HEAVY QUARKONIUM DYNAMICS IN THE QGP WITH A QUANTUM MASTER EQUATION APPROACH*

Stéphane Delorme, Thierry Gousset, Roland Katz
Pol-Bernard Gossiaux

Nantes Université, IMT Atlantique, CNRS/IN2P3, SUBATECH
44307, Nantes, France

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In recent years, a significant theoretical effort has been made towards a dynamical description of quarkonia inside the Quark–Gluon Plasma (QGP), using the open quantum systems formalism. In this framework, one can get a real-time description of a quantum system (here the quarkonium) in interaction with a thermal bath (the QGP) by integrating out the bath degrees of freedom and studying the system reduced density matrix. We investigate the real-time dynamics of a correlated heavy quark–antiquark pair inside the QGP using the novel coupled quantum master equations derived from the first QCD principles. The equations are solved numerically in 1D to lessen computing costs and are used for the first time to gain insight into the dynamics in both a static and evolving medium following a Björken-like temperature evolution with several initial conditions.

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1. Quarkonia in the Quark–Gluon Plasma

Quarkonia suppression is one of the key observables for the study of the Quark–Gluon Plasma. Different mechanisms are involved in the dissociation of heavy quark–antiquark pairs in the medium. At non-zero temperature, the interaction between the two quarks of a pair is progressively screened as temperature rises and more color charges are present. Additionally, the pair may scatter on light quarks and gluons present in the QGP, which can lead to the dissociation of the pair. Those two effects can be described by a complex potential whose real part will describe the screening effect and the imaginary part — the effect of collisions in the medium. This picture is far from complete as at RHIC or LHC energies, the important number

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of charm quarks and antiquarks produced in collisions can lead to recombination where two initially uncorrelated quarks will form a charmonium[1]. The treatment of recombination is thus crucial for phenomenology. Numerous efforts were made in the last decade to describe the real-time quantum dynamics of quarkonia using the open quantum systems formalism. In this framework, one can derive quantum master equations by exploiting the scale hierarchies of the system to describe the evolution of a system composed of a quarkonium in interaction with the QGP medium through its density operator.

2. The model

Blaizot and Escobedo [1] derived quantum master equations describing the non-Abelian dynamics of a $Q\bar{Q}$ pair in the Quark–Gluon Plasma in the quantum Brownian regime. Those equations assume a weak coupling between the quarks and the plasma constituents, described using the NRQCD formalism (magnetic interactions are neglected) and can be put in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\mathcal{D}_\mathrm{s}(s,s')\\ \mathcal{D}_\mathrm{o}(s,s')\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_\mathrm{s}(s,s')\\ \mathcal{D}_\mathrm{o}(s,s')\end{pmatrix}, \tag{1}$$

with $\mathcal{D}_\mathrm{s}$ and $\mathcal{D}_\mathrm{o}$ the density operators in the singlet and octet color channels, s and s' the relative distance between the quark and antiquark (and its conjugated variable), and $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$ a superoperator describing the different transitions. The quark and antiquark dynamics are encoded by the $\mathcal{L}_0$ operator, while the static screening due to medium is described by the $\mathcal{L}_1$ operator. The $\mathcal{L}_2$ operator represents the fluctuations of the diffusion process in the medium, while the $\mathcal{L}_3$ describes dissipation. The original equations are not positivity-preserving, which is a key property of the well-defined quantum master equations. We derived new equations[2] that are an extension to the original ones, which include an additional $\mathcal{L}_4$ operator that guarantees the preservation of the density operator positivity.

The equations are solved numerically in one dimension using the Crank–Nicolson method for the charmonium system (the charm mass is taken as $m_c = 1.4692\ \mathrm{GeV}/c^2$) and discretized on a grid of 501×501 points. The s and s' variables go from -10 to 10 fm with a spatial step $\Delta s = 0.04$ fm and the time step Δt is taken as $0.01\ \mathrm{fm}/c$. The complex potential used is a potential developed specifically for one-dimensional studies [2] and aims at reproducing, as best as possible, the temperature-dependent mass spectra and decay widths of quarkonium states obtained from [3].

[1] This effect is supposed to be rather small for bottomonia as the number of b quarks produced is much lower.

[2] Following a procedure described in [1].

3. Charmonium dynamics in the QGP

3.1. Fixed medium temperature

We consider a QGP medium with a fixed temperature $T = 300$ MeV and we prepare the system as the vacuum $1S$-like state (ground state) in the singlet color channel. As shown in Fig. 1, the evolution is marked by the initial population of the octet channel as a P-like state, due to the dipolar nature of the color transitions. Following this, both density operators delocalize along the $s = s'$ axis and progressively diagonalize. At later times, two components arise: a long-lived correlation around $|s + s'| \sim 0$ and a dissociated component for larger $|s + s'|$.

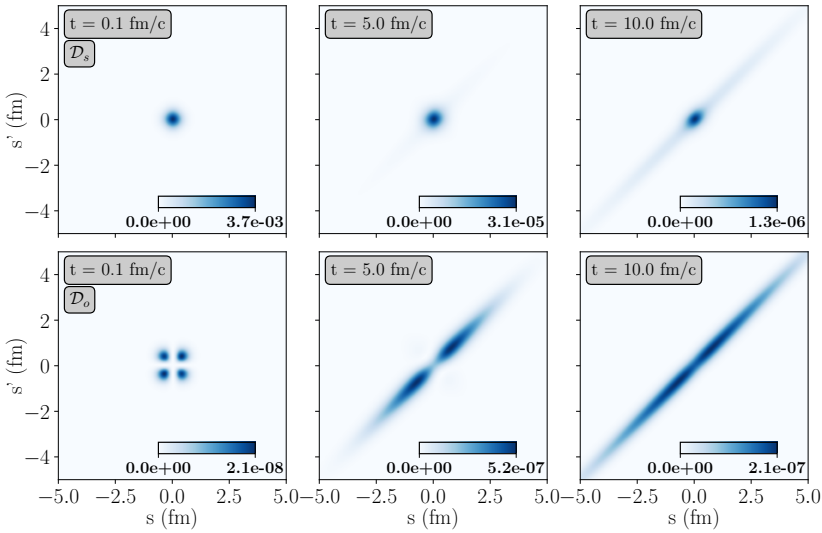

Fig. 1. (Color online) Evolution of the singlet density operator $\mathcal{D}_\mathrm{s}$ (top panels) and octet density operator $\mathcal{D}_\mathrm{o}$ (bottom panels) over time for a medium temperature $T = 300$ MeV and a $1S$-like singlet initial state. From left panel to right panel: $t =$ 0.1, 5, and 10 fm/c. The color scale changes from plot to plot for better readability.

Figure 2 shows the evolution over time of the weights of the first three vacuum eigenstates, defined as the instantaneous projections on the eigenstates. They are given by $P_n(t) = \langle n|\mathcal{D}_\mathrm{s}|n\rangle$, where the various $|n\rangle$ are the vacuum eigenstates. They thus represent the probability of finding the pair in the eigenstate $|n\rangle$ as if the plasma would freeze-out at a time t. In the initial stage of the evolution, the first ($1P$-like) and second ($2S$-like) excited states are populated from the ground state. The $1P$-like is populated later than the $2S$-like as several transitions are necessary to reach a $1P$-like state in the singlet color channel due to the dipolar color transitions. Following

this initial re-equilibration phase, the weights decrease due to the diffusion of the density operators observed in Fig. 1. The populations then finally reach a steady state with a common decay rate.

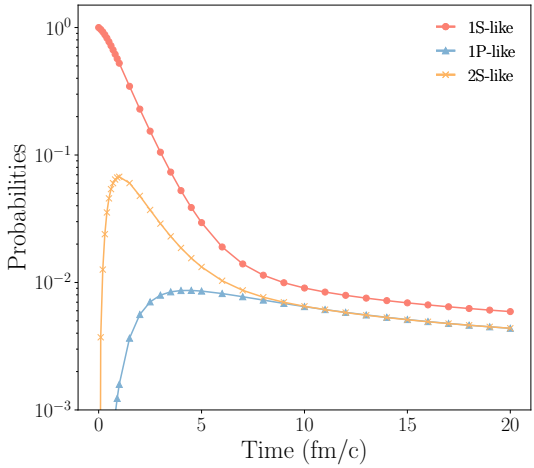

Fig. 2. Evolution of the population of the first three vacuum charmonium eigenstates as a function of time with a $1S$-like singlet initial state in a fixed temperature ($T = 300$ MeV) medium.

3.2. Time-dependent medium temperature

We now consider a more realistic scenario where we take an initial state prepared as a P-like state in the color octet channel. Additionally, we now relax the medium fixed temperature assumption and study a medium with a temperature evolving according to a simple Björken profile $T(t) = T_0(\frac{1}{1+t})^{1/3}$ with $T_0 = 600$ MeV. As can be seen in Fig. 3, the density operators diagonalize after an initial transient phase. Due to the higher initial temperature this diagonalization is faster but the system reaches a similar final state as compared to a $1S$-like singlet initial state.

The evolution of the vacuum eigenstates populations, shown in Fig. 4, can again be decomposed in two phases. In the initial stage of the evolution, bound states form quickly due to the singlet-to-octet transitions. This early formation is however facilitated by the initial closeness of the two quarks. After this first phase, the excited states exhibit the same behavior as in the previous scenario, with a slightly different common decay rate. The ground behavior is however much different, as the population of the $1S$-like state becomes almost constant after ~ 7 fm/c. This behavior can be understood as the increase of the binding energy and the decrease of the thermal forces due to the cooling of the medium, resulting in a much smaller decay rate.

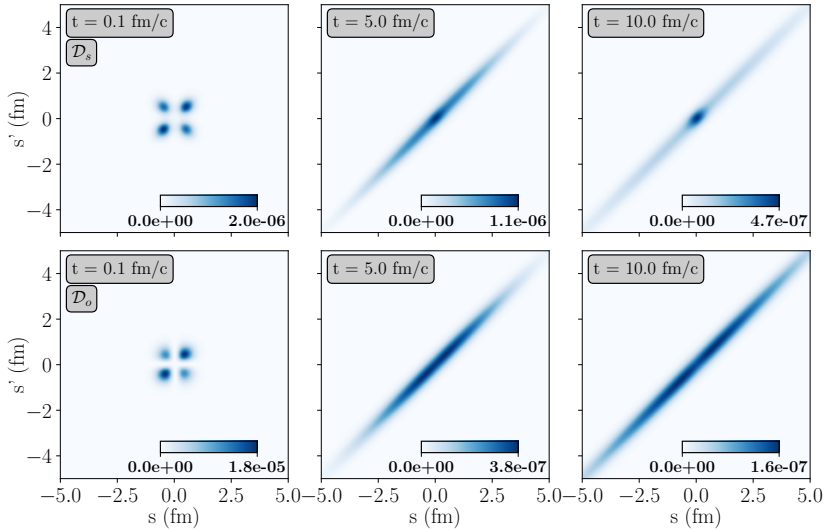

Fig. 3. (Color online) Evolution of the singlet density operator $\mathcal{D}_\mathrm{s}$ (top panels) and octet density operator $\mathcal{D}_\mathrm{o}$ (bottom panels) over time for a cooling medium ($T_0 = 600$ MeV and a P-like octet initial state. From left panel to right panel: $t =$ 0.1, 5, and 10 fm/c. The color scale changes from plot to plot for better readability.

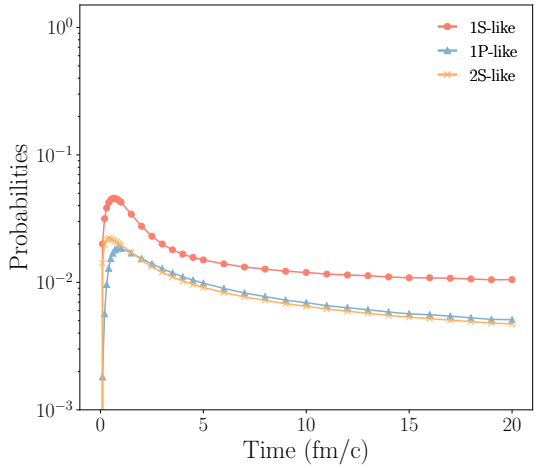

Fig. 4. Evolution of the population of the first three vacuum charmonium eigenstates as a function of time with a P-like octet initial state in a cooling medium ($T_0 = 600$ MeV).

4. Conclusion

We presented new quantum master equations describing the non-Abelian dynamics of a single quarkonium in the Quark–Gluon Plasma. Those equations include dissipative effects and are positivity-preserving and were resolved numerically in one dimension. The evolution of the density operators describing the pair is characterized by the delocalization of the initial state along the $s = s'$ direction and decoherence due to the medium thermal effects. Another interesting result is that no matter what initial state is considered, the system reaches similar final states, indicating that the influence of the QGP washes the initial-state dependence. An upcoming publication will give more details on the model and new results.

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