ASYMPTOTIC NORMALIZATION COEFFICIENTS OF $^{28}\mathrm{Si} \to {}^{27}\mathrm{Al} + p$ CONFIGURATIONS FROM THE PROTON TRANSFER REACTION*

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The differential cross section of the proton transfer in the $^{27}\mathrm{Al}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{28}\mathrm{Si}$ reaction at an energy of 41.3 MeV has been measured and analysed within the modified DWBA method. For the $^{28}\mathrm{Si}(^{28}\mathrm{Si}^*) \to {}^{27}\mathrm{Al}+p$ configurations, the new values of the squares ANCs $C^2_{^{28}\mathrm{Si}} = 1630 \pm 300\ \mathrm{fm}^{-1}$ and $C^2_{^{28}\mathrm{Si}^*} = 1590 \pm 190\ \mathrm{fm}^{-1}$, respectively, have been obtained.

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1. Introduction

Radiative capture reactions such as (p,γ) play an important role in the formation of our Universe. Exact knowledge of their cross sections (astrophysical S-factors) and reaction rates is necessary for modeling nucleosynthesis in hydrogen-burning stars. At the same time, the difficulties of measuring very small cross sections for these reactions at stellar energies are well known due to the Coulomb repulsion. Therefore, indirect methods of their determination play an important role.

To estimate the contribution of the direct mechanism to the astrophysical S-factor of radiative capture, the values of asymptotic normalization coefficients (ANCs) for bound states of a captured particle in a finite nucleus are successfully used. They can be extracted from the analysis in the framework of the modified distorted wave Born approximation (MDWBA) [1, 2] of differential cross sections (DCs) of the direct $A(x,y)B$ peripheral proton transfer reactions, where $x = y + p$ and $B = A + p$. In particular, knowledge of the ANC values of $^{28}\mathrm{Si} \to {}^{27}\mathrm{Al} + p$ is necessary to estimate the direct proton capture by the ^{27}Al nuclei in the astrophysically significant reaction $^{27}\mathrm{Al}(p,\gamma)^{28}\mathrm{Si}$. This reaction is a competing process for the MgAl hydrogen burning in the model of massive main sequence stars [3]

$$^{27}\mathrm{Al}(p,\alpha)^{24}\mathrm{Mg}(p,\gamma)^{25}\mathrm{Al}\left(\beta^+\right){}^{25}\mathrm{Mg}(p,\gamma)^{26}\mathrm{Al}(p,\gamma)^{27}\mathrm{Si}\left(\beta^+\right){}^{27}\mathrm{Al}\,.$$

The purpose of this work is to find out the suitability of the proton transfer $^{27}\mathrm{Al}(^{10}\mathrm{B},{}^{9}\mathrm{Be})^{28}\mathrm{Si}_{\mathrm{gs}}$ and $^{27}\mathrm{Al}(^{10}\mathrm{B},{}^{9}\mathrm{Be})^{28}\mathrm{Si}^*$ reactions near the Coulomb barrier to correctly determine the ANC values of the $^{28}\mathrm{Si}_{\mathrm{gs}}$ and $^{28}\mathrm{Si}^*$ from the analysis of the corresponding DC. For this, the DCs of the above reactions and elastic scattering of $^{27}\mathrm{Al} + {}^{9}\mathrm{Be}$ were measured, and the peripherality and dominance of the simple proton stripping mechanism were verified.

2. Experiment

Measurements of the DCs of the $^{27}\mathrm{Al}(^{10}\mathrm{B},{}^{9}\mathrm{Be})^{28}\mathrm{Si}$ reaction and elastic $^{27}\mathrm{Al} + {}^{9}\mathrm{Be}$ scattering were carried out on a ^{10}B ion beam of the U-200P heavy ion accelerator (Heavy Ion Laboratory, University of Warsaw) at an energy of $E = 41.3$ MeV. A self-supporting $\mathrm{Al_2O_3}$ film with a thickness of 0.15 ± 0.01 mg/cm^2 was used as a target. Charged particles — the reaction products — were registered and identified by four ΔE–E telescopes, which were installed in the experimental chamber of the multi-detector facility ICARE, which includes systems of remotely controlled platforms with telescopes for detectors and a target device [4, 5]. Ionization chambers (ΔE) and semiconductor Si (E) detectors were used. More details of the experimental setup and details of the experiment are described in [6, 7].

Figure 1 shows the energy spectra of the detected ^{10}B and ^{9}Be nuclei. The energy resolution in the spectra is 400–500 keV, and the groups of detected ^{10}B nuclei, corresponding to elastic scattering by ^{16}O and ^{27}Al, are kinematically separated only for the angles $\theta_{\rm lab} > 22°$. For smaller angles, the areas of the corresponding peaks in the overlapping peak were divided into the proportion corresponding to the DCs calculated with the global optical potentials (OP) for $^{10}{\rm B} + {}^{27}{\rm Al}$ [8, 9] and $^{10}{\rm B} + {}^{16}{\rm O}$ [7, 10] taking into account the ratio of the content of ^{27}Al and ^{16}O nuclei in the Al_2O_3 target (see Fig. 2). The experimental DCs of the elastic scattering thus obtained are in good agreement with these extracted directly from the spectra in the angular region, $\theta_{\rm lab} > 22°$, as well as with the DCs obtained in [8] at a close beam energy of ^{10}B ($E = 41.6$ MeV).

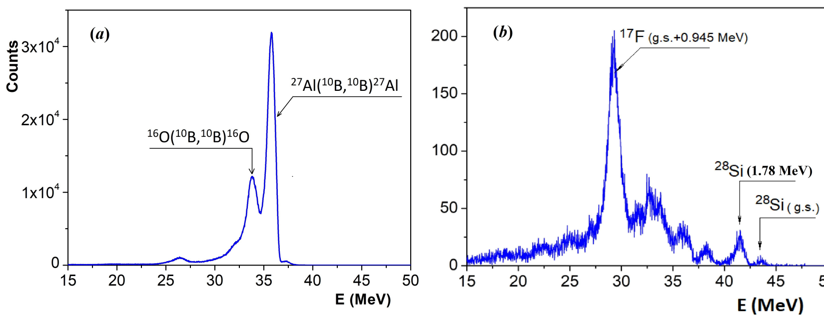

Fig. 1. Fragments of the energy spectra of scattered ^{10}B nuclei (a) and ^{9}Be nuclei from the (^{10}B, ^{9}Be) reaction (b), measured at an angle of $\theta_{\rm lab} = 22°$.

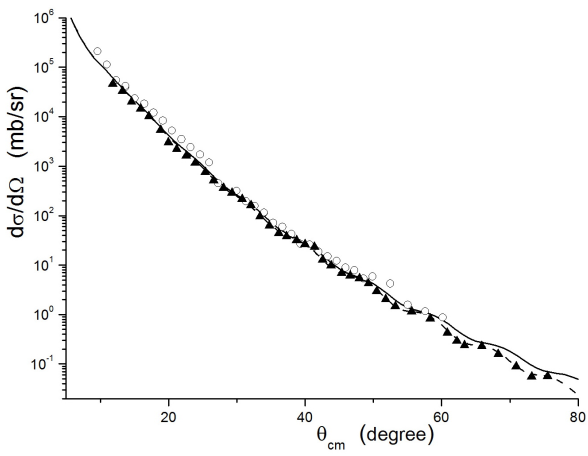

Fig. 2. Angular distribution of the $^{27}{\rm Al} + {}^{10}{\rm B}$ elastic scattering. The open circles are our experimental DCs, triangles are the data from [8], and the solid and dashed lines are optical model calculations using the OP from Refs. [8, 9].

A good agreement with the data of the above works confirms the correctness of the absolute normalization of the DC obtained in our experiment, the error of which, according to our estimates, did not exceed 10%.

In the spectra of the detected ^{9}Be nuclei from the ^{27}Al(^{10}B, ^{9}Be)^{28}Si reaction, the groups corresponding to the ground (0^+) and the first excited ($E^* = 1.778$ MeV, 2^+) states of the ^{28}Si nucleus are well distinguished (see Fig. 1 (b)). The experimental values of the cross sections were obtained in the range of angles 9.3–58.4 with absolute errors of 10–30%.

3. MDWBA analysis and obtaining the ANC

The measured DCs for the ^{27}Al(^{10}B, ^{9}Be)^{28}Si reaction were analyzed using the MDWBA in which, the DC $\mathrm{d}\sigma/\mathrm{d}\Omega$ for the peripheral proton transfer $A(x,\, y)B$ ($x = y + p$ and $B = A + p$) can be written in the form of [6]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = C^2_{Ap} \times C^2_{yp} \times R(E_i, \theta;\, b_{yp},\, b_{Ap})\,, \tag{1}$$

$$R(E_i, \theta;\, b_{yp},\, b_{Ap}) = \frac{\sigma^{\mathrm{DWBA}}_{Ap}(E_i, \theta;\, b_{yp},\, b_{Ap})}{b^2_{yp} \times b^2_{Ap}}\,. \tag{2}$$

Here, C_{Ap} and C_{yp} are the ANCs for $A + p \to B$ and $y + p \to x$, which determine the amplitudes of the tails of the radial B and x nuclei wave functions in the $(A{+}p)$ and $(y{+}p)$ channels [11]. $\sigma^{\mathrm{DWBA}}_{Ap}$ is the single-particle DWBA cross section [12]; the values b_{Ap} and b_{yp} are the single-particle ANCs for the shell-model wave functions of the two-body $B = (A + p)$ and $x = (y + p)$ bound states, which determine the amplitudes of their tails; E_i is the relative kinetic energy of the colliding particles and θ is the center-of-mass scattering angle. In our case, $A = {}^{27}$Al; $x = {}^{10}$B and $y = {}^{9}$Be. The function $R(\ldots)$ is separated from the structure of the DC formula to estimate the degree of peripherality of the particle (which is a proton in our case) transfer and to evaluate the related uncertainties of the theoretical approach in the extracted ANC. Its value must remain constant when the geometric parameters of the corresponding proton binding potentials change, *i.e.* when changing the values of b.

Assuming a simple proton stripping mechanism that occurred at the periphery of the interacting nuclei A and x, one can determine the value of the squared ANC $C^2_{B\to A+p}$ by normalizing the calculated cross section to the experimental one, which is sometimes called the “indirectly determined” ANC. Usually, the stripping mechanism dominates in the region of the main diffraction maximum of the angular distribution of the detected particle y, where it is reasonable to perform the normalization.

Selection of sets of the OP parameters for the entrance and exit channels of the reaction was performed using the literature data, including the recommended OP given in the papers with the corresponding experimental data. OPs selected according to the best description of the angular distributions of the reaction and describing elastic scattering are presented in Table 1.

Table 1. OP parameters (see references in the table) used in MDWBA calculations and obtained ANC values. V and W in MeV, r and a in fm. Symbols "V" and "D" in the sixth column mean the volume and surface form of W.

Set	Channel	V_V	r_V	a_V	W	r_W	a_W	R_C	$C^2_{^{28}\mathrm{Si}}$[fm^{-1}]	$C^2_{^{28}\mathrm{Si}*}$[fm^{-1}]	Ref.
1	$^{10}\mathrm{B} + {}^{27}\mathrm{Al}$	38.7	1.804	0.850	99.0 V	1.804	0.620	2.302			[8]
	$^{9}\mathrm{Be} + {}^{28}\mathrm{Si}$	10.0	2.268	0.643	23.4 V	2.032	0.746	2.275	1890 ± 350	1790 ± 210	[13]
2	$^{10}\mathrm{B} + {}^{27}\mathrm{Al}$	52.99	1.787	0.825	9.688 V	2.369	0.601	2.234			[9]
	$^{9}\mathrm{Be} + {}^{28}\mathrm{Si}$	259.7	1.282	0.726	16.24 V	1.639	0.600	1.555			[14]
					11.55 D	1.200	0.843	1.555	1370 ± 260	1400 ± 170	
	Averaged mean								1630 ± 300	1590 ± 190	

For the applicability of the MDWBA analysis to the experimental cross sections of the reaction, the behavior of the test function $R\,(b_{^{28}\mathrm{Si}})$ (2) in the region of the main maximum of the angular distribution of the DC was preliminarily considered. To do this, the geometric parameters r_0 and a of the Woods–Saxon potential (W–S) of the bound state of the transferred proton were varied within physically reasonable limits relative to the "standard" values of 1.25 fm and 0.65 fm, each time adjusting the potential depths to the experimental proton binding energies (well-depth procedure). It follows from the calculations that the spread of test function values remains within $\sim 4\%$ and 6% during the transfer of a proton to the ground state and the state $E^* = 1.78$ MeV of the ^{28}Si nucleus, respectively, which is less than the errors of the experimental DCs.

Thus, the proton transfer in both reaction channels can be considered peripheral, and in accordance with MDWBA, the DCs of each of the channels in the region of small angles should be normalized to the product of the squares of the corresponding ANC $C^2_{^{10}\mathrm{B}} \times C^2_{^{28}\mathrm{Si}}$ or $C^2_{^{10}\mathrm{B}} \times C^2_{^{28}\mathrm{Si}*}$. The value of ANC $C^2_{^{10}\mathrm{B}}$ required to extract experimental DCs from the analysis using formula (1) was taken equal to $C^2_{^{10}\mathrm{B}} = 4.35 \pm 0.39$ fm^{-1} from [1]. The values of the orbital and total angular momenta of the transferred proton in the ground and $E^* = 1.78$ MeV states of the ^{28}Si nucleus were taken equal to $\ell = 2$, $j = 5/2$ and 0, 1/2 (dominant), respectively.

Experimental DCs of the proton transfer to the ground (0^+) and the first excited ($E^* = 1.779$ MeV, 2^+) states of the ^{28}Si nucleus, as well as the angular distributions calculated within the framework of MDWBA with sets of OPs that give the best fit to the data, are shown in Fig. 3. It can be seen that the experimental data are well described in the region of the forward angles, including the first two diffraction maxima of the angular distributions.

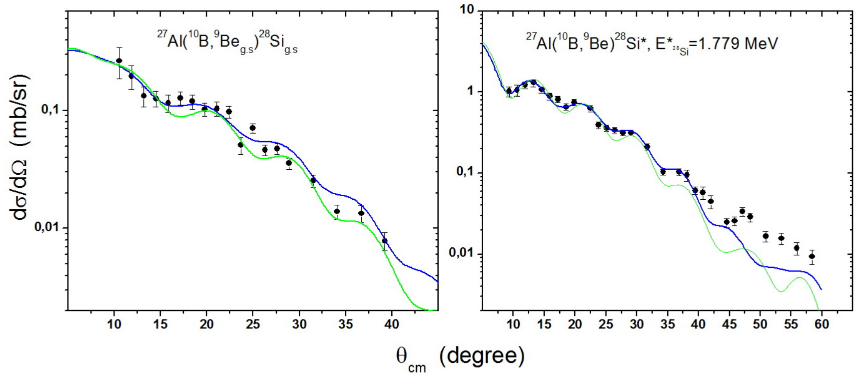

Fig. 3. (Color online) The experimental (points) and calculated angular distributions (solid lines) of ^{9}Be from the ^{27}Al(^{10}B, ^{9}Be)^{28}Si reaction with proton transfer to the $1d_{5/2}$ (left) and $2s_{1/2}$ states of ^{28}Si nuclei. Blue/black and green/gray curves are the MDWBA calculations with OP sets 1 and 2, respectively (see Table 1).

By normalizing the curves calculated for each set of the OPs to the experimental values in the region of the forward angles, the values of the ANC squares for the ground and first excited states were extracted and averaged over the results for the first 6 angles of measurement (see Table 1). The values of the ANC squares, averaged over the OP sets are given in the bottom line of the table. The presented errors include experimental errors, as well as uncertainties associated with the ambiguity of the OP parameters and the residual dependence of the function $R(b)$ (see Eq. (2)) on the uncertainty of the geometric parameters of the W–S form of the bound state potential of the transferred proton in the ^{28}Si nucleus.

We also performed the MDWBA analysis of the experimental DCs of the ^{28}Si$(n,d)^{27}$Al reaction at 6 MeV [15] and 10 MeV [16], as well as the ^{27}Al(^{3}He$,d)^{28}$Si at 25 MeV [17] and 37 MeV [18] at a proton transfer to the ground state of ^{28}Si. At that the ANCs, $C_d^2 \to n+p = 0.775$ fm^{-1} [11] and $C^2_{^3\mathrm{He}} \to d+p = 4.32$ fm^{-1} [19] were used. The OP parameters were taken from the same works, except for the exit channel ^{28}Si$\,+\,n$, where the OP was used from [20]. The values 880 fm^{-1}, 2286 fm^{-1}, and 1060 fm^{-1}, 1390 fm^{-1} of $C^2_{^{28}\mathrm{Si}}$ were obtained from the analysis of the (d,n) and (^{3}He$,d)$ reactions, respectively. Such a large discrepancy between the values and their deviations from the value found from the reactions (^{10}B, ^{9}Be) is possible due to the significant non-peripherality of these reactions, since the values of the test functions R have a spread within 20–40%. Surprisingly, the average value of these squared ANC is ~ 1400 fm^{-1}, which is rather close to the value of 1630 ± 300 fm^{-1} found from the peripheral reaction (^{10}B, ^{9}Be).

4. Conclusions

The DCs of the $^{27}\mathrm{Al} + {}^{10}\mathrm{B}$ elastic scattering and the ^{27}Al(^{10}B, ^{9}Be)^{28}Si reaction have been measured at $E_{^{10}\mathrm{B}}$ =41.3 MeV at the U-200P cyclotron (HIL, University of Warsaw). The values for the squared ANCs $C^2_{^{28}\mathrm{Si}} = 1630 \pm 300$ fm^{-1} and $C^2_{^{28}\mathrm{Si}^*} = 1590 \pm 190$ fm^{-1} were obtained from the MDWBA analysis of these data.

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