A NONLINEAR QUARK–GLUON CASCADE CONVERGES AND TRANSITS TO A CHAOTIC REGIME*

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Computer simulations of the transition of quarks to hadrons, hadrons to quark–gluon plasma, and plasma to hadrons have been carried out. Nonlinear quark–gluon dynamics is considered a quantum process within the framework of discrete mappings. The dynamic variable is the momentum fraction (x) of the QCD parton, which acts as a one-dimensional Poincaré section in the momentum phase space. The probability of finding a certain fraction of the momentum of a parton at a given moment is determined by the momentum distribution of the partons at the previous moment in time. At critical values of the control parameter, bifurcations of phase quark–gluon trajectories occur. As a result of the counteraction of the processes of emission and absorption of gluons, stable attractor quark–gluon structures are formed. The Poisson stability is determined by the Lyapunov exponents. The sequence of bifurcations converges and chaos arises. The change from regular quark–gluon dynamics to irregular chaotic one corresponds to the limit of multiple hadronic processes and the emergence of quark–gluon matter in the deconfinement state. Chaotization of the dynamical system leads to thermalization of the quark–gluon medium.

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1. Introduction

As is known, problems arise in the mathematical method of describing quantum chromodynamics (QCD) in the nonperturbative region at large distances, when the perturbation theory is inapplicable. Accounting for the contribution of gluon emission to the quark–gluon evolution leads to violation of the Bjorken scaling and is determined by the well-known linear equations DGLAP [1–3], BFKL [4–6]. Gluon emission in QCD is determined by the splitting functions $F_{g/g}(z), F_{g/q}(z), F_{q/g}(z), F_{q/q}(z)$ of the probability to find a gluon in a gluon (g/g) or a gluon in a quark (g/q) with a momentum fraction z. However, to take into account the fusion of gluons, it is necessary to introduce the phenomenological probability of the fusion of QCD quanta. Many different methods for modeling evolutionary equations with allowance for gluon recombination have been proposed [7–9]. The action Yang–Mills (Y–M) already contains cubic and quartic nonlinear interaction terms in the field strength tensor $S_{\mathrm{Y-M}} = -\frac{1}{4}\int \mathrm{d}^4x F^a_{\mu\nu}(x)F^{\mu\nu}_a(x)$.

In the quantum world, processes are probabilistic in nature. The amplitude of the transition probability from one state to another is the sum of the amplitudes of all possible trajectories and is written as a functional integral: $\psi = \int \mathrm{e}^{\frac{iS(x)}{\hbar}} Dx(t)$ with $\hbar$ being the Planck constant. The action $S(x)$ is an operator of quantum evolution. $Dx(t)$ is a conditional functional integration over all trajectories $x(t)$. Fast phase fluctuations in the imaginary exponent cancel each other out, and only trajectories with minimal action remain. In the quantum world, one can speak of well-defined amplitudes of trajectories, and the particle moves not along one chosen amplitude, but along an infinite set with the same start and end points. The particle can move along any trajectory, and the amplitude of this trajectory, in response, will be included with a certain weight.

2. Nonlinear quark–gluon cascade

Considering evolution in nonlinear dynamics as a discrete quantum process, we use the mathematical apparatus of discrete mappings instead of differential equations. The nonlinear equation is introduced [10, 11] as an evolution of the nucleon structure function $F_2(x, Q^2)$. Using the method of Poincare sections and choosing the share of momentum as a one-dimensional section of the phase space of partons momentum distribution (PDF), we have an evolution equation

$$x_{t+1} = \lambda F(x_t)\,. \tag{1}$$

Here, Bjorken's or Feynman's variable x_t is the momentum fraction at a discrete time $(t = 0, 1, 2\ldots)$, λ is a control parameter characterizing the degree of correlation of a given parton with other partons at a certain energy/temperature. The control parameter determines the nature of the evolution regimes. Switching to continuous time allows the construction, known as the Poincare section. In the framework of our quality approach, we use the renormalization-group (RG) approach to the evolution equation, allowing to recreate a physical picture of the critical behavior. The number of partons varies with the energy scale, but the total momentum is naturally conserved. For a nonlinear quark–gluon cascade, the probability of finding a certain fraction of the parton momentum x_{t+1} at a given moment $(t+1)$ is determined by the momentum distribution of partons at the previous time moment (t). The positive terms of the parton momentum distribution correspond to an increase in the number of quarks and gluons in the cascade, while the negative terms correspond to a decrease. The negative members of the distribution correspond to recombinations: quark–antiquark, quark–gluon, and gluon–gluon.

3. Numerical solution of the nonlinear equation

The numerical solution of the nonlinear cascade of parton distribution functions (PDF) showed the termination of evolution in the region of small values of the control parameter $\lambda < 0.25$. Small perturbations do not change the PDF. The increase in λ leads at first only to the excitation stable state. With a further increase in the parameter, repeated period doubling bifurcations occur. Numerical calculations show that the bifurcation sequence λ_m quickly converges at $\lambda_\infty = 0.892$ and chaos is observed. The scale of successive splittings of elements of limit cycles after each bifurcation is determined by

$$\alpha_{\mathrm{F}} = \lim_{m\to\infty}\left(\frac{x_m - x_n}{x_{m+1} - x_n}\right) \cong 2.5\,, \tag{2}$$

$$\lim_{m\to\infty}\frac{\lambda_m - \lambda_{m-1}}{\lambda_{m+1} - \lambda_m} = \delta_{\mathrm{F}} = 4.6692\,, \tag{3}$$

where α_{F} and δ_{F} are the Feigenbaum constants [12], x_m is the element of a limit cycle nearest to the element cycle x_n.

The structure of the bifurcation diagrams display of PDF self-similar and thus, the chaotic system has inherent properties of fractals. Fractal analysis of PDF F_m carried out the averaging over all k values, defined as

$$F_m = \frac{1}{N-2^m}\sum_{k=0}^{N-2^m}\left|x_{k+2^m} - x_k\right|. \tag{4}$$

In a state of dynamic chaos, two close orbits in phase space diverge exponentially with time with Lyapunov's coefficient in the exponent: $\alpha_{\mathrm{L}} =$

$\frac{\ln|\mu|}{T}$, which in a computer simulation is calculated using parallel running of two close initial conditions and examines their divergence. Computer simulation was used to study the formation of stable structures in a quark-gluon cascade. The nature of stability of fixed points (cycles) and the type of bifurcations of mappings is determined by their multipliers. In turn, multipliers are the own numbers of the Jacobian matrix perturbations. The maximum value x_{t+1} is found from $\mathrm{d}x_{t+1}/\mathrm{d}x_t = 0$. The Jacobian is $J = |\frac{\mathrm{d}x_{t+1}}{\mathrm{d}x_t}|$ and the map is stable at a point x_0 if $J(x_0) < 1$. When the coupling constant $\alpha_\mathrm{S}(Q^2)$ is small, the evolution is incoherent, if the relationship is strong enough that spontaneous synchronization quark–gluon movements can occur. The dynamics of quark–gluon systems is very sensitive to initial conditions. The Lyapunov exponent is calculated for stationary periodic and chaotic processes.

Analyzing only the structures of attractors without taking into account transient processes and iterating the mapping for different initial values with a slow change in the control parameter, we obtain the results shown in figure 1.

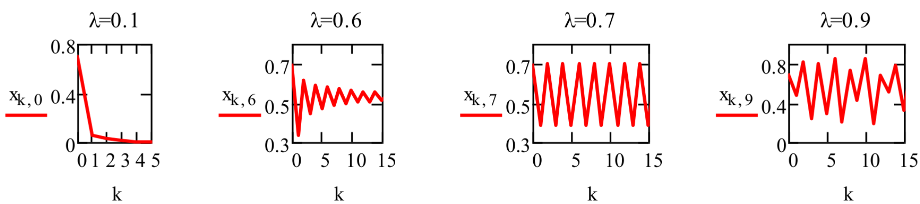

Fig. 1. Different regimes of nonlinear evolution for different values of the control parameter λ.

In a nonlinear quark–gluon cascade, the counteraction of intense radiation and absorption processes leads to the formation of asymptotically Poisson-stable states. These are the so-called attractors. At values of the control parameter close to unity, regular ordered dynamics of the quarks and gluons transforms into irregular dynamics with exponentially diverging phase trajectories. That is how a chaos arises. The transition to the chaos state means a transition to the state of deconfinement: the partons cease to be bound in the limit of hadron size and form the QGP state. The confinement–deconfinement transition, as well as the restoration of chiral symmetry, is described on the basis of the transition of the QCD partons by nonlinear regular evolution to the state of irregular dynamic chaos.

A difference between chaotic and non-chaotic modes is given in figure 2, where we select the modulus of the difference between phase trajectories with close initial values.

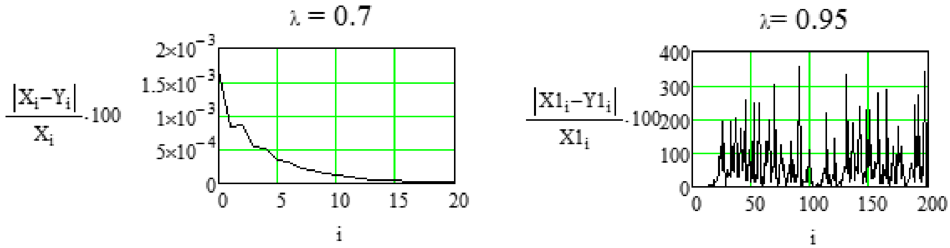

Fig. 2. Regular and chaotic modes.

The figure shows how small (10^{-3}) the regular phase trajectories differ (left) and how much diverge (10^{+2}) in the chaotic case (right).

The processes of multiple hadron production correspond to the bifurcations of phase trajectories followed by the formation of stable structures. The possibility of forming stable structures is related to the effective competition between the processes of merging and splitting of quarks and gluons. In nonlinear dynamics, an infinite cascade of bifurcations exponentially converges at a certain value of the control parameter. The cascade process of formation of stable hadronic structures passes into an irregular regime of dynamic chaos. Thus, at a sufficient energy density in nuclear and hadronic interactions, cascade multiple hadron processes cease and a chaotic irregular dynamics of quarks and gluons arises.

An increase in the collision energy of nuclei and hadrons corresponds to an increase in the value of the control parameter of nonlinear dynamics. Branching processes of formation of secondary hadrons are associated with a cascade of period-doubling bifurcations. The convergence of the sequence of period-doubling bifurcations at a critical value of the control parameter leads to the termination of multiple processes when all points of the phase trajectory become unstable. There is a chaotic irregular dynamics of quarks and gluons leading to thermalization of the state.

Thus, similarly to the convergence of the well-known Feigenbaum logistic map [12–14], the nonlinear quark–gluon cascade of bifurcations converges. This happens when the value of the control parameter is $\lambda = 0.892$. The formation of QGP through mixed hadron and quark phases at a value of the control parameter close to unity and further plasma hadronization are shown in figure 3. The presence of "voids" in the bifurcation diagram indicates the presence of "hadron-like structures". The figure shows a dependency graph of the equation

$$x_{i,j} = \lambda_j F\left(x_{i-1,j}\right) . \tag{5}$$

An increase of nuclei and hadrons collision energy corresponds to an increase of the nonlinear dynamics control parameter's value. Regular branching processes of secondary hadrons cascade production are associated with

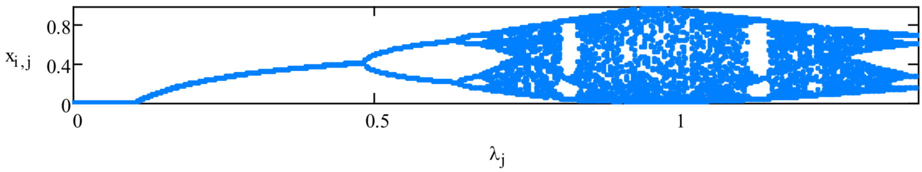

Fig. 3. Transition of partons to hadrons ($0.75 < \lambda < 0.892$), hadrons and quarks to QCP ($0.892 < \lambda < 1$) and QGP to hadrons.

a cascade of period-doubling bifurcations. It is necessary to note that particles from a vacuum are born in pairs. The convergence of the sequence of period-doubling bifurcations at a critical value of the control parameter leads to termination of multiple processes, when all points of the phase trajectory become unstable. A chaotic irregular dynamics of quarks and gluons, associated with thermalization of QGP arises. The presence of "voids" in the bifurcation diagram indicates the presence of a "hadron-like phase".

4. Conclusion and outlook

Arising in the quark–gluon cascade, the strange attractor with a fractal self-similar structure displays a new nonlinear phenomenon in the hadron physics is deterministic chaotic dynamics [15]. Dynamic quark–gluon systems are highly sensitive to the initial conditions.

Thus, the nonlinear PDF mapping represents itself the nonlinear dynamics in the phase space of a strongly correlated quark–gluon dynamical system. As a result of competing processes of creation and fusion, stable quark–gluon attractor structures are formed. As the control parameter increases, successive bifurcations of the orbits occur corresponding to the production of secondary hadrons. Bifurcation diagrams form fractal structures. The sequence of bifurcations converges exponentially at the critical point $\lambda = \lambda_\infty$. The hadrons "melt", and the nonlinear quark–gluon cascade passes from regular to chaotic dynamics. The transition of the system into dynamically determined chaos corresponds to the appearance of deconfinement QGP. Hadronization of QGP is modeled by the formation of stable quark–gluon structures.

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REFERENCES

[1] V. Gribov, L. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972).

[2] Y. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977).

[3] G. Altarelli, G. Parisi, *Nucl. Phys. B* **126**, 298 (1977).

[4] E. Kuraev, V. Fadin, *Sov. J. Nucl. Phys.* **41**, 466 (1985).

[5] L. Lipatov, *Sov. J. Nucl. Phys.* **23**, 338 (1976).

[6] L. Gribov, E. Levin, M. Ryskin, *Phys. Rep.* **100**, 1 (1983).

[7] A. Mueller, J. Qiu, *Nucl. Phys B* **268**, 427 (1986).

[8] W. Zhu, *Nucl. Phys. B* **551**, 245 (1999).

[9] M. Devee, J.K. Sarma, *Proc. Indian Natn. Sci. Acad.* **81**, 16 (2015).

[10] A. Temiraliev, I. Lebedev, in: «Proceedings of the XXIII International Baldin Seminar on High Energy Physics Problems», *Dubna*, Moscow, Russia, 19–24 September, 2016, p. 138.

[11] A. Temiraliev, I. Lebedev, A. Danlybaeva, in: «Proceedings of the 10$^{\rm th}$ International Scientific Conference on Chaos and Structures in Nonlinear Systems. Theory and Experiment», *Al-Farabi Kazakh National University, Faculty of Physics and Technology*, Almaty, Kazakhstan, 16–18 June, 2017, p. 58.

[12] M.J. Feigenbaum, *Phys. Usp.* **141**, 343 (1983).

[13] A.V. Batunin, *Phys. Usp.* **38**, 609 (1995).

[14] I.I. Kogan, D. Polyakov, *Phys. Atom. Nuclei* **66**, 2062 (2003).

[15] V. Kuvshinov, A. Kuzmin, *Phys. Part. Nucl.* **36**, 100 (2005).