PROBING FLOW FLUCTUATIONS THROUGH FACTORIZATION BREAKING OF HARMONIC FLOWS IN HEAVY-ION COLLISION*

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We study factorization-breaking coefficients between the momentum-dependent and momentum-averaged flow vectors to probe flow fluctuations caused by the initial-state fluctuations in heavy-ion collision. The coefficients for the flow vector *squared* and flow magnitude *squared* could be used for the extraction of flow angle decorrelations. We compare our model results with preliminary experimental data. We also present the predictions for the momentum-dependent correlation between mixed flow harmonics.

1. Introduction

The dense matter created in the interaction region of a heavy-ion collision expands rapidly. Due to the asymmetry of the initial source distribution, the collective expansion leads to an azimuthal momentum anisotropy of the spectra of particles

$$\frac{\mathrm{d}N}{\mathrm{d}p\,\mathrm{d}\phi} = \frac{\mathrm{d}N}{2\pi\mathrm{d}p}\left(1 + 2\sum_{n=1}^{\infty} V_n(p)\,\mathrm{e}^{in\Psi_n}\right), \tag{1}$$

where the Fourier coefficients $V_n(p)'s$ are called the harmonic flow coefficients.

The initial source fluctuates event-by-event (ebe), leading to the ebe fluctuation of the harmonic flow coefficients in the final state [1, 2]. These flow fluctuations can be probed by studying the correlation coefficient between the harmonic flow vectors in two different transverse-momentum bins [3],

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the *factorization-breaking coefficients*. The fluctuations result in the decor-relation (deviation of the correlation coefficients from 1) between these flow vectors, which manifests the combined effect of flow magnitude decorrela-tion as well as the flow angle decorrelation [4, 5]. These two effects can be separated by constructing four particle correlators between two transverse-momentum bins. However, to avoid the experimental difficulty due to the low statistics at higher momenta, the coefficients are constructed keeping two particles fixed at transverse-momentum bins (differential flow) and the other two are momentum averaged (integrated flow) [6]. We compare our model results with the preliminary results by the ALICE Collaboration. We also study the similar factorization coefficients between the mixed flow harmon-ics *e.g.* V_2^2–$V_4(p)$ or $V_2 V_3$–$V_5(p)$, which serve as measures of the non-linear hydrodynamic response of the medium.

2. Factorization breaking for harmonic flow vector, magnitude, and angle

The complex flow vector $V_n(p) = v_n(p) \, e^{in\Psi_n(p)}$ in Eq. (1) denotes the harmonic flow vector of the order of n for particles emitted at the transverse momentum p; v_n and Ψ_n are the corresponding flow magnitude and angle. In general, one can construct the four-particle correlators, measuring the factorization-breaking coefficients between two flow vectors *squared* at two different transverse-momentum bins as [3]

$$r_{n;2}(p_1, p_2) = \frac{\langle V_n^2(p_1) V_n^\star (p_2)^2 \rangle}{\sqrt{\langle v_n^4(p_1) \rangle \langle v_n^4(p_2) \rangle}} \, , \qquad (2)$$

and similarly for the flow magnitude *squared*, the factorization-breaking co-efficient can be constructed as [7]

$$r_n^{v_n^2}(p_1, p_2) = \frac{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}{\sqrt{\langle v_n^4(p_1) \rangle \langle v_n^4(p_2) \rangle}} = \frac{\langle v_n^2(p_1) v_n^2(p_2) \rangle}{\sqrt{\langle v_n^4(p_1) \rangle \langle v_n^4(p_2) \rangle}} \, . \qquad (3)$$

However, in practice, the above formulas are difficult to use in experiment due to the low statistics in higher momentum bins.

To ease this difficulty, an alternative approach proposed by the AL-ICE Collaboration, would be to keep only one of the flows in a transverse-momentum bin and the other flow averaged over the momentum range. Following this, we construct the factorization-breaking coefficient using the four-particle correlator for the flow vector squared and flow magnitude squared as [6]

$$r_{n;2}(p) = \frac{\langle V_n^2 V_n^\star(p)^2 \rangle}{\sqrt{\langle v_n^4 \rangle \langle v_n^4(p) \rangle}} \qquad \text{and} \qquad r_n^{v_n^2}(p) = \frac{\langle v_n^2 v_n^2(p) \rangle}{\sqrt{\langle v_n^4 \rangle \langle v_n^4(p) \rangle}} \, . \qquad (4)$$

We use the boost-invariant version of the hydrodynamic model MUSIC [9] for Pb+Pb collisions at 5.02 TeV. For the initial state, two models are used; a two-component Glauber Monte Carlo model [10] and the T$_R$ENTo model [11]. We use a constant shear viscosity to entropy density ratio $\eta/s = 0.08$, unless otherwise specified. In Fig. 1. there are shown the factorization-breaking coefficients between the flow vectors squared (left) and flow magnitudes squared (right) for the elliptic flow. For 0–5% centrality, our model results corroborate the ALICE preliminary data. However, for semi-central collision (30–40%), our model results do not fully describe the data. Also one can see in Fig. 1 that the decorrelation for the magnitudes, $r_2^{v_2^2}(p)$, is approximately one-half of the decorrelation for the vectors $r_{2;2}(p)$, which manifests the fact that the total flow vector decorrelation involves equal contribution from the flow magnitude and the flow angle decorrelation; $[1 - r_{n;2}(p)] \simeq 2[1 - r_n^{v_n^2}(p)]$.
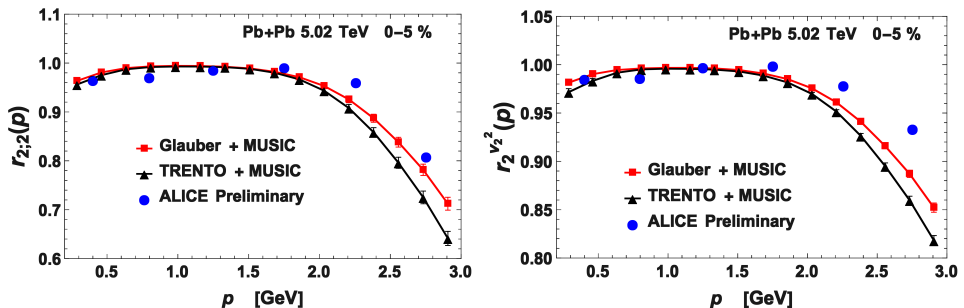

Fig. 1. (Colour on-line) The factorization-breaking coefficient between flow vectors squared $V_2(p)^2$ and V_2^2 (left) and the flow magnitudes squared $v_2(p)^2$ and v_2^2 (right) for the elliptic flow. The red squares and black triangles denote the results obtained with the Glauber model and the T$_R$ENTo model initial conditions, respectively. The experimental data of the ALICE Collaboration are represented using blue dots (figure from [8]).

Once we have the flow vector and flow magnitude factorization-breaking coefficients separately, we can estimate the flow angle decorrelation by constructing the correlator for flow angle, taking the ratio of the four-particle flow vector correlator and flow magnitude correlator Eq. (4)

$$F_n(p) = \frac{\langle V_n^2 V_n^\star(p)^2 \rangle}{\langle v_n^2 v_n^2(p) \rangle} \ . \tag{5}$$

The above correlator is an experimental measure of the flow angle decorrelation

$$F_n(p) = \frac{\langle v_n^2 v_n(p)^2 \cos\left[2n\left(\Psi_n(p) - \Psi_n\right)\right] \rangle}{\langle v_n^2 v_n(p)^2 \rangle} \ . \tag{6}$$

However, the actual angle decorrelation could be defined as [7]

$$\frac{\langle v_n^4 \cos\left[2n\left(\Psi_n(p) - \Psi_n\right)\right]\rangle}{\langle v_n^4 \rangle} \ . \tag{7}$$

The results presented in Fig. 2 indicate that the two formulas give similar results, suggesting that the experimental measure of Eq. (5) could be used to estimate the weighted flow angle decorrelation. Our model results describe the experimental angle decorrelation. On the other hand, the simple average of the decorrelation angle

$$\langle \cos\left[2n\left(\Psi_n(p) - \Psi_n\right)\right]\rangle \tag{8}$$

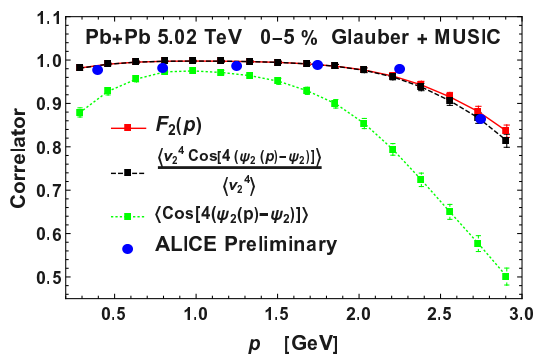is very different and cannot be estimated directly from the data.

Fig. 2. (Colour on-line) The flow angle correlation estimated as the ratio of the flow vector and the flow magnitude factorization-breaking coefficients (Eq. (5)). The symbols with the dashed lines denote the angle correlations calculated directly from the simulated events (Eq. (7)). The green squares represent the simple average of the decorrelation angle (Eq. (8)).

3. Mixed flow correlations

Similar correlations coefficients can be constructed between mixed flow harmonics as a measure of non-linear coupling between the flow harmonics. Such correlation coefficients are sensitive to the non-linear response of the hot dense medium [12]. For example, one can construct the correlation coefficients between V_2^2 and $V_4(p)$ or between $V_2 V_3$ and $V_5(p)$. However, to extract the angle decorrelation between these mixed flows, one needs to construct the correlation coefficient (or factorization-breaking coefficients) between the squares of the harmonics used. The mixed flow vector–vector, magnitude–magnitude, and angle decorrelation are given by

$$\frac{\langle V_2^4 V_4^\star(p)^2\rangle}{\sqrt{\langle v_2^8\rangle\langle v_4^4(p)\rangle}}\,,\qquad \frac{\langle v_2^4 v_4(p)^2\rangle}{\sqrt{\langle v_2^8\rangle\langle v_4^4(p)\rangle}}\qquad \text{and}\qquad \frac{\langle V_2^4 V_4^\star(p)^2\rangle}{\langle v_2^4 v_4(p)^2\rangle}\,. \tag{9}$$

In Figs. 3 and 4, there are shown the results predicted in the model for the mixed flow factorization-breaking coefficients between V_2^4 and V_4^2 for flow vector, magnitude, and angle decorrelations. Again, we notice that the flow magnitude decorrelation is approximately one-half of the flow vector decorrelation. The horizontal lines denote the baseline for the correlations between the momentum-averaged flow harmonics.
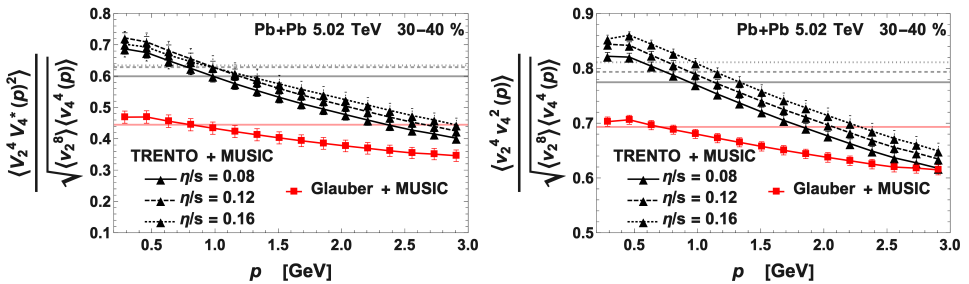

Fig. 3. The flow vector (left) and flow magnitude correlation (right) between V_2^4 and $V_4(p)^2$. For the T$_R$ENTo model initial conditions, the solid, dashed, and dotted lines represent results with $\eta/s = 0.08$, 0.12, 0.16, respectively. The horizontal lines represent the corresponding correlation coefficients for momentum averaged flow vectors V_2^4 and V_4^2 (figure from [8]).
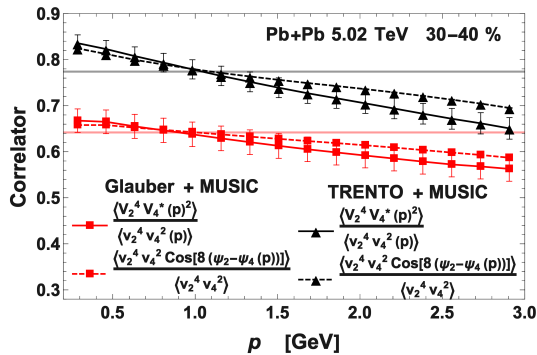

Fig. 4. The flow angle correlation between V_2^4 and $V_4(p)^2$ (figure from [8]).

4. Summary and outlook

We study the factorization-breaking coefficients between the harmonic flow coefficient in a fixed transverse-momentum bin and the average flow har-

monic. Such factorization coefficients show significant decorrelations which is a direct consequence of the initial-state fluctuation. The separate measurement of the flow vector and magnitude decorrelation lead to the extraction of flow angle decorrelation, where the magnitude and the flow angle decorrelations constitute approximately equal parts of the total flow vector decorrelation. Our model results reproduce well the ALICE preliminary data in central collisions, but deviate in the case of semi-central collisions. We also present predictions for the momentum-dependent mixed flow correlations, which measure the non-linear response of the hydro-evolution and put additional constrains on the models. Our results could be verified in experiments.

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