ASSESSING RELATIVISTIC CORRECTIONS TO THE Λ HYPERON GLOBAL SPIN POLARIZATION MEASUREMENTS*

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In this work, we present a new method to measure the global spin polarization of hyperons produced in non-central relativistic heavy-ion collisions which accounts for relativistic effects due to the frame dependence of global orbital angular momentum direction defining the spin quantization axis. Using the new correlator, we estimate the resulting numerical corrections to be reaching 10 percent for the most energetic particles.

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1. Introduction

The recent positive measurements of global spin polarization of hyperons produced in non-central relativistic heavy-ion collisions [1–6] are among the most important experimental results of the contemporary high-energy nuclear physics triggering a vast theoretical development of this field [7–10], for some recent advancements, see, for instance, Refs. [11–15]. While the experimental data unambiguously confirm the correlation between the average spin polarization of particles emitted from the system and the global orbital angular momentum (OAM) deposited in the hot and dense medium during the initial nuclear impact, a precise quantitative description of this phenomenon still requires a complete understanding of all aspects of the experimental procedure being employed.

In this work, we discuss possible relativistic effects usually being ignored in the standard measurement which result from the change of the direction

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of the global OAM (defining the spin quantization axis) when transformed into the particle rest frame where the spin polarization is being measured. Following Ref. [16] we propose an improved measurement method that naturally accounts for such effects and estimate resulting numerical corrections.

2. Global spin polarization measurement

The measurement of the spin polarization vector $\boldsymbol{P}' = P'\hat{\boldsymbol{P}}'$ of the Λ hyperon, where P' denotes its magnitude and $\hat{\boldsymbol{P}}'$ its orientation, is based on Λ parity-violating weak decay, $\Lambda \to p + \pi^-$, using the property that the outgoing proton is preferentially emitted in the direction of the Λ spin. The spin polarization vector is determined in the rest frame of the decaying Λ $S'(\boldsymbol{p}_\Lambda)$ through the angular momentum distribution of the emitted protons employing the formula

$$\begin{aligned}\frac{\mathrm{d}N_p^{\mathrm{pol}}}{\mathrm{d}\Omega'} &= \frac{1}{4\pi}\left[1 + \boldsymbol{P}'\cdot\hat{\boldsymbol{p}}'_p\right]\\ &= \frac{1}{4\pi}\left[1 + \alpha_\Lambda P'\left(\cos\left(\Phi' - \phi'_p\right)\sin\theta'_p\sin\Theta' + \cos\theta'_p\cos\Theta'\right)\right]\,, \qquad (1)\end{aligned}$$

where we parametrized the directions of the proton momentum and the Λ spin polarization vector using spherical coordinates as

$$\hat{\boldsymbol{p}}'_p = \left(\sin\theta'_p\cos\phi'_p, \sin\theta'_p\sin\phi'_p, \cos\theta'_p\right)$$

and

$$\hat{\boldsymbol{P}}' = \left(\sin\Theta'\cos\Phi', \sin\Theta'\sin\Phi', \cos\Theta'\right)\,,$$

respectively. The quantity $\alpha_\Lambda = 0.732$ denotes the Λ decay constant [17]. One can easily check that the Λ weak decay law (1) depends on $\cos\theta^* \equiv \cos(\Phi' - \phi'_p)\sin\theta'_p\sin\Theta' + \cos\theta'_p\cos\Theta'$, with θ^* denoting the angle between $\hat{\boldsymbol{p}}'_p$ and $\hat{\boldsymbol{P}}'$ in the reference frame $S^*(\boldsymbol{p}_\Lambda)$ obtained from $S'(\boldsymbol{p}_\Lambda)$ through the rotation that aligns $\boldsymbol{P}'$ with the new z axis, see the left panel of Fig. 1.

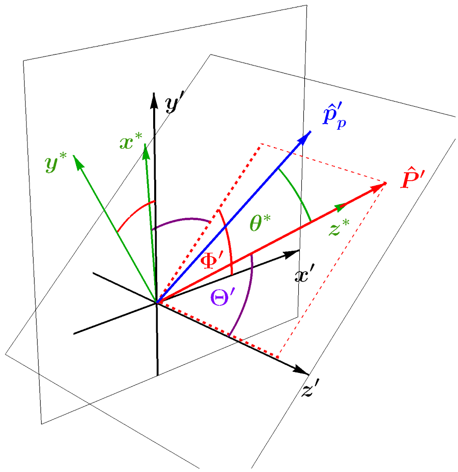
Given the proton angular momentum distribution (1) the polarization vector is given entirely by the averaged momentum components, namely

$$\boldsymbol{P}' = \frac{3}{\alpha_\Lambda}\left(\left\langle\hat{p}'_{p,x}\right\rangle, \left\langle\hat{p}'_{p,y}\right\rangle, \left\langle\hat{p}'_{p,z}\right\rangle\right)\,, \qquad (2)$$

where $\langle\cdots\rangle \equiv \int\left(\frac{\mathrm{d}N_p^{\mathrm{pol}}}{\mathrm{d}\Omega'}\right)(\cdots)\sin\theta'_p\,\mathrm{d}\theta'_p\,\mathrm{d}\phi'_p$ denotes the angular average.

The quantity which is measured in the experiment, also known as *global polarization*, is given by

$$P_H = \frac{8}{\pi\alpha_\Lambda}\left\langle\sin\phi'_p\right\rangle\,. \qquad (3)$$



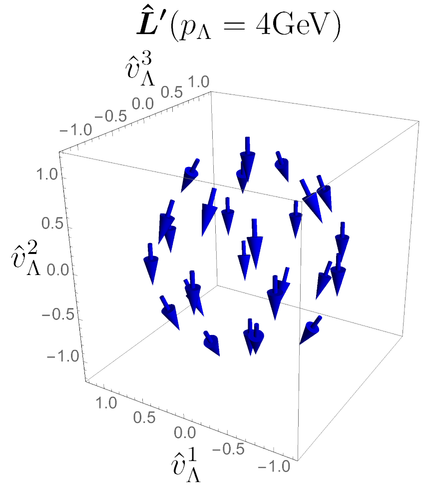


Fig. 1. Left: The directions of the proton momentum $\hat{\boldsymbol{p}}'_p$ and spin polarization vector $\hat{\boldsymbol{P}}'$ viewed in the Λ rest frames $S'(\boldsymbol{p}_\Lambda)$ and $S^*(\boldsymbol{p}_\Lambda)$. Right: Visualization of the vectors $\boldsymbol{L}'$ for Λ with different velocity directions.

Comparing this expression to Eq. (1), one can identify $P_H \equiv P' \sin\Theta' \sin\Phi'$ as a y component of the spin polarization vector in the Λ rest frame $S'(\boldsymbol{p}_\Lambda)$. The motivation for measuring P_H follows from the expectation that the initial global OAM of the system deposited in the nuclear impact which is oriented along the y direction in the center-of-mass frame

$$\hat{\boldsymbol{L}} = \frac{\boldsymbol{L}}{L} = (0, -1, 0) \tag{4}$$

may be converted, by virtue of the spin-orbit coupling, to the spin of the emitted particles resulting in a non-vanishing average spin polarization along the same direction. However, for relativistic particles (large momenta of Λ's), the y direction in the center-of-mass frame differs from the y direction in the Λ rest frame (where its polarization is determined) by a non-trivial Lorentz transformation. This effect is usually ignored in the experimental analyses with the non-relativistic assumption that the possible corrections are negligibly small. Such a choice may, however, constitute a significant problem whenever the measurement involves averaging over Λ's within a wide momentum range or one studies differential results.

3. Correlation with global orbital angular momentum

In Ref. [16], an improved measurement method of the global spin polarization was developed, which accounts for the relativistic effects discussed above. Noticing that the OAM vector components $L^k = -\frac{1}{2}\epsilon^{kij}L^{ij}$ constitute the spatial components of the respective (antisymmetric) angular momentum tensor $L^{\mu\nu}$, the changes of L^k connected with the change of frame may be straightforwardly traced employing the Lorentz transformation rules similar to that of the Faraday tensor, see right panel of Fig. 1.

As a result, instead of considering the standard projection $\hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$, one should study the quantity

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \left(1 - \left(\boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}}\right)^2\right)^{-1/2} \left(\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} \boldsymbol{v}_\Lambda \cdot \boldsymbol{P}' \;\; \boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}}\right). \tag{5}$$

The latter defines the projection of the spin polarization vector on the global OAM direction in the rest frame $S'(\boldsymbol{p}_\Lambda)$ of the Λ with momentum $\boldsymbol{p}_\Lambda = \boldsymbol{v}_\Lambda E_\Lambda = m_\Lambda \gamma_\Lambda \boldsymbol{v}_\Lambda$. Obviously, using formula (5), it is guaranteed that the spin polarization vector of each Λ is projected along the same physical axis irrespectively of Λ momentum. Hence, in our opinion, the measurement based on Eq. (5) may provide a better estimate of the spin polarization projection along the global OAM of the system.

4. Estimates of relativistic corrections

Using Eq. (5), one may estimate possible numerical corrections resulting from relativistic effects which are neglected in the standard measurement method based on $\hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$ projection. For that purpose, one has to average formula (5) over the Λ's with different momenta. For simplicity, let us consider a situation where $\boldsymbol{P}' = P'\hat{\boldsymbol{L}}$, which implies

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = P' \left(1 - v_2^2\right)^{-1/2} \left(1 - \frac{v_2^2}{1 + \sqrt{1 - v^2}}\right) \equiv P' F_P(\boldsymbol{v}), \tag{6}$$

where v_i denote components of the Λ velocity, while $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$. Let us also assume that the velocity distribution of Λ's has the Fermi–Dirac form, $F_T(v) = N\left[\exp\left(m_\lambda/(T_{\rm eff}\sqrt{1-v^2})\right) + 1\right]^{-1}$, with $T_{\rm eff}$ denoting the effective temperature and N defining a normalization constant (irrelevant for our considerations). The momentum average of the quantity (5) for Λ's with the momentum in the range of (m,n) GeV is then given by

$$\left\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \right\rangle_{m-n} = P' \frac{\int_{v_{(m)}}^{v_{(n)}} {\rm d}v \int {\rm d}\Omega \, F_P(\boldsymbol{v}) F_T(v)}{\int_{v_{(m)}}^{v_{(n)}} {\rm d}v \int {\rm d}\Omega \, F_T(v)}, \tag{7}$$

where we introduced the quantity $v_{(n)} = \tanh\left[\sinh^{-1}\left(\frac{n\,{\rm GeV}}{m_\Lambda}\right)\right]$. Our numerical estimates of the ratio $\frac{\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{m-n}}{P'}$ for two values of the effective temperature are shown in Fig. 2. In particular, for $T_{\rm eff} = 150$ MeV, we obtain the following results: $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{2-3} = 0.97\ P'$, $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{3-4} = 0.94\ P'$, $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{4-5} = 0.92\ P'$, and $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{5-6} = 0.90\ P'$. We find that the magnitude of corrections depends strongly on the considered momentum range. However, having in mind differential experimental data, we find that the relativistic effects which we account for using our method may introduce corrections of up to 10% for the most energetic Λ.

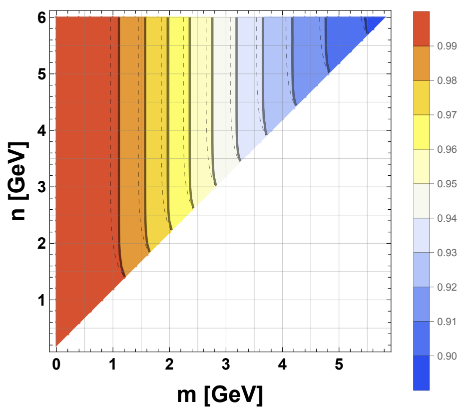


Fig. 2. The quantity $\frac{\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{m-n}}{P'}$ for Λ's within the momentum range (m,n) GeV. Results are shown for $T_{\rm eff} = 150$ MeV (solid contours and shading) and $T_{\rm eff} = 300$ MeV (dashed contours).

5. Conclusions

In this work, we discussed a new method for measuring the global spin polarization of particles in heavy-ion collisions which represents an improvement over the standard method by accounting for relativistic effects connected with the change of the global orbital angular momentum direction defining the quantization axis when being transformed to the particle rest frame where the polarization is measured. Using the new method, we estimated the possible numerical correction due to considered effects of ranging up to 10% of the currently measured signal for most energetic particles.

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