# ASTROPHYSICAL $S(0)$-FACTORS FOR THE $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$, AND $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ DIRECT CAPTURE PROCESSES IN A POTENTIAL MODEL*

S.A. Turakulov, E.M. Tursunov

Institute of Nuclear Physics, 100214, Tashkent, Uzbekistan
and
National University of Uzbekistan, 100174, Tashkent, Uzbekistan

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Astrophysical $S$-factors at zero energy for the direct nuclear capture reactions $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$, and $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ are estimated within the framework of a two-body potential cluster model on the basis of extranuclear capture approximation of Baye and Brainis. The values of $S(0)$-factors have been calculated using two different potential models for each process, which were adjusted to the binding energies and empirical values of the asymptotical normalization coefficients from the literature. New values of $S(0)$-factors have been obtained.

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## 1. Introduction

Determination of the low-energy values of the astrophysical $S$-factor for the direct radiative capture reactions $d(\alpha,\gamma)^6\mathrm{Li}$, $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$, and $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, especially at $E=0$, plays an important role in nuclear astrophysics in the both Standard Solar and Big Bang nucleosynthesis (BBN) models [1, 2]. The calculation of $S(0)$ is carried out only with the help of theoretical approaches, since the direct experimental measurements of the cross section at ultralow energies are not possible due to very small values of the cross section. In particular, for the first capture reaction at energies of 10 keV, the cross section is of the order of nanobarns. As it is well-known, the $\alpha+d\rightarrow{}^6\mathrm{Li}+\gamma$ synthesis process is the main source of the $^6\mathrm{Li}$ isotope in a period of primordial nucleosynthesis. For this reason, investigating the $\alpha+d\rightarrow{}^6\mathrm{Li}+\gamma$ synthesis is of great interest within the experimental [3–7] and theoretical studies [8–10]. The direct capture processes

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$^3$He$(\alpha,\gamma)^7$Be and $^3$H$(\alpha,\gamma)^7$Li present the main source of the primordial $^7$Li element [11]. All the above-mentioned reactions are directly related to the cosmological lithium problem [12]. Moreover, the processes $^3$He$(\alpha,\gamma)^7$Be and $^7$Be$(p,\gamma)^8$B are essential for the estimation of neutrino fluxes from the Sun [1, 2]. In the present work, we extend the theoretical model previously developed in Refs. [11, 13–16] for the determination of zero energy astrophysical $S$-factor on the basis of extranuclear capture approximation as proposed in Refs. [17, 18]. The aim of the present work is to determine the values of the $S(0)$-factor for the $^3$He$(\alpha,\gamma)^7$Be, $^3$H$(\alpha,\gamma)^7$Li, and $^7$Be$(p,\gamma)^8$B direct nuclear capture reactions within the framework of two-body potential cluster model. The method can be extended to the direct capture process $d(\alpha,\gamma)^6$Li within the "exact mass prescription". However, the last case presents only the methodical interest since the isospin-forbidden E1 astrophysical $S$-factor of the $d(\alpha,\gamma)^6$Li process can be described only within the three-body model, but not in the two-body model [9].

## 2. Theoretical model

In fact, the experimental measurements and theoretical approaches define the cross sections of the process. However, in the capture process which involves light nuclei, the cross section decreases exponentially when the energy tends to zero. Therefore, in the low-energy nuclear astrophysics, the astrophysical $S$-factors are used. This quantity is expressed with the help of the cross section as [13, 19]

$$S(E)=\sum_{l_\mathrm{f}J_\mathrm{f}}S_{l_\mathrm{f}J_\mathrm{f}}(E)=E\,\exp(2\pi\eta)\sum_{l_\mathrm{f}J_\mathrm{f}}\sum_{l_\mathrm{i}J_\mathrm{i}}\sum_{\lambda}\sigma^{\Omega\lambda}_{l_\mathrm{i}J_\mathrm{i}\to l_\mathrm{f}J_\mathrm{f}}(E)\,, \tag{1}$$

where $l_\mathrm{i}$, $J_\mathrm{i}$ ($l_\mathrm{f}$, $J_\mathrm{f}$) are the orbital and the total angular momenta of the initial (final) states, respectively, $\eta$ is the Zommerfeld parameter, $\Omega=$ E or M (electric or magnetic transition), $\lambda$ is a multiplicity of the transition. For the above radiative capture reactions, the electric dipole E1 and quadrupole E2 transitions contributions are dominant in the cross section. Thus, the cross sections for the electric transitions of the radiative capture process is expressed as [17, 19]

$$\sigma^{E\lambda}_{l_\mathrm{i}J_\mathrm{i}\to l_\mathrm{f}J_\mathrm{f}}(E)=\frac{8\pi\,\mathrm{e}^2\mu}{\hbar c}\frac{k_\gamma^{2\lambda+1}}{k^3}N_{E\lambda}\left[I_\mathrm{if}(E)\right]^2\,, \tag{2}$$

where $k=\sqrt{2\mu E}/\hbar c$ is the wave number of the colliding particles relative motion, $\mu$ is the reduced mass of the clusters involved in the capture process. $k_\gamma=E_\gamma/\hbar c$ is the wave number of the photon corresponding to energy

$E_\gamma = E_{\rm th} + E$, where $E_{\rm th}$ is the threshold energy and $N_{E\lambda}$ is

$$N_{E\lambda} = \left[Z_1\left(\frac{A_2}{A}\right)^\lambda + Z_2\left(\frac{-A_1}{A}\right)^\lambda\right]^2 \frac{\lambda(\lambda+1)[\lambda][l_{\rm i}][J_{\rm i}][J_{\rm f}]}{([\lambda]!!)^2\,[S_1][S_2]} \times \left(C^{l_{\rm f}0}_{\lambda 0 l_{\rm i}0}\right)^2 \left\{\begin{array}{ccc} J_{\rm i} & l_{\rm i} & S \\ l_{\rm f} & J_{\rm f} & \lambda \end{array}\right\}^2 . \quad (3)$$

The parameters $A_1$, $A_2$ are mass numbers of the clusters in the entrance channel, $A = A_1 + A_2$, $S_1$, $S_2$ are spins of the clusters, $S$ is a spin of reaction channel. We also use short-hand notations $[S] = (2S+1)$ and $[\lambda]!! = (2\lambda+1)!!$. The overlap integral is given as

$$I_{\rm if}(E) = \int\limits_0^\infty u_E^{(l_{\rm f} S J_{\rm f})}(r) r^\lambda u^{(l_{\rm i} S J_{\rm i})}(E,r) {\rm d}r\,, \quad (4)$$

where $u_E^{(l_{\rm f} S J_{\rm f})}(r)$ and $u^{(l_{\rm i} S J_{\rm i})}(E,r)$ are final bound and initial scattering wave functions, respectively.

At the next step, we determine the zero energy astrophysical $S(0)$-factor. In order to distinguish energy-dependent parts of the astrophysical $S$-factor, we introduce modified scattering functions [17]

$$\tilde{u}^{(l_{\rm i} S J_{\rm i})}(E,r) = E^{1/2}\exp(\pi\eta) u^{(l_{\rm i} S J_{\rm i})}(E,r)\,, \quad (5)$$

where $E^{1/2} = \frac{k\,\hbar c}{\sqrt{2\mu}}$ is the kinetic energy of the relative motion. When $E$ tends to zero, consequently $\eta$ also tends to infinity, and the regular $F_l$ and irregular $G_l$ Coulomb wave functions become unusable. Therefore, the radial scattering wave function is normalized with the help of the rescaled Coulomb functions $\mathcal{F}_l$ and $\mathcal{G}_l$ [18] as

$$\tilde{u}^{(l_{\rm i} S J_{\rm i})}(E,r) \underset{r\to\infty}{\to} \cos\delta_{(l_{\rm i} S J_{\rm i})}(E)\mathcal{F}_l(E,r) + \frac{2}{\pi}\exp(2\pi\eta)\sin\delta_{(l_{\rm i} S J_{\rm i})}(E)\mathcal{G}_l(E,r)\,, \quad (6)$$

where, $\delta_{l_{\rm i} S J_{\rm i}}(E)$ is the phase shift in the $(l,S,J)^{\rm th}$ partial wave. This normalization provides that $\tilde{u}^{(l_{\rm i} S J_{\rm i})}(E,r)$ has a finite limit when $E$ tends to zero [17, 18]. It will be convenient to make use of a function of the phase shift $\delta_{l_{\rm i} S J_{\rm i}}(E)$ defined as

$$\mathcal{D}_{l_{\rm i} S J_{\rm i}}(E) = \frac{2}{\pi}\left[\exp(2\pi\eta) - 1\right]\tan\delta_{l_{\rm i} S J_{\rm i}}(E) \quad (7)$$

which also has a finite limit when $E \to 0$ [18]. Taking into account that, at the ultralow energy, the phase shift $\delta_{l_{\rm i} S J_{\rm i}}(E)$ is very small and satisfies the condition $\exp(-2\pi\eta) \ll 1$, the asymptotic form (6) of the radial wave function becomes

$$\tilde{u}^{(l_\mathrm{i} S J_\mathrm{i})}(E,r) \underset{r\to\infty}{\to} \mathcal{F}_l(E,r) + \mathcal{D}_{l_\mathrm{i} S J_\mathrm{i}}(E)\mathcal{G}_l(E,r)\,, \tag{8}$$

which remains finite at $E = 0$.

Finally, we can rewrite the expression for the zero energy astrophysical $S(0)$-factor in the form of [17, 18]

$$S(0) = \frac{1}{2}\alpha\hbar c \left(\frac{E_\mathrm{th}}{\hbar c}\right)^{2\lambda+1} N_{E\lambda}\,\left[I_\mathrm{if}(0)\right]^2\,, \tag{9}$$

where $\alpha$ is the fine-structure constant. The zero energy overlap integral is given as

$$I_\mathrm{if}(0) = \int\limits_0^\infty u_E^{(l_\mathrm{f} S J_\mathrm{f})}(r) r^\lambda \tilde{u}^{(l_\mathrm{i} S J_\mathrm{i})}(0,r)\mathrm{d}r\,, \tag{10}$$

where

$$\tilde{u}^{(l_\mathrm{i} S J_\mathrm{i})}(0,r) \underset{r\to\infty}{\to} \mathcal{F}_l^0(r) + \mathcal{D}_{l_\mathrm{i} S J_\mathrm{i}}(0)\mathcal{G}_l^0(r)\,. \tag{11}$$

Using the above equations, one is able to estimate the astrophysical S(0)-factor of the above capture processes within the two-body cluster model.

## 3. Results and discussion

Calculations of the cross section and astrophysical $S(0)$-factor have been performed under the same conditions as in Refs. [11, 13–16]. The Schrödinger equation in the entrance and exit channels is solved with the two-body central nuclear potentials of the Gaussian form [8] with the corresponding point-like Coulomb potential for the $\alpha + d$ and $p + {}^7\mathrm{Be}$ systems as in [8, 20]. For synthesis of ${}^3\mathrm{He} + \alpha$ and ${}^3\mathrm{H} + \alpha$, the spherical form of Coulomb potential have been used. For consistency, we use the same model parameters as in the aforementioned paper [13]: $\hbar^2/2\,[\mathrm{a.m.u.}] = 20.7343$ MeV fm$^2$. The Coulomb parameter $R_\mathrm{C} = 3.095$ fm for the spherical form of potential [11]. The mass number corresponding to the first $A_1$ particle is $m_d = 2.0$ a.m.u., $m_{{}^3\mathrm{He}} = m_{{}^3\mathrm{H}} = 3.0$ a.m.u., $m_p = 1.007\ 276\ 4669$ a.m.u., and the mass number $A_2$ of the second particle is $m_\alpha = 4.0$ a.m.u., $m_{{}^7\mathrm{Be}} = 7.014735$ a.m.u., respectively.

It should be noted that for the calculations of the $\alpha + d$ capture reaction, we use the "exact mass" prescription in the two-body model. As was noted in Introduction, this case is of methodical interest, since realistic estimates of the isospin-forbidden E1 $S$-factor can be obtained only within the three-body model [9, 11, 14, 15]. Thus, within the assumption of the "exact mass" prescription, the exact experimental mass values $m_d = A_1 = 2.013553212724$ a.m.u. and $m_\alpha = A_2 = 4.001506179127$ a.m.u. [8] are used.

The obtained $S(0)$-factor values for the above capture processes are strongly dependent on the value of the asymptotical normalization coefficient (ANC). The values of ANC of the $\alpha + d \to {}^6$Li virtual transition and astrophysical $S(0)$-factors of the direct $d(\alpha,\gamma)^6$Li capture process are presented in Table 1 for the two potential models $V_\mathrm{D}$ [8] and $V_\mathrm{M}$ [13]. The initial potential $V_\mathrm{D}$ yields a value of $C_{\alpha d} = 2.53$ fm$^{-1/2}$ for the ANC. The modified potential $V_\mathrm{M}$ yields $C_{\alpha d} = 2.31$ fm$^{-1/2}$, which is more consistent with the empirical value of ANC, $C_{\alpha d} = 2.32 \pm 0.11$ fm$^{-1/2}$ extracted from the experimental data in Ref. [21].

Table 1. The values of ANC for the $\alpha+d \to {}^6$Li virtual transition and corresponding astrophysical $S(0)$-factors of the direct $d(\alpha,\gamma)^6$Li capture reaction.

| Model | $C_{\alpha d}$ [fm$^{-1/2}$] | $S(0)$ [MeV nb] |
|---|---|---|
| $V_\mathrm{D}$ | 2.53 | 1.53 |
| $V_\mathrm{M}$ | 2.31 | 1.26 |

In Table 2, the values of ANC for the ground $p_{3/2}$ and the first excited $p_{1/2}$ bound states of the $^7$Be(= $^3$He + $\alpha$) and $^7$Li(= $^3$H + $\alpha$) nuclei and calculated results for corresponding astrophysical $S(0)$-factors are given within two potential models $V_\mathrm{D}^n$ and $V_\mathrm{M1}^n$ [11, 15]. The parameters of these potential models are given in Ref. [15].

Table 2. Values of ANC for the ground $p_{3/2}$ and first excited $p_{1/2}$ bound states of the $^7$Be and $^7$Li nuclei and corresponding astrophysical $S(0)$-factors.

| Reaction | Model | $C_{p_{3/2}}$ [fm$^{-1/2}$] | $C_{p_{1/2}}$ [fm$^{-1/2}$] | $S(0)$ [keV b] |
|---|---|---|---|---|
| $^3$He$(\alpha,\gamma)^7$Be | $V_\mathrm{D}^n$ | 4.34 | 3.71 | 0.56 |
| | $V_\mathrm{M1}^n$ | 4.79 | 4.24 | 0.58 |
| $^3$H$(\alpha,\gamma)^7$Li | $V_\mathrm{D}^n$ | 3.72 | 3.12 | 0.10 |
| | $V_\mathrm{M1}^n$ | 4.10 | 3.55 | 0.09 |

These models differ from each other only with values of ANC for the bound states of $^7$Be and $^7$Li nuclei. They have been adjusted to the values of ANC extracted from the analysis of the low-energy experimental astrophysical $S$-factors of the $^3$He$(\alpha,\gamma)^7$Be and $^3$H$(\alpha,\gamma)^7$Li reactions in Refs. [22, 23]. The obtained theoretical estimations of the zero energy $S(0)$-factor for the $^3$He$(\alpha,\gamma)^7$Be process with the potentials $V_{\rm D}^n$ and $V_{\rm M1}^n$ are very consistent with the new data $S(0) = 0.56 \pm 0.04$ keV b of the Solar Fusion II Collaboration [1]. In addition, obtained results for the $^3$H$(\alpha,\gamma)^7$Li capture process in the frame of the proposed couple of potential models are in good agreement with the result $S(0) = 0.10 \pm 0.02$ keV b of the NACRE Collaboration [19].

In Table 3, the values of ANC for the bound state of the $^8$B$(= p + {}^7$Be$)$ nucleus and calculated results for corresponding astrophysical $S(0)$-factors are given for two potential models $V_{\rm D}$ and $V_{\rm M}$ from Ref. [16], where the full set of parameters of these potential models is given. As can be seen from the table, the obtained theoretical estimates for the zero energy $S(0)$-factor of the $^7$Be$(p,\gamma)^8$B process are consistent with the new data $S(0) = 20.8 \pm 2.10$ eV b of the Solar Fusion II Collaboration [1].

Table 3. The values of ANC for the $p + {}^7\mathrm{Be} \to {}^8\mathrm{B}$ virtual transition and corresponding astrophysical $S(0)$-factors of the direct $^7$Be$(p,\gamma)^8$B capture reaction.

| Model | $C_{\alpha d}$ [fm$^{-1/2}$] | $S(0)$ [eV b] |
|---|---|---|
| $V_{\rm D}$ | 0.70 | 18.32 |
| $V_{\rm M}$ | 0.73 | 19.61 |

## 4. Conclusions

In the present work, the zero energy astrophysical $S$-factors for the $^3$He$(\alpha,\gamma)^7$Be, $^3$H$(\alpha,\gamma)^7$Li, and $^7$Be$(p,\gamma)^8$B radiative capture reactions have been estimated in the framework of the two-body potential cluster model on the basis of extranuclear capture approximation [17, 18]. For the astrophysical $S(0)$-factors of the $^3$He$(\alpha,\gamma)^7$Be, $^3$H$(\alpha,\gamma)^7$Li, and $^7$Be$(p,\gamma)^8$B capture reactions, the obtained estimates are consistent with new data sets of the Solar Fusion II and NACRE collaborations.

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## REFERENCES

[1] E.G. Adelberger *et al.*, *Rev. Mod. Phys.* **83**, 195 (2011).

[2] B.D. Fields, *Annu. Rev. Nucl. Part. Sci.* **61**, 47 (2011).

[3] R.G.H. Robertson *et al.*, *Phys. Rev. Lett.* **47**, 1867 (1981); *Erratum ibid.* **75**, 4334 (1995).

[4] J. Kiener *et al.*, *Phys. Rev. C* **44**, 2195 (1991).

[5] P. Mohr *et al.*, *Phys. Rev. C* **50**, 15 (1994).

[6] LUNA Collaboration (M. Anders *et al.*), *Phys. Rev. Lett.* **113**, 042501 (2014).

[7] LUNA Collaboration (D. Trezzi *et al.*), *Astropart. Phys.* **89**, 57 (2017).

[8] S.B. Dubovichenko, *Phys. Atom. Nuclei* **73**, 1526 (2010).

[9] D. Baye, E.M. Tursunov, *J. Phys. G: Nucl. Part. Phys.* **45**, 085102 (2018).

[10] A.S. Solovyev, *Phys. Rev. C* **106**, 014610 (2022).

[11] E.M. Tursunov, S.A. Turakulov, A.S. Kadyrov, *Phys. Rev. C* **97**, 035802 (2018).

[12] M. Asplund *et al.*, *Astrophys. J.* **644**, 229 (2006).

[13] E.M. Tursunov, S.A. Turakulov, P. Descouvemont, *Phys. Atom. Nuclei* **78**, 193 (2015).

[14] E.M. Tursunov, S.A. Turakulov, A.S. Kadyrov, *Nucl. Phys. A* **1000**, 121884 (2020).

[15] E.M. Tursunov, S.A. Turakulov, A.S. Kadyrov, *Nucl. Phys. A* **1006**, 122108 (2021).

[16] E.M. Tursunov, S.A. Turakulov, A.S. Kadyrov, L.D. Blokhintsev, *Phys. Rev. C* **104**, 045806 (2021).

[17] D. Baye, P. Descouvemont, M. Hesse, *Phys. Rev. C* **58**, 545 (1998).

[18] D. Baye, E. Brainis, *Phys. Rev. C* **61**, 025801 (2000).

[19] C. Angulo *et al.*, *Nucl. Phys. A* **656**, 3 (1999).

[20] S.B. Dubovichenko, N.A. Burkova, A.V. Dzhazairov-Kakhramanov, A.S. Tkachenko, *Nucl. Phys. A* **983**, 175 (2019).

[21] L.D. Blokhintsev *et al.*, *Phys. Rev. C* **48**, 2390 (1993).

[22] Q.I. Tursunmahatov, R. Yarmukhamedov, *Phys. Rev. C* **85**, 045807 (2012).

[23] S.B. Igamov, R. Yarmukhamedov, *Nucl. Phys. A* **781**, 247 (2007).