KINETICS OF THE CHIRAL PHASE TRANSITION IN A QUARK–MESON σ MODEL*

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Using the two-particle irreducible (2PI) Φ-functional formalism for self-consistent approximations of a linear-σ model for quarks and mesons in and out of equilibrium, the build-up of fluctuations of the net-baryon number during the time evolution of an expanding fireball is studied within a kinetic theory for the order parameter (σ field) and quark distribution functions. Initializing the system with purely Gaussian fluctuations, a fourth-order cumulant is temporarily built up due to the evolution of the σ field. This is counterbalanced, however, by the dissipative evolution due to collisions between quarks, anti-quarks, mesons, and the mean field, depending on the speed of the fireball expansion.

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1. Introduction

One important motivation for ultra-relativistic heavy-ion experiments, as conducted, *e.g.*, with the Large Hadron Collider at CERN, the Relativistic Heavy Ion Collider (RHIC) at BNL, and in the future at the Facility for Antiproton and Ion Research (FAIR) is the understanding of the phase diagram of strongly interacting matter under extreme conditions of temperature and density. For small baryo-chemical potentials, μ_B, lattice-QCD calculations [1, 2] show that the transition between a quark–gluon plasma and a hadron-resonance gas as well as the chiral transition is a smooth crossover at a transition temperature $T_{\rm c} \simeq 155$ MeV. Based on effective models such as the Nambu–Jona-Lasinio model, quark–meson models with constituent quarks [3–6], and their Polyakov-loop extended versions [7–10], at larger μ_B, one

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expects a 1st-order transition line ending in a critical point with a 2nd-order transition [11–14]. The main challenge is that this phase structure must be reconstructed from the observables, which reflect the state of this medium at the end of the fireball evolution (thermal freeze-out), which lasts only for a very short time at the order of some $10\,\text{fm}/c \simeq 10^{-23}\,\text{s}$. A challenging theoretical question therefore is whether "grand-canonical" higher-order cumulants of the net-baryon density can develop and survive the rapid time evolution of the finite-size fireball, expected to occur when the medium is undergoing a 1st- or especially a 2nd-order phase transition, and whether corresponding quantitative signatures of a possible critical point can be observed.

In this contribution, we study this, employing a set of coupled equations for the quarks, anti-quarks, and mesons as well as the order parameter, σ, of the chiral symmetry within a linear quark–meson σ model, derived from the two-particle irreducible functional (Φ functional) formalism [15].

2. The kinetic equations

We start from an O(4) linear-σ model for σ-mesons, pions, and u- and d-quarks

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{i=1} \bar{\psi}_i \left[\mathrm{i}\partial\!\!\!/ - g\left(\sigma + \mathrm{i}\gamma_5 \vec{\pi}\cdot\vec{\tau}\,\right)\right]\psi_i \\ & + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi}\,\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0\,, \quad (1)\end{aligned}$$

where $\lambda = 20$, $f_\pi = 93$ MeV, $m_\pi = 138$ MeV, $\nu^2 = f_\pi^2 - m_\pi^2/\lambda$, and $U_0 = m_\pi^4/(4\lambda) - f_{\pi^2} m_{\pi^2}$ are chosen to lead to the right pion phenomenology in the vacuum. The quark–meson-coupling constant g is varied in the range between 2–5, leading to cross-over as well as 1st- and 2nd-order chiral phase transitions at finite T and μ_B.

For the kinetic equations to describe both the equilibrium state as well as the off-equilibrium kinetic evolution of this model, we use the 2PI Φ-derivable approximation, defined in terms of the corresponding Feynman diagrams in Fig. 1. Solving the corresponding self-consistent equations for the propagators and the mean σ field in thermal equilibrium indeed leads to a phase diagram with a cross-over transition at lower μ_B and a first-order transition line ending in a critical point at $(T, \mu_B) = (108, 157)$ MeV (for a quark–meson coupling, $g = 3.3$).

For the derivation of coupled kinetic equations of motion for the mean σ field and the generalized Boltzmann equations for the quark- and meson-phase-space-distribution functions, the diagrams are evaluated within the

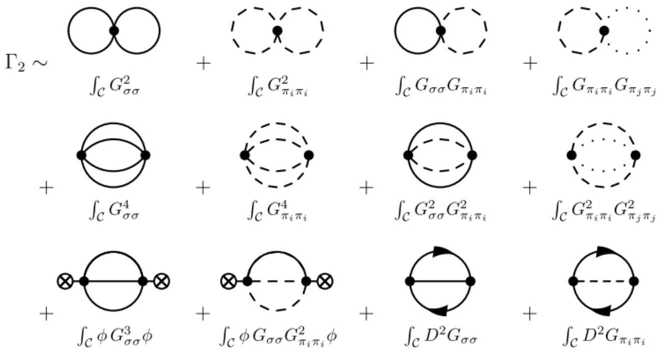


Fig. 1. Included 2PI part of the effective action with 1st line: Hartree diagrams, 2nd line: basketball diagrams, 3rd line: sunset diagrams, where solid lines stand for the σ propagator, dashed and pointed lines for the pion propagators, and solid lines with arrows for the fermion propagator. The circle with a cross represents a σ mean field.

Schwinger–Keldysh real-time formalism, leading to corresponding Kadanoff–Baym equations. Then a first-order gradient-expansion approximation to the Wigner transforms of Green's functions as well as an "on-shell approximation" with self-consistent dispersion relations has been applied. This results in a non-Markovian dissipative equation for the mean field, σ, and a Boltzmann equation with a collision integral including the scattering processes depicted in Fig. 2.

3. Simulation of a heavy-ion collision

To simulate the formation of higher-order cumulants of net-baryon-density fluctuations in momentum bins, we describe the fireball of strongly interacting quark–meson matter by an expanding homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker metric, $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)$. This only leads to a modification in the drift terms of the mean-field and kinetic equations. For the mean field, the "Hubble expansion" adds an additional dissipation term, $3H\partial_t\sigma$, with the "Hubble constant" $H = \dot{a}/a$, as well as an additional term of the form of $-Hp\partial_p f(t,p)$ in the drift terms for the particle phase-space distribution. Note that due to the assumed spatial homogeneity and isotropy, the fs only depend on t and $p = |\vec{p}|$.

To initialize the fireball, a spherically symmetric bubble of radius $R_0 = 5$ fm is considered, which then is expanding according to the above defined FLRW expansion with $a = vt$. The medium within this bubble is initialized in thermal equilibrium with a temperature T_0 and baryon chemical potential μ_{B0}. Then the initial net-quark number is Monte-Carlo sampled correspond-

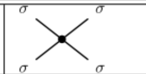
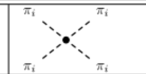
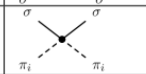
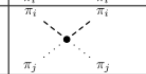
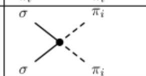
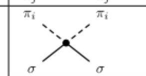
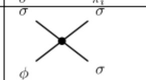
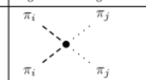
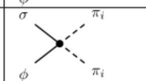
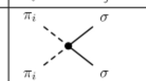
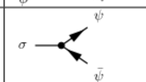
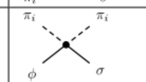
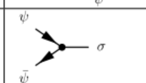
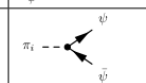
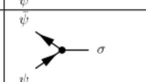
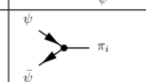
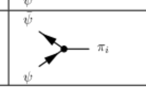
collision integral	diagram	collision integral	diagram
$\mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}$	
$\mathcal{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}$	
$\mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma}$	
$\mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}$	
$\mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$	
$\mathcal{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}$		$\mathcal{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma}$	
$\mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma}$		$\mathcal{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$	
$\mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma}$		$\mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$	
		$\mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}$	

Fig. 2. The scattering processes in the collision integrals of the kinetic equation. A full σ-line indicates a mean-field contribution, while a full ϕ-line describes a scattering process involving a σ meson.

ing to a Gaussian distribution with the mean determined by the thermal initial state and a standard deviation of $\sigma_{q,\text{net}} = \langle N_{q,\text{net}}\rangle/10$. To mimic the expected fluctuations in a heavy-ion collision within a given "centrality bin", we keep the parameters R_0 and T_0 fixed and adjust μ_q such that the fireball contains the net-quark number $N_{q,\text{net}}$ specified by the Monte-Carlo sampling.

With these initial conditions, the coupled mean-field and kinetic integro-differential equations of motion are solved numerically on a momentum grid. It has been checked that the total net-quark number is conserved within a few percent numerical accuracy.

In Fig. 3, we show the results for the cumulant ratio, $R_{4,2} = \kappa_4/\kappa_2$, for initial conditions adjusted such that the system undergoes cross-over, second-order, and first-order transition, respectively. The fluctuations are plotted in different momentum intervals and for different expansion velocities, v, as a function of vt. I turns out that the most pronounced fluctuations occur at the critical time scales $\tau_{m_\sigma,\text{min}}$ (dynamical minimum of the σ mass) and $\tau_{\sigma \to q\bar{q}}$ ($q\bar{q}$-pair production from σ decay). The fluctuations become largest for the smallest expansion velocity of $v = 0.05c$, corresponding to a quasi-adiabatic expansion, where the system stays for the longest time close to the critical region. However, in relativistic heavy-ion collisions this intermediate build-up of fluctuations related with the critical region of the phase diagram cannot be observed but only those surviving until the thermal freeze-out, which corresponds in our model to $vt \geq 6$ fm and a fireball radius of $R \geq 11$ fm.

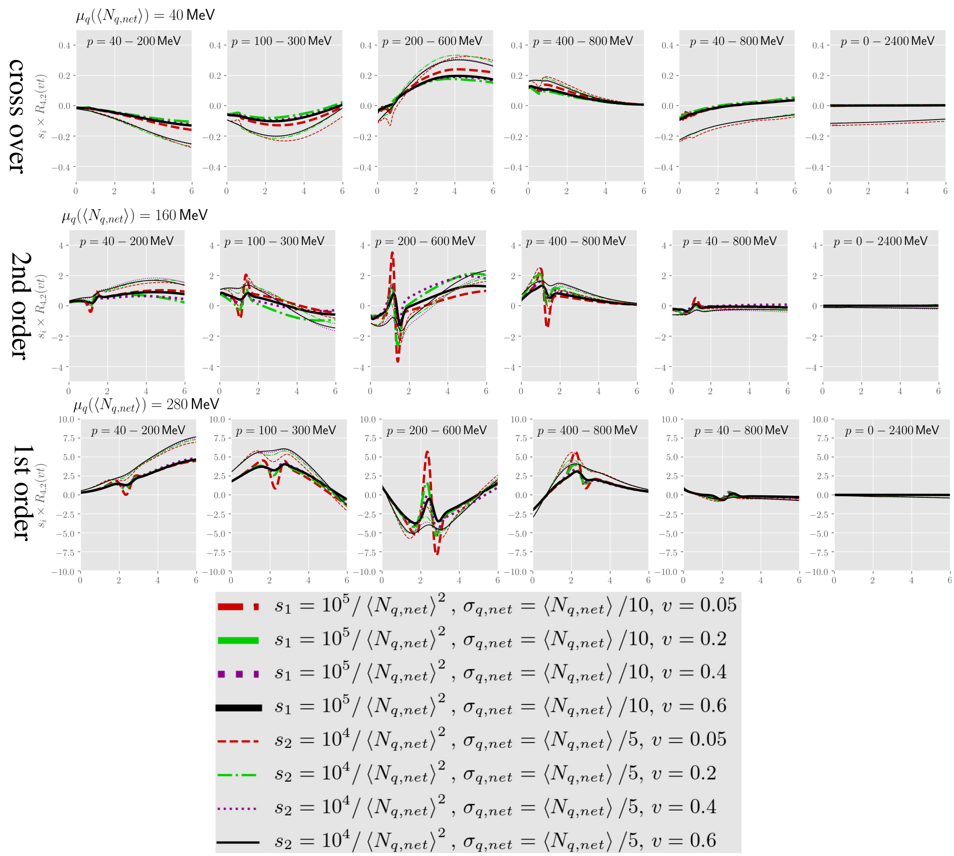


Fig. 3. Results for the rescaled cumulant ratio $R_{4,2}$ for different initial conditions where the fireball evolves through a cross-over, second-order, or first-order transition.

In the most interesting case, $\mu_q = 160$ MeV, where the system evolves close to the critical point of a 2^{nd}-order phase transition, the largest cumulant ratio in the final state is observed for intermediate expansion velocities $v = 0.2$–$0.4c$, while for the case when the system goes through a 1^{st}-order phase transition, the final fluctuations are rather insensitive to the expansion velocity (for the most interesting momentum range $p = 200$–600 MeV). This allows, in principle, to distinguish between different types of the phase transition and indicates the expected longer relaxation times ("critical slowing down") around a critical point in the phase diagram, *i.e.*, the system needs longer to equilibrate and thus the fluctuations survive until the thermal freeze-out. The absolute magnitude of the cumulant ratio increases with an increasing net-baryon number (note the scaling factors $s_1, s_2 \sim 1/\langle N_{q,\mathrm{net}} \rangle^2$ in the plots of Fig. 3).

4. Conclusions

Although the fluctuations of net-baryon numbers in an expanding finite system are less pronounced compared to the expectations from an equilibrated infinite strongly interacting matter, our simulations suggest that a significant deviation from the crossover behavior is observable through higher-order cumulant ratios in different momentum bins, providing a positive candidate for an experimental signature of the chiral phase transition and a possible critical region in the phase diagram of strongly interacting matter.

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