INTERACTIONS ENCODED IN PHASE SHIFT*

Pok Man Lo

University of Wrocław, 50-204 Wrocław, Poland

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I describe how interactions can be included via a model phase shift.

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1. Introduction

The S-matrix formulation of statistical mechanics [1, 2] offers a robust approach to determine the bulk properties of the medium based on the interactions among constituents. The connection is established by the density of states (DoS) [3, 4], which can be expressed in terms of the S-matrix via

$$\begin{aligned} B(E) &= \frac{1}{2}\mathrm{Im}\,\mathrm{Tr}\left[S^{-1}\frac{\mathrm{d}}{\mathrm{d}E}S - \left(\frac{\mathrm{d}}{\mathrm{d}E}S^{-1}\right)S\right] \\ &= \frac{\mathrm{d}}{\mathrm{d}E}\,\mathrm{Im}\,\ln\det S(E) \\ &= 2\frac{\partial}{\partial E}\mathcal{Q}(E)\,, \end{aligned} \tag{1}$$

where $\mathcal{Q}(E)$ is the scattering phase shift. The change in thermal pressure due to interaction is given by [3, 5]

$$\Delta P \approx T\int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}E'}{(2\pi)}\,\mathrm{e}^{-\beta\left(m_{\mathrm{tot}}+\frac{P^2}{2m_{\mathrm{tot}}}+E'\right)}\,B\left(E'\right)\,. \tag{2}$$

While I focus here on the non-relativistic case, analogous equations can be written for the relativistic case.

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2. Resonant scattering

Consider first the case in which the interaction is dominated by a single resonance of mass $m_{\rm res}$ and width γ. The resonant phase shift can be written as

$$\mathcal{Q}(E) = \tan^{-1} \frac{\gamma(E)/2}{m_{\rm res} - E} \, . \tag{3}$$

The effective spectral function B assumes the standard Breit–Wigner form upon neglecting the energy dependence of the numerator $\gamma(E) \to \gamma_{\rm BW}$:

$$\begin{aligned} B_{\rm res}(E) &= 2 \frac{\mathrm{d}}{\mathrm{d}E} \, \mathcal{Q}_{\rm res}(E) \\ &\approx \frac{\gamma_{\rm BW}}{(E - m_{\rm res})^2 + \gamma_{\rm BW}^2/4} \\ &= -2 \, \mathrm{Im} \, \frac{1}{E - m_{\rm res} + i \, \gamma_{\rm BW}/2} \, . \end{aligned} \tag{4}$$

The partial pressure (2) becomes that of a free gas of resonances: treated as if they were a fundamental degree of freedom. In addition, when the width is narrow the pointlike gas result [6] is recovered as $B_{\rm res}(E) \to 2\pi\delta(E - m_{\rm res})$.

3. Non-resonant scattering

On the other extreme, I consider a structureless, non-resonant scattering. Here, one obtains [3]

$$2\mathcal{Q}(E) \approx 2 \, q(E) \, f \approx -\phi \, T_{\rm nr} \, , \tag{5}$$

where $q(E) = \sqrt{2 m_{\rm red} E}$ is the relative momentum in the CoM frame, f is the forward scattering amplitude

$$f(E) = \frac{\mathrm{e}^{i\mathcal{Q}}}{q} \, \sin \mathcal{Q} \, ; \tag{6}$$

and ϕ is the (non-relativistic) phase space

$$\phi(E) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, 2\pi\delta\left(E - \frac{q^2}{2 m_{\rm red}}\right) = \frac{m_{\rm red} \, q(E)}{\pi} \, . \tag{7}$$

This correctly identifies the non-relativistic T-matrix, $T_{\rm nr}$, as

$$T_{\rm nr} \approx -\frac{4\pi f}{2 m_{\rm red}} \, . \tag{8}$$

Note that we consider only the real part in this approximation. The thermal pressure in Eq. (2) reads

$$
\begin{aligned}
\Delta P &\approx \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}E'}{(2\pi)}\,\mathrm{e}^{-\beta\left(m_{\mathrm{tot}}+\frac{P^2}{2m_{\mathrm{tot}}}+E'\right)}2\mathcal{Q}\left(E'\right) \\
&= \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}^3q}{(2\pi)^3}\,\mathrm{e}^{-\beta\left(m_{\mathrm{tot}}+\frac{P^2}{2m_{\mathrm{tot}}}+\frac{q^2}{2m_{\mathrm{red}}}\right)}\left(-T_{\mathrm{nr}}\right) \\
&\approx N_{\mathrm{th}}^A\,N_{\mathrm{th}}^B \times \left(-T_{\mathrm{nr}}\right)\,.
\end{aligned}
\tag{9}
$$

The same result can be obtained when an in-medium mass shift of species A, due to their interactions with species B, is imposed

$$
\begin{aligned}
\Delta P &\approx T\int \frac{\mathrm{d}^3p_A}{(2\pi)^3}\,\mathrm{e}^{-\beta\left(m_A+\frac{p_A^2}{2m_A}\right)}\left(-\beta\Delta m_A\right) \\
&= -\Delta m_A N_{\mathrm{th}}^A = N_{\mathrm{th}}^A\,N_{\mathrm{th}}^B \times \frac{4\pi f}{2m_{\mathrm{red}}}\,,
\end{aligned}
\tag{10}
$$

where the shift in mass is given by [7–12]

$$
\Delta m_A \approx \int \frac{\mathrm{d}^3k_B}{(2\pi)^3}\,\mathrm{e}^{-\beta\left(m_B+\frac{k_B^2}{2m_B}\right)}\,T_{\mathrm{nr}}\,. \tag{11}
$$

The generalization to quantum statistics is straightforward. This demonstrates how “in-medium” effects can be included via “vacuum” phase shifts. After all, the Hamiltonian should contain all the necessary information [13]. Note that when the relevant experimental results are available to quantify the DoS, the thermal observables computed become model independent. This provides a useful framework to analyze the observables in heavy-ion collision experiments, such as hadron yields and the momentum distributions of light hadrons [14–21].

4. Going further

There are many other aspects in which the S-matrix phenomenology can improve a thermal model. For example, the roots in the complex plane of S-matrix encode details of non-resonant interactions. Their effects on the thermal trace are as important as those from the poles. Note that the empirical partial-wave amplitudes cannot be reconstructed from a list of resonances alone.

Another aspects is the ability to handle coupled-channel effects, which would be essential to reliably describe higher resonances. As a rule, multiple channels open up, and many of these states do not result in a strong enhancement in the phase shift and thus the partial pressure would not be well approximated by the free gas [22].

Theoretically, it remains challenging to understand the S-matrix elements in terms of quarks and gluons degrees of freedom: presumably, they are forbidden in the open channels, and at low temperatures, the S-matrix scheme should yield a gas of pions. Realizing this, in the S-matrix approach could yield novel insights into describing the thermal properties of interacting hadrons and, eventually, the deconfinement phase transition in QCD.

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