Phys.* **1610**, 100 (2016).
[44] P. Arnold, T. Gorda, S. Iqbal, *J. High Energy Phys.* **2011**, 053 (2020).
[45] P. Arnold, T. Gorda, S. Iqbal, *J. High Energy Phys.* **2204**, 085 (2022).
[46] P. Arnold, O. Elgedawy, arXiv:2202.04662 [hep-ph].
[47] P. Arnold, *J. High Energy Phys.* **2203**, 134 (2022).
[48] S. Iqbal, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[49] P.B. Arnold, G.D. Moore, L.G. Yaffe, *J. High Energy Phys.* **0301**, 030 (2003).
[50] S. Schlichting, I. Soudi, *J. High Energy Phys.* **2107**, 077 (2021).
[51] T. Dai, J.-F. Paquet, D. Teaney, S.A. Bass, *Phys. Rev. C* **105**, 034905 (2022).
[52] W. Ke, X.-N. Wang, *J. High Energy Phys.* **2105**, 041 (2021).

[53] W. Ke, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[54] T. Dai, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[55] P.B. Arnold, G.D. Moore, L.G. Yaffe, *J. High Energy Phys.* **0305**, 051 (2003).
[56] J. Ghiglieri, G.D. Moore, D. Teaney, *J. High Energy Phys.* **1803**, 179 (2018).
[57] B. Müller, *Phys. Rev. D* **104**, L071501 (2021).
[58] X. Du, S. Schlichting, *Phys. Rev. D* **104**, 054011 (2021).
[59] X. Du, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[60] J. Brewer, B. Scheihing-Hitschfeld, Y. Yin, *J. High Energy Phys.* **2205**, 145 (2022).
[61] A.N. Mikheev, A. Mazeliauskas, J. Berges, *Phys. Rev. D* **105**, 116025 (2022).
[62] B.S. Scheihing-Hitschfeld, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[63] S. Plumari *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A27 (2023), this issue.
[64] D. Almaalol, A. Kurkela, M. Strickland, *Phys. Rev. Lett.* **125**, 122302 (2020).
[65] D. Almaalol *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A28 (2023), this issue.
[66] Y. Fu, J. Ghiglieri, S. Iqbal, A. Kurkela, *Phys. Rev. D* **105**, 054031 (2022).
[67] Y. Fu *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A31 (2023), this issue.
[68] S. Mrówczyński, *Phys. Lett. B* **314**, 118 (1993).
[69] P. Romatschke, M. Strickland, *Phys. Rev. D* **68**, 036004 (2003).
[70] P.B. Arnold, J. Lenaghan, G.D. Moore, *J. High Energy Phys.* **0308**, 002 (2003).
[71] S. Hauksson, S. Jeon, C. Gale, *Phys. Rev. C* **105**, 014914 (2022).
[72] A. Ipp, D.I. Müller, D. Schuh, *Phys. Lett. B* **810**, 135810 (2020).
[73] M.E. Carrington, A. Czajka, S. Mrówczyński, *Phys. Rev. C* **105**, 064910 (2022), `arXiv:2202.00357 [nucl-th]`.
[74] A. Czajka, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[75] D. Schuh, poster presented at the Quark Matter 2022 Conference, Kraków, Poland, 4–10 April, 2022, not included in the proceedings.
[76] M.E. Carrington, A. Czajka, St. Mrówczyński, *Acta Phys. Pol. B Proc. Suppl.* **16**, 1-A118 (2023), this issue.