

DEVELOPMENT OF TRANSVERSE FLOW FOR SMALL AND LARGE SYSTEMS IN CONFORMAL KINETIC THEORY*

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We employ an effective kinetic description to study the space-time dynamics and development of the transverse flow of small and large collision systems. By combining analytical insights in the few interactions limit with numerical simulations at higher opacity, we are able to describe the development of transverse flow from very small to very large opacities, realised in small and large collision systems. Surprisingly, we find that deviations between kinetic theory and hydrodynamics persist even within the limit of very large interaction rates, which can be attributed to the presence of the early pre-equilibrium phase.

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1. Introduction

Modern frameworks for simulations of heavy-ion collisions typically feature an extended hydrodynamic phase that dominates the spacetime evolution of the system. This is where most of the commonly observed signatures of thermalization and collective behaviour emerge, notably also transverse flow, which refers to anisotropies in the transverse momentum spectrum of measured particles, that build up in response to spatial anisotropies in the initial state.

However, the picture is not as clear in the case of small systems. If the system is too dilute or if the gradients are too large, hydrodynamics may lose its applicability. Still, some signatures of collective behaviour, most notably transverse flow, are also experimentally observed in small systems, which

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calls for an alternative description. Another way to model the dynamics of a hadronic collision is via kinetic theory, a model describing the time evolution of phase-space distributions based on microscopic interactions. This description is applicable to the whole range from very small to very large systems, with hydrodynamics being its limit for large interaction rates.

This motivates the work that we present here. To test the applicability of hydrodynamics, we employ a simplified kinetic theory description for the full opacity range and perform comparisons to hydrodynamics on the basis of transverse flow and related observables.

2. Model and setup

We consider a simplified boost-invariant system of massless particles with an initially vanishing longitudinal pressure and transverse momentum anisotropy. If we describe only energy-weighted degrees of freedom, the initial condition is then fully determined by the initial energy density $\epsilon(\tau_0, \mathbf{x}_\perp)$. We choose the initial condition to be of a simple Gaussian form, introducing only one anisotropic mode with n -fold rotational symmetry, whose magnitude is characterized by the eccentricity ϵ_n (see Refs. [1, 2] for details).

In our simplified kinetic theory description, the system evolves according to the relativistic Boltzmann equation in the relaxation-time approximation (RTA)

$$p^\mu \partial_\mu f = C_{\text{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R} (f - f_{\text{eq}}), \quad (1)$$

where f is the distribution function, p^μ is the particle momentum, $f_{\text{eq}} = (e^{p \cdot u/T} - 1)^{-1}$ is the Bose–Einstein equilibrium distribution, u^μ is the fluid four-velocity, $\tau_R = 5(\eta/s)T^{-1}$ is the relaxation time, T is the local temperature, and η/s the specific shear viscosity.

In conformal RTA, the system’s time evolution depends only on one parameter, the opacity $\hat{\gamma}$, defined as [3]

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{30}{\pi^2 \nu_{\text{eff}}} \frac{1}{\pi R^2} \frac{dE_\perp^{(0)}}{d\eta}\right)^{1/4} R^{3/4}. \quad (2)$$

Here, $dE_\perp^{(0)}/d\eta$ is the initial energy per rapidity, R is the system size, and ν_{eff} is the effective number of degrees of freedom. For $\eta/s = 0.16$, the opacity takes typical values of 0.88 in minimum-bias pp collisions and 9.2 in central PbPb collisions.

We will present numerical results for kinetic theory from a Relativistic Lattice Boltzmann approach [4]. The hydrodynamic results that we will compare them to are obtained using the **vHLL**E code [5] with transport coefficients matched to RTA (see Ref. [1] for details).

3. Time evolution and opacity dependence in kinetic theory

We characterize the transverse-plane dynamics using the transverse energy $dE_{\perp}/d\eta$ and the energy-weighted flow harmonics v_n^E , computed as

$$\frac{dE_{\perp}}{d\eta} = \tau \int d^2x_{\perp} \int d^3p p_{\perp} f, \quad v_n^E = \frac{\int d^2x_{\perp} \int d^3p p_{\perp} e^{in\phi_p} f}{\int d^2x_{\perp} \int d^3p p_{\perp} f}, \quad (3)$$

where $(p_{\perp}, \phi_p)$ characterize the transverse-plane projection $\mathbf{p}_{\perp}$ of p^{μ} .

We examined these observables on the full parametric range in opacity and eccentricity [1]. Here, we present a selection of interesting results. Figure 1 (a) shows the time evolution of $dE_{\perp}/d\eta$ for various values of $\hat{\gamma}$. At small values of τ/R , the system is in the free-streaming regime and $dE_{\perp}/d\eta$ remains constant. During equilibration, longitudinal pressure is generated and $dE_{\perp}/d\eta$ is decreased due to work done by the system against longitudinal expansion. Since the transverse expansion is negligible at early times, the longitudinal cooling of the system can be effectively described by 0+1D Bjorken dynamics locally at each point in the transverse plane. Corresponding results shown with dashed gray lines and empty circles are in excellent agreement with the full numerical simulations. Finally, when $\tau/R \gtrsim 1$, transverse expansion becomes dominant and the cooling stops.

In terms of the harmonic response coefficients, we present results for the opacity dependence of the response coefficients v_n^E/ϵ_n to a small initial-state harmonic anisotropy of eccentricity ϵ_n . Our numerical results shown in

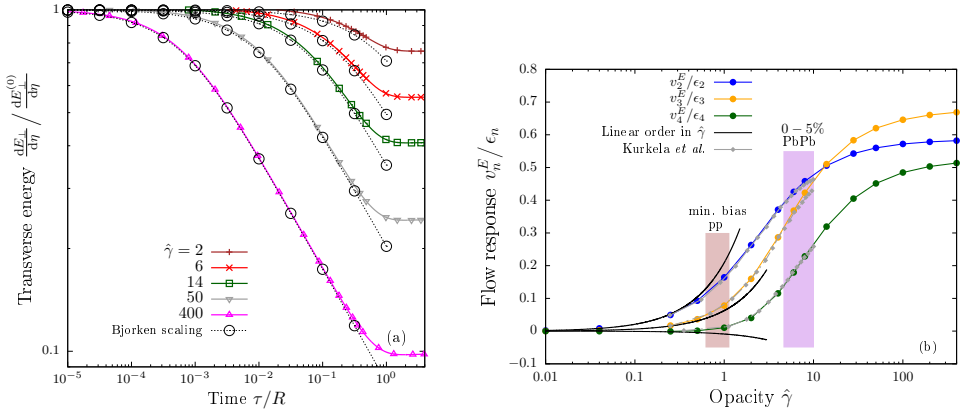


Fig. 1. (Colour on-line) (a) Time dependence of the transverse energy $dE_{\perp}/d\eta$ for various opacities. Our numerical results are shown with continuous lines and symbols, while the Bjorken scaling estimation is shown with dotted lines and empty circles. (b) Opacity dependence of the flow responses v_n^E/ϵ_n obtained from numerical simulations (coloured lines), using a linear order approximation (black lines), and from Ref. [2] (grey lines).

Fig. 1(b) indicate that for small values of $\hat{\gamma}$, these coefficients scale linearly with the opacity $\hat{\gamma}$. Linearized results of v_2^E and v_3^E are good approximations for pp opacities. The numerical coefficients are reported in Eqs. (86)–(88) of Ref. [1]. In the intermediate opacity range, we validated our results by comparison with the results reported in Ref. [2] in an identical setup. Finally, at opacities several times larger than those of central PbPb collisions, the late-time values of v_n^E/ϵ_n saturate. We will examine this further in the next section.

4. Comparing results from kinetic theory and hydrodynamics

For a better understanding of the large- $\hat{\gamma}$ limit of our kinetic theory results, we performed a comparison with hydrodynamic simulations. In order to facilitate this comparison, in this section, we will consider observables that can be computed directly from the energy-momentum tensor $T^{\mu\nu}$

$$\frac{dE_{\text{tr}}}{d\eta} = \tau \int d^2x_{\perp} (T^{xx} + T^{yy}) , \quad \epsilon_p = \frac{\int d^2x_{\perp} (T^{xx} - T^{yy} + 2iT^{xy})}{\int d^2x_{\perp} (T^{xx} + T^{yy})} , \quad (4)$$

where $dE_{\text{tr}}/d\eta$ and ϵ_p act as proxies for $dE_{\perp}/d\eta$ and v_2 .

As it turns out, there is some subtlety in defining the hydrodynamic analogue to a given setup in kinetic theory. Naively initializing hydrodynamics using the same energy profile used for the kinetic theory simulations leads to significant inconsistencies at the level of $dE_{\text{tr}}/d\eta$, as shown in Fig. 2(a). Furthermore, the large- $\hat{\gamma}$ limit of the elliptic flow coefficient ϵ_p/ϵ_2 differs in kinetic theory and hydrodynamics and, more importantly, both differ from the prediction in ideal hydrodynamics. This is due to differences in the pre-equilibrium behaviour, as discussed in the following.

In the regime of large opacities $\hat{\gamma} \gg 1$, equilibration sets in before the transverse expansion and at early times, the system can be effectively described as undergoing a homogeneous Bjorken flow at each point in the transverse plane. During the equilibration period, the system establishes a longitudinal pressure and begins to cool by performing work against the longitudinal expansion [6]. Due to the conformal behaviour, the hotter regions will undergo this equilibration faster than the colder ones, and hence the hotter regions also start to cool earlier than the colder regions. Due to this *inhomogeneous cooling*, the initial eccentricity decreases before the onset of the transverse expansion, and only a decreased spatial eccentricity will be converted to flow. Since the equilibration period is absent in ideal hydrodynamics, and described differently in viscous hydrodynamics and kinetic theory [7], the effect on the eccentricity also differs between these descriptions. In Ref. [1], we showed that the magnitudes of the eccentricity decrease match the discrepancies arising in ϵ_p .

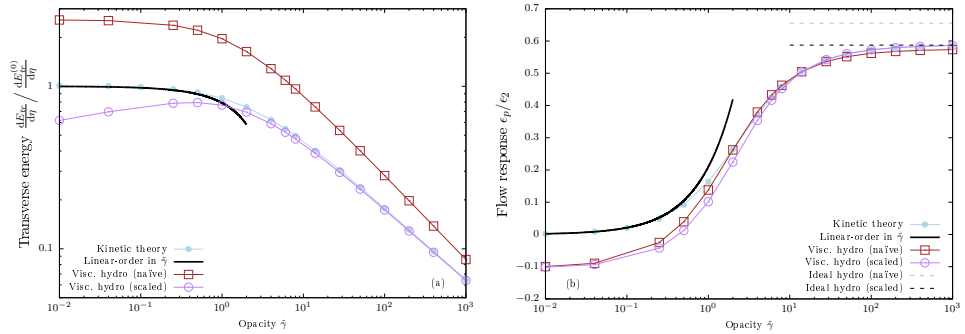


Fig. 2. (Colour on-line) Opacity dependence of (a) $dE_{\text{tr}}/d\eta$ and (b) ϵ_p/ϵ_2 measured at $\tau = 4R$ obtained in kinetic theory (blue line with filled circles), viscous with naive (brown line with empty squares), and scaled (purple line with empty circles). The ideal hydrodynamics limit for ϵ_p/ϵ_2 is shown with dotted gray and black lines for the naive and scaled initializations, respectively.

We will now present a modification of hydrodynamic setups that can alleviate the problem with pre-equilibrium. The idea we follow is to counteract the observed eccentricity decay by rescaling the hydro initial condition. As $\tau\epsilon$ rises in magnitude in hydrodynamics, its initial condition is scaled down such that it will come into agreement with kinetic theory only after pre-equilibrium, assuming local Bjorken flow dynamics. As cooling proceeds inhomogeneously, the scaling factor is chosen locally, which will change the eccentricity in the hydro initial state such that it agrees with kinetic theory at later times. This is explained in more detail in [8, 9].

This scheme relies on the timescale separation between equilibration and transverse flow. At small opacities, equilibration will be interrupted by the onset of transverse expansion and cannot be encapsulated in the local Bjorken flow description. In these cases, we expect our scaling scheme to lead to unphysical results.

Figure 2 now compares the kinetic theory results for the opacity dependences of $dE_{\text{tr}}/d\eta$ and ϵ_p with hydrodynamics results based on the naive and scaled initializations. For consistency, we characterize the simulation results based on the values of $\hat{\gamma}$, $dE^{(0)}/d\eta$, and ϵ_2 corresponding to the kinetic-theory setup. When the initial conditions are scaled as discussed above, the viscous and ideal hydrodynamics results seamlessly converge to the kinetic-theory results at large $\hat{\gamma}$. The kinetic-theory and viscous hydrodynamics results stay in good agreement down to $\hat{\gamma} \gtrsim 10$. For small opacities, the hydrodynamics results for ϵ_p obtained using the scaled and naive initial conditions behave similarly. When the naive initialization is used, $dE_{\text{tr}}/d\eta$ increases as $\hat{\gamma}$ is decreased up to a plateau value, indicating that the time scale for transverse expansion becomes shorter than the pre-equilibrium one,

such that the system never equilibrates. Since our scaling scheme reduces the initial energy density to account for the increase of $dE_{\text{tr}}/d\eta$ over the whole free-streaming evolution, its final value will be increasingly underestimated as $\hat{\gamma}$ is decreased.

5. Conclusions

Our kinetic theory description is able to produce accurate results for cooling and flow for all opacities, ranging from linear behaviour at small opacities to saturation to ideal fluid behaviour at large opacities. In a naive comparison to hydrodynamics, we found sizeable discrepancies even at large opacities, which we determined to be due to different behaviour in the pre-equilibrium stage, where eccentricities decay by differing amounts.

We presented a prescription to bring hydrodynamics into agreement with kinetic theory based on the idea of scaling the initial energy density in hydrodynamics in such a way that it agrees with kinetic theory only after the pre-equilibrium period in the case of purely longitudinal expansion. With this setup, we obtained agreement in the large-opacity limit.

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