NET BARYON CUMULANTS IN VISCOUS HYDRODYNAMICS*

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A valuable tool used in the search for QCD's critical point is the computation of cumulants of conserved charge. Near this point, it is expected a sharp increase in this quantity due to the divergence of correlation lengths. This calculation requires high statistics, which poses a challenge to hydrodynamics simulations, which tend to be computationally expensive. The issue can be ameliorated by means of a procedure called oversampling, *i.e.* one repeats the Monte Carlo step of the particlization many times for a single hydro event. However, this has the drawback of removing the effects of fluctuations caused during the particlization. We use a toy model to demonstrate a method to compute cumulants (developed originally by Grassi, Hirayama, and Ollitrault) in a scenario where the oversampling procedure is employed and proceed to compute it for several collision energies.

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1. Introduction

One of the questions that ultrarelativistic nucleus–nucleus collisions aims to answer is the nature of the phase transition between the confined and deconfined matter. It was proposed that cumulants of conserved charges could be used as a probe for this phenomenon [1, 2].

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To obtain precise values for the aforementioned cumulants, a large number of events should be generated. Albeit the necessary statistics is feasible for experiments, it is a challenge for hydrodynamics-based simulations due to its computational cost. The usual workaround of oversampling particles during the Cooper–Frye procedure (see *e.g.* [3, 4]) introduces biases in observables which are not based on averages.

In this work, we build an initial condition with non-zero baryonic density by smearing the partonic cascade of AMPT [5] in a similar fashion as was done in Ref. [6]. We evolved the generated initial conditions in three distinct energies in full (3+1)D hydrodynamics with up to second-order viscous corrections [7–9]. We perform the oversampling of the Cooper–Frye procedure [4, 10] and employ the two-step averaging proposed in Ref. [11] to calculate the cumulants of protons, anti-protons, and net-protons for central Au+Au collisions in energies from 19.6 GeV to 200 GeV.

2. Initial-condition model and hydrodynamic simulation

The main requirement for a desirable initial condition in this work is the presence of non-zero baryonic density. To this end, we smear partons from the AMPT [5] events on the string-melting mode, in a procedure we will detail below.

The AMPT model uses HIJING [12] to simulate the nucleus–nucleus collisions, followed by a partonic cascade. At the end of this partonic cascade, partons are hadronized and a hadronic cascade is then simulated. We intercept partons during the partonic cascade at a hypersurface defined by the hyperbola $\tau_0 = \sqrt{t^2 - z^2}$. The energy-momentum tensor as well as a baryon-density profile is built by smearing the partons in a similar procedure as in Ref. [6]

$$T^{\mu\nu}(x, y, \eta_s) = \sum_{i=\text{partons}} \frac{p_i^\mu p_i^\nu}{p_i^\tau} \phi_i(x, y, \eta_s) \,, \tag{1}$$

$$\rho_B(x, y, \eta_s) = \sum_{i=\text{partons}} \frac{Q_i}{K} \phi_i(x, y, \eta_s) \,, \tag{2}$$

$$\phi_i(x, y, \eta_s) = \frac{K}{(2\pi)^3 \sigma_r^2 \tau \sigma_s} \, \mathrm{e}^{-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_r^2} - \frac{(\eta_s-\eta_i)^2}{2\sigma_\eta^2}} \,, \tag{3}$$

where p_i^μ, Q_i, x_i, y_i, and η_i refer to the momentum, baryonic charge, and position (in hyperbolic coordinates) of the i^{th} parton. K, σ_r, and σ_η are adjustable parameters. The values for the parameters σ_r and σ_η as well as the time τ_0 in which we intercept the partons are the same as in Ref. [6].

Once the energy-momentum tensor is obtained, we perform the Landau-matching procedure as described in Ref. [13]

$$T^{\mu}_{\ \nu}u^{\mu} = \varepsilon u^{\nu}\,, \tag{4}$$

$$\Pi = \frac{\varepsilon - T^{\mu}_{\ \mu}}{3} - P(\varepsilon)\,, \tag{5}$$

$$\pi^{\mu\nu} = \frac{u^{\mu}u^{\nu}}{3}(T^{\alpha}_{\ \alpha} - 4\varepsilon) + \frac{1}{3}(\varepsilon - T^{\alpha}_{\ \alpha})g^{\mu\nu} + T^{\mu\nu}\,. \tag{6}$$

Once the hydrodynamic components are obtained, we evolve them with (3+1)D relativistic viscous hydrodynamics [7–9]. We use the same parameters for transport coefficients as in Ref. [3] and the NEOS equation of state [14]. These are parameters more appropriate to LHC energies and it is one of our main limitations, since we are interested in RHIC energies.

3. Data generation and analysis

Before we proceed to compute the cumulants, we adjust two other parameters. The constant K present in Eqs. (2) and (3) is chosen to reproduce $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$ and the energy density used to determine the particlization hypersurface is chosen to approximately reproduce $\mathrm{d}N_{p}/\mathrm{d}y$ and $\mathrm{d}N_{\bar{p}}/\mathrm{d}y$. This procedure was done for central Au+Au collisions at energies of $\sqrt{s_{NN}} = 19.6, 62.4$, and 200 GeV.

Once these two final parameters are tuned, we generated 100 events for centrality 0–5%, with each event being oversampled a thousand times. We are then in a position to compute the cumulants of proton, antiprotons, and net-protons. The procedure is done as in [11], which we briefly describe here for completeness.

First, one writes the cumulants in terms of the moments. As an example, C_2 can be written as $C_2 = \mu_2 - \mu_1^2$. The n^{th} moment can be obtained from the moment generating function $\mathrm{MGF}_X(z) = \langle e^{zX}\rangle$ by evaluating the n^{th} derivative $\mathrm{MGF}_X^{(n)}(0)$. The average $\langle\cdot\rangle$ is an average over each sample, and over the many samples generated. The novelty of the procedure proposed by Hirayama *et al.* [11] was the proposition to decompose this average into an inner average over samples (from the same hydrodynamic event) and an outer average over hydrodynamic events. One can then propose a probability for a given sample to create the particle of interest in a given kinematic region and evaluate the inner average analytically.

As in [11], we assume that the total number of protons (antiprotons) that may be generated during a hydrodynamic event to be a constant N_{max} ($\bar{N}_{\mathrm{max}}$). Then, the number N ($\bar{N}$) of protons (antiprotons) that will be observed inside a given kinematic region will follow a binomial distribution, with a detection probability α ($\bar{\alpha}$).

The values for $N_{\rm max}$ ($\bar{N}_{\rm max}$) and α ($\bar{\alpha}$) are estimated as

$$N_{\rm max} = \langle N_{p,{\rm total}} \rangle_{\rm freeze-out} \qquad \bar{N}_{\rm max} = \langle \bar{N}_{p,{\rm total}} \rangle_{\rm freeze-out} \,, \tag{7}$$

$$\alpha = \frac{\langle N \rangle_{\rm freeze-out}}{N_{\rm max}} \qquad \bar{\alpha} = \frac{\langle \bar{N} \rangle_{\rm freeze-out}}{\bar{N}_{\rm max}} \,. \tag{8}$$

One important remark that must be done is that statistical errors in the determination of parameters of the distribution may lead to biases in the cumulant calculations. Hence, it is paramount to use a large enough number of samples to ensure any biases introduced to be smaller than the statistical errors. We tested with a toy model that the used number of samples per hydrodynamic event is enough in our case.

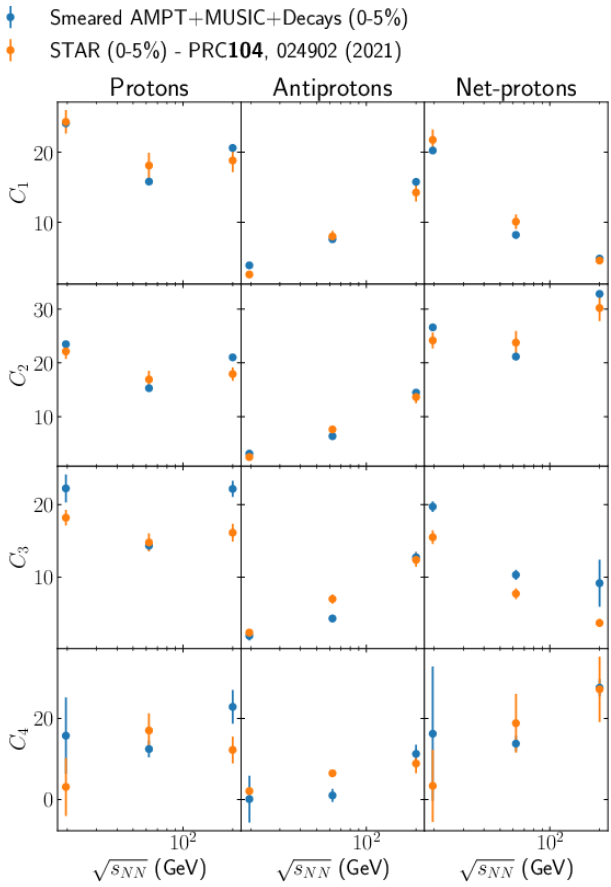

Fig. 1. Cumulants of proton, antiprotons, and net-protons in three different collisions energies of Au+Au in central collisions (0–5%) computed using hydrodynamic-based simulations and its comparison with experimental data from [15].

Results for the cumulants C_1, C_2, C_3, and C_4 for protons and antiprotons are presented in Fig. 1. We draw attention that error bars are competitive in size with experimental ones (statistical plus systematic).

4. Conclusions

In this work, we showed that it is possible to compute cumulants of conserved charges using (3+1)D viscous hydrodynamics with very few hydrodynamic events. To this end, we built initial conditions that include all hydrodynamic components based on the smearing of the AMPT parton cascade. Despite the hydrodynamic transport being tuned to LHC energies, we managed to get reasonable (albeit not accurate) even at energies two-order of magnitudes below (without transport).

The fact that we obtained reasonable results in a simulation which does not contain any kind of critical fluctuations suggests that the critical point is not within the region probed. We intend in the future to study the effects on the cumulants of a critical point in the equation of state in the probed regions of density and temperature.

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