SPACE-TIME EVOLUTION OF CRITICAL FLUCTUATIONS IN AN EXPANDING SYSTEM*

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We construct a framework to describe the dynamics of the critical fluctuations in high-energy nuclear collisions. We consider the relaxation time τ_{R} for the baryon diffusion current and the coupling of the chiral condensate fluctuation $\delta\sigma$ to the baryon density fluctuation δn. We apply this framework to a one-dimensionally expanding system with the QCD critical point and investigate the effects of the relaxation time τ_{R} and the mode coupling on the correlation of baryon density fluctuations δn as a function of the rapidity interval. We show that the non-zero relaxation time makes the signal propagate at a finite speed, which results in a time lag in the response of correlation evolution of baryon number fluctuations δn from chiral fluctuations.

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1. Introduction

The search for the QCD critical point is one of the main topics in high-energy nuclear collisions. A number of experimental programs, including the Beam Energy Scan (BES) programs at the Relativistic Heavy-Ion Collier (RHIC) [1] and future experiments FAIR, NICA, J-PARC-HI, and HIAF aim to find signals of the QCD critical point and the first-order phase transition. To experimentally detect the signals, it is important to eventually develop a dynamical model that can quantitatively describe the evolution of the dense QCD matter created in the nuclear collisions. As a first step toward this ultimate goal, we investigate the qualitative nature of the dynamics of the critical fluctuations in the longitudinally expanding systems.

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Critical phenomena associated with the QCD critical point are characterized by enhanced fluctuations of the chiral condensate $\sigma = \langle \bar{q}q \rangle$ and the baryon number density $n_B = \langle \bar{q}\gamma^0 q \rangle$. In the idealized systems close to equilibrium, the fluctuations of (non-conserved) chiral condensate can be integrated out as a fast mode so that the critical dynamics is governed by the fluctuations of the baryon number density and other hydrodynamic modes. However, it is non-trivial whether we may eliminate the σ mode as a fast mode in the realistic situations where the time-scale separation is unclear. In addition, the standard dynamical models are based on the second-order hydrodynamics where non-hydrodynamic modes of the second-order correction of the dissipative currents play an important role in causality and stability. For a consistent extension of the second-order model, it is important to revive the dynamical σ mode and examine its effect. In this study, we construct a model of critical fluctuations by coupling the chiral condensate fluctuation $\delta\sigma \equiv \bar{q}q - \langle \bar{q}q \rangle$ to the baryon number density fluctuation $\delta n \equiv \bar{q}\gamma^0 q - \langle \bar{q}\gamma^0 q \rangle$ and the dynamical baryon diffusion current $\boldsymbol{\nu}$. We then analyze the two-point correlation in a one-dimensionally expanding system.

2. Model

We extend the coupled Langevin equation [2, 3] introducing the relaxation time τ_{R} and the dissipative current $\boldsymbol{\nu}$

$$\begin{aligned} \frac{\mathrm{d}(\delta\sigma)}{\mathrm{d}t} &= -\mathit{\Gamma}\frac{\delta F}{\delta(\delta\sigma)} + \tilde{\lambda}\nabla^2\frac{\delta F}{\delta(\delta n)} + \xi_\sigma \,, & (1)\\ \tau_{\mathrm{R}}\frac{\mathrm{d}\boldsymbol{\nu}}{\mathrm{d}t} + \boldsymbol{\nu} &= \tilde{\lambda}\nabla\frac{\delta F}{\delta(\delta\sigma)} + \lambda\nabla\frac{\delta F}{\delta(\delta n)} + \boldsymbol{\xi}_n \,, & (2)\\ \frac{\mathrm{d}(\delta n)}{\mathrm{d}t} &= -\nabla\cdot\boldsymbol{\nu}\,. & (3) \end{aligned}$$

The noise terms ξ_σ and $\boldsymbol{\xi}_n$ are randomly sampled by the Gaussian distribution satisfying the fluctuation–dissipation relations (FDR)

$$\begin{aligned} \left\langle \xi_\sigma(x)\xi_\sigma\left(x'\right)\right\rangle &= 2T\mathit{\Gamma}\delta^4\left(x-x'\right)\,, & (4)\\ \left\langle \xi_\sigma(x)\xi_{n,i}\left(x'\right)\right\rangle &= 2T\tilde{\lambda}\delta^4\left(x-x'\right)\,, & (5)\\ \left\langle \xi_{n,i}(x)\xi_{n,j}\left(x'\right)\right\rangle &= 2T\lambda\delta_{ij}\delta^4\left(x-x'\right)\,, & (6) \end{aligned}$$

where $\mathit{\Gamma}, \lambda$, and $\tilde{\lambda}$ are the transport coefficients. The Ginzburg–Landau functional F is given by

$$\begin{aligned} F\left[\delta\sigma,\delta n\right] &= \int \mathrm{d}^3\boldsymbol{x}\left[\frac{a}{2}(\partial_i\delta\sigma)^2 + b\partial_i\delta\sigma\partial_i\delta n + \frac{c}{2}(\partial_i\delta n)^2 + V(\delta\sigma,\delta n)\right]\,, & (7)\\ V(\delta\sigma,\delta n) &= \frac{A}{2}\delta\sigma^2 + B\delta\sigma\delta n + \frac{C}{2}\delta n^2\,. & (8) \end{aligned}$$

We relate the parameters A, B, and C to the susceptibility χ as

$$\frac{A(T)C(T) - B(T)^2}{A(T)} = \frac{1}{\chi(T)}\,, \tag{9}$$

so that the potential has a flat direction at the critical temperature T_c. We parameterize the susceptibility $\chi(T)$ as in Ref. [4].

3. Results

We follow the space-time evolution of $\delta\sigma$ and δn in a Bjorken-expanding system. The time dependence of temperature is parameterized as $T(\tau) = T_0\left(\frac{\tau_0}{\tau}\right)^{c_\mathrm{s}^2}$. We set the initial time τ_0, initial temperature T_0, and sound velocity c_s^2 as 0.6 fm, 220 MeV, and 0.15, respectively. We choose the critical temperature T_c as 160 MeV. We introduce the conserved baryon density in τ–η_s space as $\delta\tilde{n} \equiv \tau\delta n$.

In Fig. 1, we illustrate the Green functions at $\tau = 0.85$ fm, obtained from the initial conditions of the smeared delta function $\delta\sigma(\eta_\mathrm{s}, \tau_0) = \frac{1}{\sqrt{2\pi w^2}}\exp(-\frac{\eta_\mathrm{s}^2}{2w^2})$ with $w = 0.1$ and vanishing baryon density fluctuation $\delta\tilde{n}(\eta_\mathrm{s}, \tau_0) = 0$, and without the noise terms. In panel (a), the effect of the relaxation time τ_R on $\delta\sigma$ is found to be small. In panel (b), we see that the baryon density fluctuation is induced by the coupling between $\delta\sigma$ and δn. We also observe that $\delta\tilde{n}$ diffuses to large η_s instantly with the vanishing relaxation time (solid red) while it diffuses at a finite speed with the finite relaxation times (long-dashed green and dashed blue). The diffusion for $\tau_\mathrm{R} = 2.0$ fm is slower than that for $\tau_\mathrm{R} = 1.0$ fm.

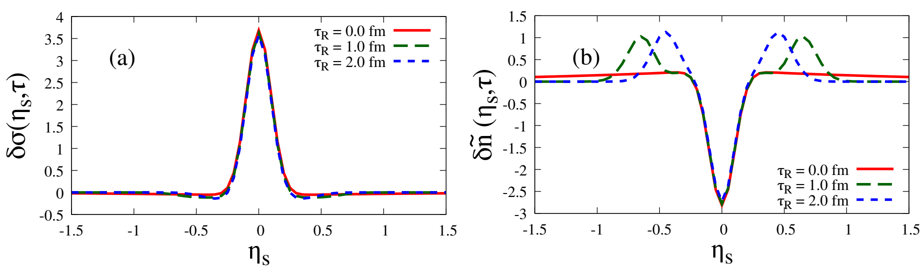

Fig. 1. (Color online) Distribution of (a) $\delta\sigma(\eta_\mathrm{s}, \tau)$ and (b) $\delta\tilde{n}(\eta_\mathrm{s}, \tau)$ at $\tau = 0.85$ fm with different relaxation times τ_R. The solid red line is for the vanishing relaxation time, and the long-dashed green and dashed blue are for $\tau_\mathrm{R} = 1.0$ fm and 2.0 fm, respectively.

We next analyze the correlation function $C(\Delta\eta_\mathrm{s}, \tau)$ with different relaxation times

$$C\left(\Delta\eta_\mathrm{s}, \tau\right) = \frac{\langle\delta\tilde{n}(\eta_\mathrm{s}, \tau)\delta\tilde{n}(\eta_\mathrm{s} + \Delta\eta_\mathrm{s}, \tau)\rangle}{\langle\delta\tilde{n}(\eta_\mathrm{s}, \tau_0)^2\rangle}\,. \tag{10}$$

The delta function in FDR (4)–(6) is smeared by the Gaussian of the width 0.1. We initialize $\delta\tilde{n}(\eta_s, \tau_0)$ with the thermal distribution and calculate $C(\Delta\eta_s, \tau)$ by only considering $\delta\tilde{n}$. In Fig. 2 (a), we see a peak at $T_c = 160$ MeV, which is shown in blue (star). With a larger relaxation time, the peak shifts to a lower temperature ~ 150 MeV shown in magenta (open circles), which means that the finite relaxation time causes a time lag in the response.

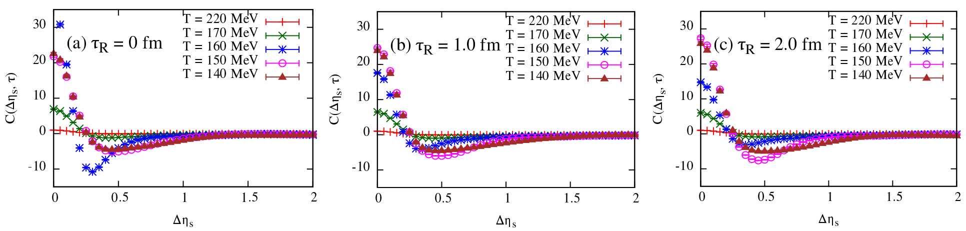

Fig. 2. (Color online) Correlation function $C(\Delta\eta_s, \tau)$ (a) for vanishing relaxation time, (b) $\tau_R = 1.0$ fm, and (c) $\tau_R = 2.0$ fm.

4. Conclusion

We have constructed a dynamical model for the critical dynamics in an expanding system. We coupled the baryon density fluctuation to the chiral condensate fluctuation and also introduced the baryon relaxation time. Within this model, we analyzed 1+1D space-time evolution of these fluctuations and their correlations. We showed that the non-zero relaxation time makes the signal propagate at a finite speed, which results in a time lag in the response of correlation evolution. As an outlook, we plan to analyze the effect of critical fluctuations on experimental observables by implementing it in a fluctuating hydrodynamic model in the future.

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